CHAPTER – 3

ANALYSIS OF CREEP IN AN ISOTROPIC UNIFORM COMPOSITE CYLINDER

3.1. GENERAL

In applications such as pressure vessel for industrial gases or a media transportation of high-pressurized fluids and piping of nuclear reactors, the cylinder has to operate under severe mechanical and thermal loads, causing significant creep hence reduced service life (Gupta and Pathak, 2001; Tachibana and Iyoku, 2004; Hagihara and Miyazaki, 2008). As an example, in the high temperature engineering test reactor, the temperature reaches of the order of 900°C (Tachibana and Iyoku, 2004). The piping of reactor cooling system are subjected to high temperature and pressure and may be damaged due to high heat generated from the reactor core (Hagihara and Miyazaki, 2008).

A number of studies pertaining to creep behaviour of the cylinder assume the cylinder to be made of monolithic material. However, under severe thermo-mechanical loads cylinder made of monolithic materials may not perform well. The weight reduction achieved in engineering components, resulting from the use of aluminum/aluminum base alloys, is expected to save power and fuel due to a reduction in the payload of dynamic systems. However, the enhanced creep of aluminum and its alloys may be a big hindrance in such applications. Aluminum matrix composites offer a unique combination of properties, unlike many monolithic materials like metals and alloys, which can be tailored by modifying
the content of reinforcement. Experimental studies on creep under uniaxial loading have demonstrated that steady state creep rate is reduced by several orders of magnitude in aluminum or its alloys reinforced with ceramic particles/whiskers like silicon carbide as compared to pure aluminum or its alloys (Nieh, 1984; Nieh et al, 1988). A significant improvement in specific strength and stiffness may also be attained in composites based on aluminum and aluminum alloys containing silicon carbide particles or whiskers. In addition, a suitable choice of variables such as reinforcement geometry, size and content of reinforcement in these composites can be used to make the cost-effective components with improved performance.

With these forethoughts, it is decided to investigate the steady state creep in a cylinder made of Al-SiCp composite and subjected to high pressure and high temperature. A mathematical model has been developed to describe the steady state creep behaviour of the composite cylinder. The developed model is used to investigate the effect of material parameters viz particle size and particle content, and operating temperature on the steady state creep response of the composite cylinder.

### 3.2 SELECTION OF CREEP LAW

In aluminum based composites, undergoing steady state creep, the effective creep rate ($\dot{\varepsilon}_e$) is related to the effective stress ($\sigma_e$) through the well documented threshold stress ($\sigma_o$) based creep law given by (Mishra and Pandey, 1990; Park et al, 1990; Mohamed et al, 1992; Pandey and Mishra, 1992; Gonzalez and Sherby, 1993; Pandey et al, 1994; Park and Mohamed, 1995; Cadek et al,
1995; Li and Mohamed, 1997; Li and Langdon, 1997, 1999; Yoshioka et al., 1998; Tjong and Ma, 2000; Ma and Tjong, 2001).

\[ \dot{\varepsilon}_e = A \left( \frac{\sigma_e - \sigma_0}{E} \right)^n \exp \left( \frac{-Q}{RT} \right) \]  

where the symbols \( A' \), \( n \), \( Q \), \( E \), \( R \) and \( T \) denote respectively the structure dependent parameter, true stress exponent, true activation energy, temperature-dependent Young’s modulus, gas constant and operating temperature.

The values of true stress exponent \( (n) \) appearing in Eqn. (3.1) is usually selected as 3, 5 and 8, which correspond to three well-documented creep cases for metals and alloys: (i) \( n = 3 \) for creep controlled by viscous glide processes of dislocation, (ii) \( n = 5 \) for creep controlled by high temperature dislocation climb (lattice diffusion), and (iii) \( n = 8 \) for lattice diffusion-controlled creep with a constant structure (Tjong and Ma, 2000). Though, some of the research groups (Mishra and Pandey, 1990; Pandey and Mishra, 1992; Gonzalez and Sherby, 1993; Pandey et al., 1994) have used a true stress exponent of 8 to describe the steady state creep in Al-SiCp,w (subscript ‘p’ for particle and ‘w’ for whisker) composites but a number of other research groups (Park et al., 1990; Mohamed et al., 1992; Park and Mohamed, 1995; Cadek et al., 1995; Yoshioka et al., 1998; Li and Mohamed, 1997; Li and Langdon, 1997, 1999) have observed that a stress exponent of either ~3 or ~5, rather than 8, is a better choice to describe steady state creep data of discontinuously reinforced Al-SiC composites. Keeping this in view, a stress exponent of 5 is used to describe the steady state creep behaviour of Al-SiCp composite cylinder in this study.
3.3 ESTIMATION OF CREEP PARAMETERS

The creep law given by Eqn. (3.1) may alternatively be expressed as,

\[ \dot{\varepsilon}_e = [M(\sigma_e - \sigma_o)]^n \]  

(3.2)

where \( M = \frac{1}{E} \left( A \exp \left( -\frac{Q}{RT} \right) \right)^{1/n} \) and the stress exponent \( n = 5 \).

The creep parameters \( M \) and \( \sigma_o \) given in Eqn. (3.2) are dependent on the type of material and are also affected by the temperature \( (T) \) of application. In a composite, the dispersoid size \( (P) \) and the content of dispersoid \( (V) \) are the primary material variables affecting these parameters. In the present study, the values of \( M \) and \( \sigma_o \) have been extracted from the uniaxial creep results reported for Al-SiCp by Pandey et al (1992). Though, Pandey et al (1992) used a stress exponent of 8 to describe steady state creep in these composites. But due to the objections raised by several researchers (Park et al, 1990; Mohamed et al, 1992; Park and Mohamed, 1995; Cadek et al, 1995; Li and Mohamed, 1997; Yoshioka et al, 1998; Li and Langdon, 1997, 1999), we have chosen a stress exponent of 5 to describe steady state creep in Al-SiCp composite.

In order to extract the values of creep parameters for Al-SiCp, the individual set of creep data reported by Pandey et al (1992) have been plotted as \( \dot{\varepsilon}^{1/5} \) versus \( \sigma \) on linear scales as shown in Figs. 3.1(a)-(c). From the slope and intercepts of these graphs, the values of creep parameters \( M \) and \( \sigma_o \) have been obtained and are reported in Table 3.1. This approach of determining the threshold stress \( (\sigma_o) \) is known as linear extrapolation technique (Lagneborg and Bergman,
To avoid variation due to systematic error, if any, in the experimental results, the creep results from a single source have been used.

The $\dot{\epsilon}^{1/5}$ versus $\sigma$ plots corresponding to the observed experimental data points of Al-SiCp composites (Pandey et al., 1992), for various combinations of particle size, particle content and operating temperature, exhibit an excellent linearity as evident from Figs. 3.1(a)-(c). The coefficient of correlation for these plots has been reported in excess to 0.916 as given in Table 3.1. In the light of these results, the choice of stress exponent $n = 5$, to describe the steady state creep behaviour of Al-SiCp composite, is justified.

### Table 3.1: Creep parameters for Al-SiCp composites (Pandey et al., 1992)

<table>
<thead>
<tr>
<th>$P$ ($\mu$m)</th>
<th>$T$ (°C)</th>
<th>$V$ (vol %)</th>
<th>$M$ ($s^{1/5}$/MPa)</th>
<th>$\sigma_o$ (MPa)</th>
<th>Coefficient of correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7</td>
<td>350</td>
<td>10</td>
<td>$4.35 \times 10^{-3}$</td>
<td>19.83</td>
<td>0.945</td>
</tr>
<tr>
<td>14.5</td>
<td>350</td>
<td>10</td>
<td>$8.72 \times 10^{-3}$</td>
<td>16.50</td>
<td>0.999</td>
</tr>
<tr>
<td>45.9</td>
<td>350</td>
<td>10</td>
<td>$9.39 \times 10^{-3}$</td>
<td>16.29</td>
<td>0.998</td>
</tr>
<tr>
<td>1.7</td>
<td>350</td>
<td>20</td>
<td>$4.35 \times 10^{-3}$</td>
<td>19.83</td>
<td>0.945</td>
</tr>
<tr>
<td>1.7</td>
<td>350</td>
<td>30</td>
<td>$2.63 \times 10^{-3}$</td>
<td>32.02</td>
<td>0.995</td>
</tr>
<tr>
<td>1.7</td>
<td>400</td>
<td>20</td>
<td>$4.14 \times 10^{-3}$</td>
<td>29.79</td>
<td>0.974</td>
</tr>
<tr>
<td>1.7</td>
<td>450</td>
<td>20</td>
<td>$5.92 \times 10^{-3}$</td>
<td>29.18</td>
<td>0.916</td>
</tr>
</tbody>
</table>

The accuracy of creep response of the composite cylinder, to be estimated in subsequent sections, will depend on the accuracy associated with the prediction of creep parameters $M$ and $\sigma_o$ for various combinations of material parameters.
and operating temperature. To accomplish this task, the creep parameters given in Table 3.1 have been substituted in the constitutive creep model, Eqn. (3.2), to estimate the strain rates corresponding to the experimental stress values reported by Pandey et al (1992) for Al-SiCp composite corresponding to various combinations of material parameters and temperature as given in Table 3.1. The estimated strain rates have been compared with the strain rates observed experimentally by Pandey et al (1992). Figs. 3.2(a)-(c) show an excellent agreement between the strain rates estimated from Eqn. (3.2) and those observed experimentally, to inspire confidence in the creep parameters estimated in this study.

3.4. CREEP ANALYSIS OF COMPOSITE CYLINDER

Considering a long thick-walled cylinder made of Al-SiCp with closed end and having inner and outer radii $a$ and $b$ respectively. The cylinder is subjected to internal pressure $p$ and external pressure $q$, Fig. 3.3. The axes $r$, $\theta$ and $z$ are taken respectively taken along radial, tangential and axial direction of the cylinder. For the purpose of analysis following assumptions are made in the present work:

i. Material of the cylinder is incompressible, isotropic and has uniform distribution of SiCp in aluminum matrix.

ii. Pressure is applied gradually on the cylinder and held constant during the loading history.

iii. Stresses at any point in the cylinder remain constant with time \textit{i.e.} steady state condition of stress is assumed.

iv. Elastic deformations are small and are neglected as compared to creep deformations.
The geometric relationships between radial \( \dot{\varepsilon}_r \) and tangential \( \dot{\varepsilon}_\theta \) strain rates are,

\[
\dot{\varepsilon}_r = \frac{d\dot{u}_r}{dr}
\]

\[
\dot{\varepsilon}_\theta = \frac{\dot{u}_r}{r}
\]

where \( \dot{u}_r = \frac{du}{dt} \) is the radial displacement rate and \( u \) is the radial displacement.

Eliminating \( \dot{u}_r \) from Eqs. (3.3) and (3.4), we get the deformation compatibility equation given by,

\[
r \frac{d\dot{\varepsilon}_\theta}{dr} = \dot{\varepsilon}_r - \dot{\varepsilon}_\theta
\]

The boundary conditions for a cylinder subjected to both internal and external pressures are,

\( (i) \) At \( r = a \), \( \sigma_r = -p \) \hspace{1cm} (3.6)

\( (ii) \) At \( r = b \), \( \sigma_r = -q \) \hspace{1cm} (3.7)

The negative sign of \( \sigma_r \) in Eqs. (3.6) and (3.7) implies the compressive nature of radial stress.

The equilibrium equation for a thick-walled cylindrical vessel subjected to uniform internal and external pressures may be obtained by considering the equilibrium of forces acting on an element of the composite cylinder, confined between radius \( r \) and \( r + dr \) as shown in Fig. 3.4. The axial length of the cylinder is assumed to be \( l \).

By considering equilibrium of forces along the radial direction, we get,
\[(\sigma_r + d\sigma_r)(r + dr)d\theta l - \sigma_r rd\theta l - 2\sigma_\theta dr l \sin \frac{d\theta}{2} = 0 \quad (3.8)\]

In deriving the above equilibrium equation the weight of the cylinder is ignored in comparison to external loads acting on the cylinder.

The Eqn. (3.8) may be simplified by neglecting the higher order terms and noting that \(d\theta\) is very small \(i.e. \sin d\theta/2 \approx d\theta/2\). Therefore, the equilibrium equation becomes,

\[r \frac{d\sigma_r}{dr} = \sigma_\theta - \sigma_r \quad (3.9)\]

Under the assumption of incompressibility, one may write,

\[\dot{\varepsilon}_r + \dot{\varepsilon}_\theta + \dot{\varepsilon}_z = 0 \quad (3.10)\]

where \(\dot{\varepsilon}_z\) is the axial strain rate.

The present analysis assumes that material of the cylinder is isotropic and yields according to von-Mises yield criterion (von Mises, 1913). The effective stress for isotropic material may be expressed as,

\[f(\sigma_y) = \frac{1}{4} [ (\sigma_{14} - \sigma_{23})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 ] \quad (3.11)\]

where \(f(\sigma_y)\) is the potential function and \(\sigma_{11}, \sigma_{22}\) and \(\sigma_{33}\) are the principal stresses.

The strain rate increment \((d\varepsilon_y)\) is related to the potential function through the associated flow rule as given below,

\[d\varepsilon_y = d\lambda \frac{\partial f(\sigma_y)}{\partial \sigma_y} \quad (3.12)\]
where $d\lambda$ is the proportionality factor that must depend on $\sigma_{ij}$, $d\sigma_{ij}$ and $\varepsilon_{ij}$ apart from strain history because of strain hardening (Backofen, 1972).

Using the yield criterion given by Eqn. (3.11) into Eqn. (3.12), one may obtain the following constitutive equations in terms of principal strain increments $d\varepsilon_{11}, d\varepsilon_{22}, d\varepsilon_{33}$ and principal stresses $\sigma_{11}, \sigma_{22}, \sigma_{33}$.

\[
\begin{align*}
\sigma_{11} - \frac{\sigma_{22} + \sigma_{33}}{2} & = d\lambda \\
\sigma_{22} - \frac{\sigma_{11} + \sigma_{33}}{2} & = d\lambda \\
\sigma_{33} - \frac{\sigma_{11} + \sigma_{22}}{2} & = d\lambda
\end{align*}
\] (3.13)

The von-Mises effective stress ($\sigma_{e}$) and effective strain rate increment ($d\varepsilon_{e}$) are given by,

\[
\sigma_{e} = \frac{1}{\sqrt{2}} \left[ (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + (\sigma_{11} - \sigma_{22})^2 \right]^{1/2} \] (3.14)

\[
d\varepsilon_{e} = \frac{\sqrt{3}}{2} \left[ (d\varepsilon_{11} - d\varepsilon_{22})^2 + (d\varepsilon_{22} - d\varepsilon_{33})^2 + (d\varepsilon_{33} - d\varepsilon_{11})^2 \right]^{1/2} \] (3.15)

The effective strain increment ($d\varepsilon_{e}$) may be obtained by substituting the values of $d\varepsilon_{11}, d\varepsilon_{22}$, and $d\varepsilon_{33}$ from Eqs. (3.13) in Eqn. (3.15) and correlating the resulting equation with the effective stress given by Eqn. (3.14) to obtain,

\[
d\varepsilon_{e} = \sigma_{e} d\lambda \] (3.16)

Using Eqs. (3.13) and (3.16), we get,
\[
\begin{align*}
\frac{d\varepsilon_{11}}{\sigma_e} &= \frac{d\varepsilon_e}{\sigma_e} \left[ \sigma_{11} - \frac{(\sigma_{22} + \sigma_{33})}{2} \right] \\
\frac{d\varepsilon_{22}}{\sigma_e} &= \frac{d\varepsilon_e}{\sigma_e} \left[ \sigma_{22} - \frac{(\sigma_{11} + \sigma_{33})}{2} \right] \\
\frac{d\varepsilon_{33}}{\sigma_e} &= \frac{d\varepsilon_e}{\sigma_e} \left[ \sigma_{33} - \frac{(\sigma_{11} + \sigma_{22})}{2} \right] 
\end{align*}
\] (3.17)

Integrating the above set of Eqs. (3.17), we obtain,

\[
\begin{align*}
\dot{\varepsilon}_{11} &= \frac{\dot{\varepsilon}_e}{2\sigma_e} \left[ 2\sigma_{11} - \sigma_{22} - \sigma_{33} \right] \\
\dot{\varepsilon}_{22} &= \frac{\dot{\varepsilon}_e}{2\sigma_e} \left[ 2\sigma_{22} - \sigma_{11} - \sigma_{33} \right] \\
\dot{\varepsilon}_{33} &= \frac{\dot{\varepsilon}_e}{2\sigma_e} \left[ 2\sigma_{33} - \sigma_{11} - \sigma_{22} \right] 
\end{align*}
\] (3.18)

The set of Eqs. (3.18) are termed as constitutive equations for creep in an isotropic material in principal stress space.

The generalized constitutive Eqs. (3.18) for creep in an isotropic composite takes the following form when reference frame is along the principal directions of \( r, \theta \) and \( z \).

\[
\begin{align*}
\dot{\varepsilon}_r &= \frac{\dot{\varepsilon}_e}{2\sigma_e} \left[ 2\sigma_r - \sigma_\theta - \sigma_z \right] \\
\dot{\varepsilon}_\theta &= \frac{\dot{\varepsilon}_e}{2\sigma_e} \left[ 2\sigma_\theta - \sigma_z - \sigma_r \right] \\
\dot{\varepsilon}_z &= \frac{\dot{\varepsilon}_e}{2\sigma_e} \left[ 2\sigma_z - \sigma_r - \sigma_\theta \right] 
\end{align*}
\] (3.19-3.21)
where \( \dot{\epsilon}_r, \dot{\epsilon}_\theta, \dot{\epsilon}_z \) and \( \sigma_r, \sigma_\theta, \sigma_z \) are the strain rates and the stresses respectively along \( r, \ \theta \) and \( z \) directions, as indicated by the subscripts.

The von Mises yield criterion given by Eqn. (3.14), when reference frame is along principal directions of \( r, \ \theta \) and \( z \), becomes,

\[
\sigma_e = \frac{1}{\sqrt{2}} \left[ (\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2 + (\sigma_r - \sigma_\theta)^2 \right]^{1/2}
\]  
   (3.22)

For a cylinder made of incompressible material and having closed ends, the plain strain condition exists \( i.e. \ \dot{\epsilon}_z = 0 \) (Bhatnagar and Arya, 1974; Popov, 2001). Therefore, under plain strain condition \( (\dot{\epsilon}_z = 0) \), from Eqs. (3.3), (3.4) and (3.10), the radial displacement rate may be obtained as,

\[
\dot{u}_r = \frac{C}{r} \tag{3.23}
\]

where \( C \) is a constant of integration.

Substituting Eqn. (3.23) into Eqs. (3.3) and (3.4), we get,

\[
\dot{\epsilon}_r = -\frac{C}{r^2} \tag{3.24}
\]

\[
\dot{\epsilon}_\theta = \frac{C}{r^2} \tag{3.25}
\]

Under plain strain condition, \( (\dot{\epsilon}_z = 0) \), Eqn. (3.21) gives,

\[
\sigma_z = \frac{\sigma_r + \sigma_\theta}{2} \tag{3.26}
\]

Therefore, the axial stress at any point in the cylinder is arithmetic mean of the radial and tangential stresses at the corresponding location.
Substituting Eqn. (3.26) into Eqn. (3.22), the effective stress in the cylinder is given by,

\[
\sigma_e = \frac{\sqrt{3}}{2}(\sigma_\theta - \sigma_r)
\]  
(3.27)

Using Eqs. (3.24) and (3.26) in Eqn. (3.19), we get,

\[
\sigma_\theta - \sigma_r = \frac{4}{3}\left(\frac{\sigma_e C}{\hat{\varepsilon}_e r^2}\right)
\]  
(3.28)

Substituting \(\hat{\varepsilon}_e\) and \(\sigma_e\) respectively from Eqs. (3.2) and (3.27) into above equation and simplifying, we get,

\[
\sigma_\theta - \sigma_r = \frac{I_1}{r^{2/n}} + I_2
\]  
(3.29)

where,

\[
I_1 = \left[\frac{4}{3}\left(\frac{2}{n}\right)^{n+1}\right] \cdot \left(\frac{C^{1/n}}{M}\right) \quad \text{and} \quad I_2 = \frac{2}{\sqrt{3}}\sigma_o.
\]

Substituting Eqn. (3.29) into equilibrium Eqn. (3.9) and integrating the resulting equation, we get,

\[
\sigma_r = -\frac{n}{2} \cdot \frac{I_1}{r^{2/n}} + I_2 \ln r + C_1
\]  
(3.30)

where \(C_1\) is another constant of integration.

Using the boundary conditions given in Eqs. (3.6) and (3.7) into Eqn. (3.30), the values of \(C_1\) and \(I_1\) are obtained as,

\[
C_1 = \frac{n}{2} I_1 b^{-2/n} - I_2 \ln b - q
\]
\[ I_1 = \frac{2}{n} X \]

where,

\[ X = \left[ \frac{p - q - I_2 \ln(b/a)}{a^{2/n} - b^{-2/n}} \right] \]

The values of \( C_i \) and \( I_1 \) obtained above are substituted in Eqn. (3.30) to get the radial stress,

\[ \sigma_r = -X \left( r^{-2/n} - b^{-2/n} \right) + I_2 \ln(b/r) + q \]  \hspace{1cm} (3.31)

Using Eqn. (3.31) in Eqn. (3.29), the tangential stress is obtained,

\[ \sigma_\theta = X \left[ b^{-2/n} - \left( 1 - \frac{2}{n} \right) r^{-2/n} \right] + I_2 \left[ 1 - \ln(b/r) \right] - q \]  \hspace{1cm} (3.32)

Substituting Eqs. (3.31) and (3.32) into Eqn. (3.26), the axial stress is obtained as,

\[ \sigma_z = X \left[ b^{-2/n} - \left( 1 - \frac{1}{n} \right) r^{-2/n} \right] - I_2 \ln(b/r) - 0.5 - q \]  \hspace{1cm} (3.33)

Using Eqs. (3.26) and (3.27) in Eqs. (3.19) and (3.20), the radial and tangential strain rates in the composite cylinder are obtained in terms of effective strain rate,

\[ \dot{\varepsilon}_r = \dot{\varepsilon}_\theta = 0.87 \dot{\varepsilon}_e \]  \hspace{1cm} (3.34)

Therefore, the radial and tangential strain rates in the composite cylinder are 87% of the effective strain rate at the corresponding radial location.

### 3.5 NUMERICAL CALCULATIONS

Based on the analysis presented in previous section, a computer program has been developed to calculate the steady state creep response of the composite
cylinder for various combinations of size and content of the reinforcement (SiCp), and operating temperature. For the purpose of numerical computation, the inner and outer radii of the cylinder are taken 25.4 mm and 50.8 mm respectively, and the internal and external pressures are assumed as 85.25 MPa and 42.6 MPa respectively. The dimensions of cylinder and the operating pressure chosen in this study are similar to those used in earlier work (Johnson et al., 1961) on thick-walled cylinder made of aluminum alloy (RR59). The radial, tangential and axial stresses at different radial locations of the cylinder are calculated respectively from Eqs. (3.31), (3.32) and (3.33). The distributions of radial and tangential strain rates in the cylinder are computed from Eqs. (3.19) and (3.20) respectively. The creep parameters $M$ and $\sigma_0$ required during the computation process are taken from Table 3.1, for the desired combination of particle size, particle content and operating temperature.

3.6 RESULTS AND DISCUSSION

Numerical calculations have been carried out to obtain the steady state creep response of the composite cylinder for different particle size, particle content and operating temperature.

3.6.1 Validation

Before discussing the results obtained, it is necessary to check the accuracy of analysis carried out and the computer program developed. To accomplish this task, the tangential, radial and axial stresses have been computed from the current analysis for a copper cylinder, the results for which are available in literature (Johnson et al., 1961). The dimensions of the cylinder, operating
pressure and temperature, and the values of creep parameters used for the purpose of validation are summarized in Table 3.2.

Table 3.2: Summary of data used for validation (Johnson et al, 1961)

<table>
<thead>
<tr>
<th>Cylinder Material</th>
<th>Copper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder dimensions:</td>
<td>$a = 25.4 \text{ mm}$, $b = 50.8 \text{ mm}$</td>
</tr>
<tr>
<td>Internal Pressure</td>
<td>$23.25 \text{ MPa}$</td>
</tr>
<tr>
<td>External Pressure</td>
<td>$0$</td>
</tr>
<tr>
<td>Operating Temperature</td>
<td>$250 ^\circ \text{C}$</td>
</tr>
<tr>
<td>Creep parameters estimated</td>
<td>$M = 3.271 \times 10^{-4} \text{ s}^{-1/5}/\text{MPa}$, $\sigma_o = 11.32 \text{ MPa}$</td>
</tr>
</tbody>
</table>

To estimate the values of parameters $M$ and $\sigma_o$ for copper cylinder, firstly $\sigma_e$ have been calculated at the inner and outer radii of the cylinder by substituting the values of $\sigma_e$, $\sigma_r$, $\sigma_\theta$, and $\sigma_z$ in Eqn. (3.22) at these locations, as reported in the study of Johnson et al (1961). The values of stresses $\sigma_r$, $\sigma_\theta$, and $\sigma_z$ and the tangential strain rate ($\dot{\varepsilon}_\theta$) reported by Johnson et al (1961) at the inner and outer radii are substituted in Eqn. (3.20) to estimate the effective strain rates ($\dot{\varepsilon}_e$) at the corresponding radial locations. The effective stresses and effective strain rates thus estimated at the inner radius ($\sigma_e = 189.83 \text{ MPa}$ and $\dot{\varepsilon}_e = 2.168 \times 10^{-6} \text{ s}^{-1}$) and at the outer radius ($\sigma_e = 116 \text{ MPa}$ and $\dot{\varepsilon}_e = 1.128 \times 10^{-9} \text{ s}^{-1}$) of the copper cylinder are substituted in creep law, Eqn. (3.2), to obtain the creep parameters $M$ and $\sigma_o$ for copper cylinder as given in Table 3.2. These creep parameters have been used in the developed software to compute the distribution of tangential strain rate in the copper cylinder. The tangential strain rates, thus obtained, have been compared with those reported by Johnson et al (1961). A nice agreement is observed in Fig. 3.5 verifies the accuracy of analysis presented and software developed in the current study.
3.6.2 Effect of Particle Size

Figure 3.6 shows the distribution of radial, tangential, axial and effective stresses in the composite cylinder for varying size of SiCp reinforcement from 1.7 \( \mu m \) to 45.9 \( \mu m \). The radial stress remains compressive over the entire radius, with maximum and minimum values respectively at the inner and outer radii, under the imposed boundary conditions given in Eqs. (3.6) and (3.7). The tangential stress varies from maximum compressive value at the inner radius to reach a maximum tensile value at the outer radius of the cylinder. The axial stress, which is the average of radial and tangential stresses, Eqn. (3.26), exhibits a variation similar to that observed for tangential stress. The variation in size of reinforcement (SiCp) does not exhibit a sizable effect on the values of stresses in the cylinder, except for some marginal variation observed in tangential stress near the inner and outer radii. The maximum variation observed for tangential stress is about 4% at the inner as well as at the outer radius whereas for axial stress the variation observed is less than 1% at the inner radius and about 2% at the outer radius of the cylinder having coarser SiCp of size 45.9 \( \mu m \) as compared to those observed for cylinder having finer SiCp of size 1.7 \( \mu m \). The tangential stresses, both compressive (near the inner radius) as well as tensile (near the outer radius), in the cylinder having finer sized SiCp (1.7 \( \mu m \)) are marginally higher than those observed in cylinders with relatively coarser SiCp (\emph{i.e.} 14.5 \( \mu m \) and 45.9 \( \mu m \)). The effective stress decreases on moving from the inner to the outer radius of the cylinder. With the increase in SiCp size from 1.7 \( \mu m \) to 45.9 \( \mu m \), the effective stress exhibits a marginal increase near the inner radius but it decreases marginally towards the outer radius.
The strain rates, given by Eqs. (3.19) and (3.20), are dependent on the effective strain rate \( \dot{\varepsilon}_e \), which ultimately depends upon \( \sigma_e - \sigma_o \), the difference of effective stress \( \sigma_e \) and threshold stress \( \sigma_o \), as evident from creep law given by Eqn. (3.2). Therefore, to investigate the effect of reinforcement (SiCp) size on the creep rates, the variation of stress difference \( \sigma_e - \sigma_o \) is plotted with radial distance in Fig. 3.7. It is noticed that the stress difference \( \sigma_e - \sigma_o \) decreases significantly over the entire radius with decreasing SiCp size from 45.9 \( \mu m \) to 1.7 \( \mu m \). The decrease observed may be attributed to the increase in threshold stress with decrease in particle size as evident from Table 3.1. As a consequence, the effective strain rate \( \dot{\varepsilon}_e \) also reduces significantly over the entire radius with decreasing particle (SiCp) size, Fig. 3.8. The decrease observed is about two orders of magnitude with decrease in SiCp size from 45.9 \( \mu m \) to 1.7 \( \mu m \). It is quite evident from Eqn. (3.2) that the decrease observed in effective strain rate is due to decrease in parameter \( M \) and increase in threshold stress \( \sigma_o \), with decreasing size of SiCp, as revealed from Table 3.1. The radial and tangential strain rates are equal in magnitude but have opposite nature due to the incompressibility condition, Eqn. (3.10), and the assumption of plane strain condition \( \dot{\varepsilon}_z = 0 \). The radial (compressive) and tangential (tensile) strain rates in the cylinder for a given size of SiCp reinforcement are 13% lower than the corresponding effective strain rates, Eqn. (3.34), as is also evident from Fig. 3.8. The effect of particle size on strain rates is similar to that observed for effective strain rate. Therefore, it may be concluded that the steady state creep rates in the composite cylinder could be significantly reduced by employing finer size of SiCp reinforcement in aluminum matrix. For the same volume fraction of
reinforcement, the smaller size particles will be larger in number, and therefore lead to more load transfer to the reinforcement with a corresponding reduction in the level of effective stress shared by the matrix material, which enhances the substructure strength (Li and Langdon, 1993; Peng et al, 1998; 1999; Han and Langdon, 2002) and ultimately helps in restraining the creep flow of composite cylinder.

3.6.3 Effect of Particle Content

Figure 3.9 shows the variation of stresses in composite cylinder containing different amount (vol%) of SiCp i.e. 10%, 20% and 30%. The radial stress does not exhibit sizable variation on modifying the content of SiCp, except for a small increase observed somewhere in the middle region of the cylinder with increase in particle content from 10% to 30%. Unlike particle size, the increase in particle content induces some sizable variation in the tangential and axial stresses as observed in Fig. 3.9. By increasing the amount of SiCp from 10% to 30%, the tangential stress, compressive near the inner and tensile near the outer radius increases over the entire radius of the cylinder. The maximum increase observed in tangential stress is about 25% at the outer radius. The axial stress (compressive) increases near the inner radius but decreases towards the outer radius, with the increase in the content of SiCp from 10% to 30%. At the inner radius, the axial stress increases by about 4% but at the outer radius it decreases by about 12% with the increase in particle content from 10% to 30%. Unlike axial stress, the effective stress decreases near the inner radius but increases near the outer radius with the increase in amount of SiCp from 10% to 30%. The maximum decrease (at the inner radius) and increase (at the outer radius) observed are respectively about 5% and 6%. The stress difference \( \sigma_e - \sigma_o \) shown in Fig. 3.10, decreases
significantly over the entire radial distance with increasing SiCp content from 10% to 30%. The decrease observed is relatively more towards the inner radius. As expected, the effective strain rate, Fig. 3.11, decreases significantly with the increase in amount of SiCp. The effective strain rate decreases by about four orders of magnitude throughout the cylinder on increasing the content of SiCp from 10% to 30%. The decrease observed in effective strain rate may be attributed to decrease in creep parameter $M$ and increase in threshold stress ($\sigma_o$) with the increase in content of SiCp, as evident from Table 3.1. The impact of varying particle content on the tangential and radial creep rates is similar to those noticed for effective strain rate. By increasing the amount of SiCp in the composite cylinder, the inter-particle spacing decreases that causes the increase in threshold stress (Li and Langdon, 1999) but decrease in creep parameter $M$ (Table 3.1). Both these factors are responsible for significant reduction in strain rates. Mishra and Pandey (1990) in their review of uniaxial creep data of Nieh (1984), Nieh et al (1988) and Morimoto et al (1988) have also noticed that creep rate in SiC (whisker) reinforced aluminum alloy (6061Al) composite could be significantly reduced by increasing the content of reinforcement. A similar effect of increasing SiC (particle) content on strain rate has been noticed by Pandey et al (1992) for Al-SiCp composite under uniaxial creep.

### 3.6.4 Effect of Temperature

The creep in a material is significantly influenced by the operating temperature. Therefore, this section brings out the effect of varying operating temperature on the stresses and strain rates in a thick cylinder made of aluminum matrix composite reinforced with 20 vol% of SiCp.
Figure 3.12 shows the variation of radial, tangential, axial and effective stresses in a composite cylinder operating at three different temperatures i.e. 350 \(^\circ\)C, 400 \(^\circ\)C and 450 \(^\circ\)C. Similar to the effect observed for particle size (Fig. 3.6), the radial, tangential and axial stresses do not exhibit sizable variation with the increase in operating temperature from 350 \(^\circ\)C to 450 \(^\circ\)C. The effect of temperature on the stresses is not significant, except for a slight variation noticed for effective stress. With the decrease in operating temperature from 450 \(^\circ\)C to 350 \(^\circ\)C, the compressive value of tangential stress increases a little (around 2.5\%) near the inner radius. It is interesting to observe that in the middle of cylinder, the nature of tangential stress changes from compressive to tensile. Further, this tensile value of tangential stress increases on moving towards the outer radius of cylinder with the decrease in operating temperature from 450 \(^\circ\)C to 350 \(^\circ\)C. The maximum increase observed is around 3\%. The variation of axial stress is similar to that observed for radial stress. It remains compressive throughout the cylinder, with maximum value at the inner radius and minimum value at the outer radius. On the other hand, the effective stress, near the inner radius, decreases marginally with the decrease in temperature from 450 \(^\circ\)C to 350 \(^\circ\)C; but it exhibits a marginal increase towards the outer radius with the decrease in temperature. The stress difference \((\sigma_e - \sigma_o)\) shown in Fig. 3.13, decreases throughout the cylinder with decreasing temperature. The maximum decrease observed in stress difference is around 12\% over the entire radius when the operating temperature decreases from 450 \(^\circ\)C to 350 \(^\circ\)C.

The effective, radial and tangential strain rates in the cylinder decreases by about two orders of magnitude with the decrease in operating temperature from 450 \(^\circ\)C to 350 \(^\circ\)C. With decrease in operating temperature, the threshold stress \(\sigma_o\).
increases and the creep parameter $M$ decreases (Table 3.1), as a result of which the strain rates in the composite cylinder decrease to a significant extent. The effect of temperature on the creep rate observed in this study are similar to those reported by Pandey et al (1992) for Al-SiCp composites under uniaxial creep.

### 3.7 SELECTION OF MATERIAL PARAMETERS

It is evident from the above discussion that the creep stresses in a thick composite cylinder do not vary significantly by varying size and content of reinforcement (SiCp) as compared to the variation observed in strain rates. From the point of view of designing a composite pressure vessel, operating under elevated temperature, the strain rates are considered to be primary design parameters. In order to reduce the steady state creep rates in the composite cylinder, working under a given set of operating conditions (i.e. operating pressure and temperature) any of the following three options could be employed: (i) using finer size of reinforcement (SiCp) without varying its content, (ii) incorporating higher amount of dispersoids (SiCp) without altering its size, and (iii) simultaneously decreasing the size and increasing the amount of SiCp reinforcement. The selection of optimum size and content of the reinforcement in a composite pressure vessel, working under a given set of operating conditions, can be decided by simultaneously optimizing the cost of composite and the value of maximum strain rate in the composite cylinder for different combinations of particle size and particle content within the specified range.
Fig. 3.1: Variation of $\dot{\varepsilon}^{1/5}$ versus $\sigma$ in Al-SiCp composite for different (a) particle sizes of SiCp, (b) vol% of SiCp and (c) temperatures (Pandey et al, 1992).
Fig. 3.2: Comparison of experimental (Pandey et al, 1992) and estimated strain rates in Al-SiCp composite for different (a) particle sizes of SiCp, (b) vol% of SiCp and (c) temperatures.
Fig. 3.3: Schematic of closed end, thick-walled composite cylinder subjected to internal and external pressures.
Fig. 3.4: Free body diagram of an element of the composite cylinder.
Fig. 3.5: Comparison of tangential strain rates in copper cylinder as estimated from current analysis and reported by Johnson et al (1961).
Fig. 3.6: Variation of creep stresses for varying particle size of SiC ($V = 10 \text{ vol}\%, T = 350 \, ^\circ\text{C}$).
Fig. 3.7: Variation of stress difference for varying particle size of SiC ($V = 10 \text{ vol}\%$, $T = 350 \, ^\circ\text{C}$).
Fig. 3.8: Variation of strain rates for varying particle size of SiCp

\( V = 10\text{vol\%}, \, T = 350 \, ^\circ\text{C} \).
Fig. 3.9: Variation of creep stresses for varying particle content of SiCp

\( P = 1.7 \, \mu m, \ T = 350 \, ^{\circ}C \).
Fig. 3.10: Variation of stress difference for varying particle content of SiCp

\[ (\sigma_e - \sigma_0) \text{ (MPa)} \]

Legend:
- 10 vol% SiCp
- 20 vol% SiCp
- 30 vol% SiCp

\[ (P = 1.7 \mu m, T = 350 ^\circ C) \]
Fig. 3.11: Variation of strain rates for varying particle content of SiCp

\( P = 1.7 \mu m, T = 350 ^\circ C \).
Fig. 3.12: Variation of creep stresses in composite cylinder for varying temperature ($V = 20 \text{ vol}\%, P = 1.7\mu m$).
Fig. 3.13: Variation of stress difference in composite cylinder for varying operating temperature ($V = 20\;\text{vol\%}, P = 1.7\;\mu m$).
Fig. 3.14: Variation of strain rates in composite cylinder for varying operating temperature ($V = 20 \text{ vol\%}, P = 1.7 \mu m$).