CHAPTER – 2

REVIEW OF LITERATURE

2.1 GENERAL

Nowadays the application of composite materials in engineering, particularly in structural applications, is increasing in scope. As a result, new aspects of materials, such as the long-term behaviour under mechanical, thermal and chemical/environmental loadings are being addressed. Composite materials provide solutions to variety of technical problems faced these days by many industries. Composite materials have become more popular, because of increased competition in the global market for lightweight components of greater strength and stiffness. Among many materials, composite materials have the potential to replace widely used steel and aluminum, and enhance performance. As an example, by replacing steel components with composite components, one can achieve 60% to 80% saving in component weight, while 20% to 50% weight saving occurs if aluminum parts are replaced by composite parts (Mazumdar, 2002). The salient applications of composite materials are outlined in Table 2.1.

Table 2.1: Applications of Composites (Peters, 1998)

<table>
<thead>
<tr>
<th>Major Areas</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Automobiles</td>
<td>Combustion chambers (SiC-SiC), Engine cylinder liners (Al-SiC), CNG storage cylinders, Diesel Engine pistons (SiCw/Al-alloy), Brake rotors, Leaf springs (E-glass/epoxy), Drive shafts (Al-C), Flywheels, Racing car brakes (Al-SiC), Motorcycle drive</td>
</tr>
<tr>
<td>Sub-Marine</td>
<td>sprocket, Pulleys, Torque converter reactor, Shock absorbers (SiCp/Al-alloy), Radiator end caps.</td>
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<tr>
<td>Propulsion shaft</td>
<td>(Carbon and glass fibers), Cylindrical pressure hull (Graphite/Epoxy), Sonar domes (Glass/Epoxy), Composite piping system, Scuba diving cylinders (Al-SiC), Floats, Boat hulls.</td>
</tr>
<tr>
<td>Commercial and Industrial</td>
<td>Pressure vessels, Fuel tanks, Cutting tool inserts, Laptop cases, Wind turbine blades, Electric motors, Firefighting air bottles, Artificial ligaments, MRI scanner cryogenic tubes, Wheelchairs, Hip joint implants, Eyeglass frames, camera tripods, Musical instruments, Drilling tubes, Drilling motor shaft, Drill casing, Crane components, High pressure hydraulic pipe, X-ray tables, Heart valves, Helmets, Crucibles, Beams.</td>
</tr>
<tr>
<td>Aerospace equipment and structures</td>
<td>Rocket nozzle (TiAl-SiC fibers), Heat exchanger panels, Engine parts (Be-Al), Wind tunnel blades, Spacecraft truss structure, Reflectors, Solar panels, Camera housing, Hubble space telescope metering truss assembly, Turbine rotor, Turbine wheels (operating above 40,000 rpm), Nose caps and leading edge of missiles and Space shuttle.</td>
</tr>
<tr>
<td>Aircraft</td>
<td>Wings, Rotary launchers, Engine casing, Rings (Al₂O₃ /Al-alloy), Drive shaft, Propeller blades, Landing gear doors, Thrust reverser(Carbon/Bismaleimide), Helicopter components viz. Rotor drive shaft, Mast mount, Main rotor blades (Carbon/Epoxy).</td>
</tr>
</tbody>
</table>
The successful design of any structural components requires efficient and safe use of materials. Therefore, the ability to predict the yield strength of a material, i.e., the onset of plastic yielding, under loading conditions, is of great importance for design purposes. In the field of plasticity, it is important to establish mathematical relations for determining the conditions at which plastic yielding begins when a material is subjected to any possible combination of stresses. Further, under uniaxial loading, the plastic flow begins at yield stress as in case of simple tensile test. The yielding under a state of combined stresses can be related to some particular combination of principal stresses. Several relations have been proposed as yield criteria that give us information about the strength of a material under multiaxial loading.

The yield criteria may be expressed in terms of specific quantities, such as the stress state, the strain state, a strain energy quantity, or others. A yield criterion is usually written in mathematical form by means of a yield function $f(\sigma_y, Y)$, where $\sigma_y$ defines the state of stress and $Y$ is the yield strength under uniaxial tension (or compression). The yield function is defined such that the yield criterion is satisfied when $f(\sigma_y, Y) = 0$. When $f(\sigma_y, Y) < 0$, the stress state is elastic. The condition $f(\sigma_y, Y) > 0$ is undefined. To develop a yield function, the components of the multiaxial stress state are combined into a single quantity known as the effective stress ($\sigma_e$). The effective stress is then compared with the
yield stress $Y$, in some appropriate form, to check inset of yielding. The term $f(\sigma_y)$ is the yield function, a function of six stress components of the stress tensor that defines the yield surface in the six dimensional stress spaces separating the elastic and plastic domains.

### 2.2.1 Yield Criteria Used in the Present Study

The present work deals with the analysis of composite cylinder, which is assumed to be either isotropic or anisotropic. In addition, the effect of residual stress is also taken into account in a segment of the study. This section briefly describes the yield criteria applicable for these three cases.

For isotropic materials, the yielding would depend only on the magnitudes of principal stresses $\sigma_1, \sigma_2$, and $\sigma_3$. For these materials the well known von Mises (1913) yield criterion, also known as distortion-energy theory, might be expressed as,

$$\sigma_e = \frac{1}{\sqrt{2}} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{1/2} \quad (2.1)$$

where $\sigma_e$ is equivalent stress.

The von-Mises criterion implies that the yielding under both uniaxial tension and compression starts at the same level of tensile and compressive stresses. However, Arsenault (1987) pointed out that even in an isotropic metal matrix composite yielding does not begins at the same level of tensile and compressive stresses under uniaxial loading. Badini (1990) also noticed that the compressive yield strength of 15 vol% SiCw/6061Al composite is higher than its yield strength in tension. The processing of metal matrix composites often involves cooling from higher temperature, which results in residual tensile thermal stress in the matrix due to restraint caused by ceramic reinforcements. To account
for the effect of different yield stresses in compression and tension, some researchers (Schellekens and De Borst, 1990a, b; Bicanic et al., 1994; Moin, 1996) have employed the isotropic form of Hoffman yield criterion given below,

\[
(\sigma_i^2 + \sigma_j^2 + \sigma_k^2) - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) + Q(\sigma_1 + \sigma_2 + \sigma_3) - 1 = 0
\]

(2.2)

where \( Q \) is a material parameter.

A number of other investigators (Bicanic et al., 1994; Moin, 1996) have preferred following form of Hoffman’s yield criterion that employs uniaxial compression and tensile yield stresses expressed by \( f_c \) and \( f_t \) respectively,

\[
(\sigma_i^2 + \sigma_j^2 + \sigma_k^2) - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) + (f_c - f_t)(\sigma_1 + \sigma_2 + \sigma_3) - f_c f_t = 0
\]

(2.3)

In both the yield criteria described above (i.e. von-Mises’s and Hoffman’s yield criteria) it is assumed that the material is isotropic. This assumption may hold good at the start of plastic deformation. But this would not be a valid assumption after the material undergoes appreciable elongation. Further, the mechanical strength of whisker reinforced composites are dependent on the orientation of whiskers in the matrix (Ledrich and Shastry, 1982; Crove et al., 1985; McDenels, 1985). Badini (1990) observed that the compressive yield strength in the longitudinal direction of 15 vol% SiCw/6061Al composite was higher than its yield strength in the transverse direction. Moreover, fiber-reinforced composite or components that are fabricated by processes such as forging, rolling or extrusion exhibits anisotropic properties. Therefore, the use of von-Mises yield criterion given by Eqn. 2.1 would not be an appropriate choice for such cases.

Hill (1948) modified the von-Mises yield criterion applicable for isotropic material to incorporate the effect of anisotropy. Hill assumed the material to be
orthotropic i.e. there exists only three mutually orthogonal planes of symmetry at every point inside material. The intersection of these planes is referred as the principal axes of anisotropy. The Hill’s yield criterion does not consider Bauschinger effect and also assumes that the hydrostatic stresses do not affect yielding.

Hill’s criterion takes the following form when the principal axes of anisotropy are taken along $x$, $y$ and $z$ directions,

$$F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + H(\sigma_x - \sigma_y)^2$$

$$+ 2L\tau_{x\z}^2 + 2M\tau_{y\z}^2 + 2N\tau_{xy}^2 = 1$$

(2.4)

where the parameters $F$, $G$, $H$, $L$, $M$ and $N$ are material constants defining the degree of anisotropy. For principal axes of orthotropic symmetry, the Hill’s criterion given by Eqn. (2.4) becomes,

$$F(\sigma_2 - \sigma_3)^2 + G(\sigma_3 - \sigma_1)^2 + H(\sigma_1 - \sigma_2)^2 = 1$$

(2.5)

where $\sigma_1, \sigma_2$ and $\sigma_3$ are the principal stresses.

In limiting case, when anisotropic constants are equal (i.e. $F = G = H$), the Hill’s criterion given by Eqn. (2.5) reduces to von-Mises criterion for isotropic materials, Eqn. (2.1).

2.3 CREEP AND CREEP MECHANISMS

Creep is the progressive time-dependent inelastic deformation under constant load and temperature. Due to creep, a structural component undergoes time-dependent changes in state of stress and strain such as progressive deformations, relaxation and redistribution of stresses, local reduction of material strength, change of material behaviour from isotropic to anisotropic etc. Kraus
(1980) pointed that the creep behaviour is a function of stress, temperature, time, stress history, temperature history and material. Creep behaviour also includes the phenomenon of relaxation, which is the reduction of stress in a structure with time while the total strain remains constant. Further, it also includes recovery, which is characterized by the reduction of inelastic strain with time after the stress has been removed.

The phenomenon of creep is observed in most of the materials. The operating temperature in various industrial and structural applications is sufficiently high to cause significant creep as observed in chemical industries, nuclear power plants, missiles, aeroengines, gas turbines etc. The experimental testing and evaluation of creep in composite materials are quite complex, costly as well as time consuming. Therefore, the prediction and analysis of creep properties for assessing service life of components made of composite materials subjected to severe thermo-mechanical loadings is of great practical importance. Accordingly, a thorough understanding and development of methods for analyzing stress in components undergoing creep is extremely important.

2.3.1 Creep under Uniaxial State of Stress

Uniaxial creep tests are the basic experiments to evaluate creep behaviour of material. In this test, a standard cylindrical specimen is heated to a constant temperature \( T \) of the order of 0.3 to 0.5\( T_m \) (\( T_m \) is the melting temperature of the material) and subjected to a constant tensile force \( F \). The normal stress induced in the specimen is usually much less than the yield limit of the material. The axial strain is noticed with respect to time till fracture of the material. A typical creep curve obtained by plotting axial strain with time is shown schematically in Fig. 2.1.
Fig. 2.1: A typical uniaxial creep curve.

As soon as the specimen is loaded, an instantaneous deformation, which could be elastic or elasto-plastic depending upon the magnitude of the load, is observed. It is represented by portion OA of the curve. The remaining curve represents the creep deformation, which increases slowly with time. For analytical convenience, it is divided into the following three regions: (i) primary or non-steady state creep (AB) in which the creep rate is decreasing, (ii) secondary or steady state creep (BC) in which the creep rate is approximately constant and (iii) tertiary creep (CD) in which the creep rate increases and ultimately leads to rupture at the point D. If the load is removed at any time, curve similar to EF is
obtained; some amount of strain is instantaneously recovered. After that there is a
certain amount of strain recovery that becomes asymptotic with time axis and then
no more strain is recovered. The creep strain may therefore include permanent as
well as recoverable strain. To analyze the engineering structures and components
undergoing creep, mathematical models of creep are needed. For this purpose,
data obtained from the tensile tests serve as the main source of information about
creep.

The creep deformation is strongly influenced by temperature, stress and
time. The creep response of material can be written as,

\[ \varepsilon = f(\sigma, t, T) \]  

(2.6)

where \( \varepsilon \) is the creep strain, \( \sigma \) is the applied stress, \( t \) is the time period and \( T \) is the temperature of application. The Eqn. (2.6) can also be written in the
following form,

\[ \varepsilon = f_1(\sigma).f_2(t).f_3(T) \]  

(2.7)

As the temperature during creep test is constant, therefore, Eqn. 2.7
becomes,

\[ \varepsilon = f_1(\sigma).f_2(t) \]  

(2.8)

Many empirical expressions, as a function of \( f_1(\sigma) \) and \( f_2(t) \), have been
developed in the past. The most commonly used stress function law is Norton’s
law given as \( f_1(\sigma) = B\sigma^n \), where \( B \) and \( n \) are the material constants. The
approximation of a creep curve to a straight line is possible if secondary creep
region is predominant and elastic as well as primary creep is negligibly small.
Under such conditions, it can be assumed that creep strain rate depends upon
stress function only. The time dependence of creep has been expressed in terms of
Bailey’s empirical law given by: \( f_2(t) = A_t^m \), where \( A_t \) and \( m \) are the material constants.

### 2.3.1.1 Uniaxial Creep under Variable Stress

In the previous section, creep has been described under constant uniaxial state of stress. However, in most of the engineering components subjected to creep, the stress is variable due to two reasons. Firstly, the load applied on the structure may vary and secondly, the stress distribution may continuously vary with time under constant load. Therefore, the creep laws derived for constant one dimensional stress condition cannot be applied to variable stress conditions and hence must be modified. Time hardening and strain hardening are two popular theories, which take into account the variable stress and variable time.

The time hardening theory states that for a constant temperature and variable stress condition, creep rate \( \dot{\varepsilon}_c \) is a function of stress and time.

\[
\dot{\varepsilon}_c = f(\sigma, t)
\]  

(2.9)

However, in case of strain hardening theory it is assumed that the creep rate is a function of stress and accumulated strain.

\[
\dot{\varepsilon}_c = f(\sigma, \varepsilon)
\]  

(2.10)

The particular forms of these laws can be obtained by assuming that the creep curve can be represented by Bailey-Norton law, which is a common representation of creep in the primary and secondary creep ranges under isothermal conditions and is given below,

\[
\varepsilon_c = A \sigma^n t^m
\]  

(2.11)

where \( A, m \) and \( n \) are constants whose values depend upon the material type.
On differentiating Eqn. (2.11) with respect to time, the time hardening law can be obtained as below,

$$\dot{\varepsilon}_c = \frac{d\varepsilon_c}{dt} = A\sigma^n t^{m-1}$$

(2.12)

From the above equation it can be seen that the creep rate decreases with time since $0 < m < 1$. Further, Eqn. (2.12) can be written in a form independent of time $t$ by eliminating $t$ between Eqs. (2.11) and (2.12).

$$\dot{\varepsilon}_c = \frac{mA^{(1/m)} \sigma^{(n/m)}}{\varepsilon_c^{(1-m)/m}}$$

(2.13)

Equation (2.13) indicates that the creep strain rate decreases with increasing creep strain ($\varepsilon_c$) i.e. with the progression of creep strain, the material hardens.

Though, both the laws are derived from the same equation, but it is observed that for varying stress, the time and strain hardening laws give different creep rates. This difference is procedural and not phenomenological. Quite often the strain hardening models give more accurate predictions of experimental results for stepwise changes of stress. Unfortunately, strain hardening models do not always yield accurate predictions, particularly when several step changes in stress occur during the same test (Robotnov, 1969). Pickel et al (1971) also noticed that the strain hardening model is unable to accurately predict the creep behaviour resulting from structural instabilities. But for structurally stable materials, the predictions by strain hardening model are fairly reliable. However, in the case of gradually varying stress, both the laws give approximately similar predictions.

2.3.2 Creep under Multiaxial State of Stress

The uniaxial creep tests allow us to establish the basic features of creep behaviour and to establish relationship between stress, strain rate, temperature,
time, etc. However, in actual practice, most of the structural components operate under multiaxial stress conditions. Metallurgical models to describe creep under multiaxial stress creep are limited and the available models of multiaxial creep are mainly phenomenological in form (Kraus, 1980; Boyle and Spence, 1983; Gooch and How, 1986). Therefore, in order to analyze the influence of the stress state on the time dependent material behaviour, multiaxial creep analysis is required. So it is important to generalize our conclusions regarding creep deformations to include the multiaxial state of stress. Finnie and Hellar (1959) proposed the generalized constitutive equations for describing creep in isotropic materials under the influence of multiaxial stress as given below,

\[
\begin{align*}
\dot{\varepsilon}_x &= \frac{\dot{\varepsilon}_e}{\sigma_e} \left[ \sigma_x - \frac{1}{2} (\sigma_y + \sigma_z) \right] \\
\dot{\varepsilon}_y &= \frac{\dot{\varepsilon}_e}{\sigma_e} \left[ \sigma_y - \frac{1}{2} (\sigma_z + \sigma_x) \right] \\
\dot{\varepsilon}_z &= \frac{\dot{\varepsilon}_e}{\sigma_e} \left[ \sigma_z - \frac{1}{2} (\sigma_x + \sigma_y) \right] \\
\dot{\varepsilon}_{xz} &= \frac{3\dot{\varepsilon}_e}{\sigma_e} \tau_{xz} \\
\dot{\varepsilon}_{xy} &= \frac{3\dot{\varepsilon}_e}{\sigma_e} \tau_{xy} \\
\dot{\varepsilon}_{yz} &= \frac{3\dot{\varepsilon}_e}{\sigma_e} \tau_{yz}
\end{align*}
\]

(2.14)

where \(\sigma_x, \sigma_y, \sigma_z\) and \(\tau_{xy}, \tau_{yz}, \tau_{xz}\) are respectively the normal and shear stress components in \(x, y\) and \(z\) directions. Similarly \(\dot{\varepsilon}_x, \dot{\varepsilon}_y, \dot{\varepsilon}_z\) and \(\dot{\varepsilon}_{xy}, \dot{\varepsilon}_{yz}, \dot{\varepsilon}_{xz}\) are respectively the normal and shear strain rate components in \(x, y\) and \(z\) directions.
respectively. The effective stress \((\sigma_e)\) and effective strain rate \((\dot{\varepsilon}_e)\) given above are expressed by,

\[
\sigma_e = \frac{1}{\sqrt{2}} \left[ (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + (\sigma_x - \sigma_y)^2 + 6(\tau_{yz}^2 + \tau_{zx}^2 + \tau_{xy}^2) \right]^{1/2}
\]

\[
\dot{\varepsilon}_e = \frac{1}{\sqrt{2}} \left[ (\dot{\varepsilon}_y - \dot{\varepsilon}_z)^2 + (\dot{\varepsilon}_z - \dot{\varepsilon}_x)^2 + (\dot{\varepsilon}_x - \dot{\varepsilon}_y)^2 + 6(\dot{\varepsilon}_{yz}^2 + \dot{\varepsilon}_{zx}^2 + \dot{\varepsilon}_{xy}^2) \right]^{1/2}
\]

The Eqs. (2.14) - (2.16), are based on the assumption that the material is isotropic \(i.e.\) the properties of material remain same in all the directions. But in actual practice there are some materials which exhibits different properties in different directions and are known as anisotropic materials \(e.g.\) wood, rolled metal sheets, laminate or sandwich structures. As a special case, some of the anisotropic materials have similar properties in a plane known as plane of symmetry. Such materials are called planar anisotropic materials and the resulting anisotropy is called planar anisotropy. On the other hand, some of the anisotropic materials show identical mechanical properties about three orthogonal planes at a point. These materials are said to be orthotropic materials and the lines of intersection of three orthogonal planes are known as axes of anisotropy.

The creep deformations occur along preferred orientations, which result into mechanical properties that are direction dependent (Taira et al, 1965; 1966). As a consequence, anisotropy is induced in a material undergoing creep deformations. Taira et al (1965) demonstrated that the specimens taken out from thick-walled tube made of 0.19\% carbon steel at 450 °C, 470 °C and 500 °C exhibits different creep rate in different directions. The creep rate in axial direction of the tube is observed to be 10\% higher than the tangential creep rate. Murakami and Yamada (1974) have also noticed similar results. Therefore, an initially isotropic material becomes anisotropic during creep deformations and the
difference between the predictions based on isotropic and anisotropic theories of
creep goes on increasing with the progression of creep deformation. In this light, it
is extremely important for the designer of structural components to consider
anisotropy of the material during their stress analysis for a more realistic
approach.

Bhatnagar and Gupta (1967) proposed a theoretical framework to
incorporate the effect of initial anisotropy on
the creep behaviour of material. The
problems of creep in the presence of anisotropy have been carried out by some

Constitutive equations of creep in an anisotropic material as proposed by
Bhatnagar and Gupta (1967) are given below,

\[
\dot{\varepsilon}_x = \frac{\dot{\varepsilon}_e}{2\sigma_e} \left[ (G + H)\sigma_x - H\sigma_y - G\sigma_z \right]
\]

\[
\dot{\varepsilon}_y = \frac{\dot{\varepsilon}_e}{2\sigma_e} \left[ (H + F)\sigma_y - F\sigma_z - H\sigma_x \right]
\]

\[
\dot{\varepsilon}_z = \frac{\dot{\varepsilon}_e}{2\sigma_e} \left[ (F + G)\sigma_z - G\sigma_x - F\sigma_y \right]
\]

\[
\dot{\varepsilon}_{yz} = \frac{\dot{\varepsilon}_e}{\sigma_e} (L\tau_{yz})
\]

\[
\dot{\varepsilon}_{zx} = \frac{\dot{\varepsilon}_e}{\sigma_e} (M\tau_{zx})
\]

\[
\dot{\varepsilon}_{xy} = \frac{\dot{\varepsilon}_e}{\sigma_e} (N\tau_{xy})
\]

(2.17)

The effective stress (\(\sigma_e\)), following Hill’s criterion (Hill, 1950), is given by,

\[
\sigma_e = \frac{1}{\sqrt{2}} \left[ F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + H(\sigma_x - \sigma_y)^2 + 2(L\tau_{yz}^2 + M\tau_{zx}^2 + N\tau_{xy}^2) \right]^{1/2}
\]

(2.18)
where \( F, G, H, L, M \) and \( N \) are the anisotropic constants.

The set of constitutive equations given by Eqn. (2.17) can be transformed in terms of cylindrical polar coordinates \( r, \theta \) and \( z \) as,

\[
\dot{\varepsilon}_r = \frac{\dot{\varepsilon}}{2\sigma_e} \left[ (G + H)\sigma_r - H\sigma_\theta - G\sigma_z \right]
\]

\[
\dot{\varepsilon}_\theta = \frac{\dot{\varepsilon}}{2\sigma_e} \left[ (H + F)\sigma_\theta - F\sigma_z - H\sigma_r \right]
\]

\[
\dot{\varepsilon}_z = \frac{\dot{\varepsilon}}{2\sigma_e} \left[ (F + G)\sigma_z - G\sigma_r - F\sigma_\theta \right]
\]

\[
\dot{\varepsilon}_\phi = \frac{\dot{\varepsilon}}{\sigma_e} (L\tau_{\phi})
\]

\[
\dot{\varepsilon}_{\tau_r} = \frac{\dot{\varepsilon}}{\sigma_e} (M\tau_{\tau_r})
\]

\[
\dot{\varepsilon}_{\tau_\phi} = \frac{\dot{\varepsilon}}{\sigma_e} (N\tau_{\tau_\phi})
\]

Similarly the effective stress becomes,

\[
\sigma_e = \frac{1}{\sqrt{2}} \left[ F(\sigma_\theta - \sigma_z)^2 + G(\sigma_z - \sigma_r)^2 + H(\sigma_r - \sigma_\theta)^2 + 2(L\tau_{\phi}^2 + M\tau_{\tau_r}^2 + N\tau_{\tau_\phi}^2) \right]^{1/2}
\]

In case of vanishing isotropy i.e. \( F=G=H=L=M=N \), the Eqs. (2.19) - (2.20) reduce to those reported for isotropic case as given in Eqs. (2.17) - (2.18).

### 2.3.3 Creep Law for Al/Al-Alloy Based MMCs

In aluminum based composites, undergoing steady state creep, the effective creep rate \( \dot{\varepsilon}_r \) is related to the effective stress \( \sigma_e \) through well documented threshold stress \( \sigma_o \) based creep law (Park et al, 1990; Mishra and Pandey, 1990; Mohamed et al, 1992; Pandey et al, 1992; Gonzalez and Sherby,
where the symbols $A$, $n$, $Q$, $E$, $R$ and $T$ denote respectively structure dependent parameter, true stress exponent, true activation energy, temperature-dependent Young’s modulus, gas constant and operating temperature.

The threshold stress based creep law given by Eqn. (2.21) may alternatively be expressed as,

$$\dot{\varepsilon}_c = [M(\sigma_e - \sigma_o)]^n$$ \hspace{1cm} (2.22)

where

$$M = \frac{1}{E}(A \exp \frac{Q}{RT})^{1/n}$$

The creep parameters $M$ and $\sigma_o$ appearing in Eqn. (2.22) are dependent on the type of material and the temperature ($T$) of application. In a composite, the dispersoid size ($P$) and dispersoid content ($V$) are the primary variables affecting these parameters. Therefore, these parameters are functions of dispersoid size ($P$), volume content of dispersoid ($V$) and operating temperature ($T$). But the functional relations, describing dependence of $M$ and $\sigma_o$ on $P$, $V$ and $T$, are not known and needs to be established. The values of $M$ and $\sigma_o$ can be extracted from the experimental creep results reported for Al-SiCp composite under uniaxial loading.

### 2.3.4 Value of Stress Exponent in Creep Law

The true stress exponent ($n$) appearing in Eqn. (2.21) is usually selected as 3, 5 and 8, which corresponds to three well-documented creep cases for metals and alloys: (i) $n = 3$ for creep controlled by viscous glide processes of dislocation,
(ii) \( n = 5 \) for creep controlled by high temperature dislocation climb (lattice diffusion), and (iii) \( n = 8 \) for lattice diffusion-controlled creep with a constant structure (Tjong and Ma, 2000).

Pandey and coworkers investigated high-temperature creep behaviour of SiCp/Al (Pandey et al, 1990, 1992) and TiB\(_2\)p/Al (Pandey et al, 1994) composites. The observed creep data was explained by using substructure invariant model given by Sherby et al (1977). Pandey et al (1992, 1994) argued that during creep in these composites the subgrain boundaries are pinned by reinforced particulates like Al\(_2\)O\(_3\) and TiB\(_2\), thereby yielding a stress exponent of 8 and an activation energy of the order of lattice self diffusion.

Gonzalez and Sherby (1993) reanalyzed the creep data of a series of SiC/Al composites and demonstrated that the creep in SiC/Al composites may be described by using substructure invariant model. It is also pointed out by them that the barrier spacing plays a key role during creep of SiC/Al composites.

Mishra and Pandey (1990) analyzed the steady state creep data of SiC/6061Al composites, as reported by Nieh (1984), Nieh et al (1988) and Morimoto et al (1988), and noticed that the substructure invariant model \((n = 8)\) could explain the entire set of data. It is observed that the basic steady state creep mechanism remains unaltered for different shapes and volume fractions of SiC reinforcement, whose influence is confined to threshold stress.

Ma et al (1999) analyzed high temperature creep behaviour of 15 vol% TiB\(_2\)/Al composites in the temperature range 523 K and 673 K and observed that a stress exponent of 8 yields the best linear fit between \( \dot{\varepsilon}^{1/n} \) and \( \sigma \) for this composite. However, for \( n = 3 \) and 5, the data points exhibit a clear curvature at low applied stress. It is further noted that the data points at all the temperatures
may be well described by the substructure invariant model. In a similar way, the
creep behaviour of 20 vol% SiCw/Al, 20 vol% (Al₂O₃+TiB₂)/Al and 20 vol%
(Al₂O₃+TiB₂)/Al-Cu composites may also be reasonably explained by means of
the substructure invariant model (Tjong et al, 1999; Ma and Tjong, 1998; Ma et
al, 1999).

Park et al (1990) in their study on creep behaviour of 30 vol% SiCp/6061Al composite demonstrated that a stress exponent of 5 rather than 8 exhibits the best linear fit between \( \dot{\gamma}^{1/n} \) and \( \tau \), where \( \dot{\gamma} \) and \( \tau \) denote shear creep rate and applied shear stress, respectively. Mohamed et al (1992) in their study on similar composite also revealed that the value of true exponent \( (n) \approx 5 \) and a true activation energy for creep \( (Q) \approx 208 \text{ kJ/mol} \). Therefore, the prevailing creep mechanism in these composites is climb-controlled dislocation creep corresponding to stress exponent of 5. Li and Langdon (1998a) also made similar observations.

Cadek et al (1994a, 1995) reanalyzed the creep data 20 vol% SiCw/6061Al and 20 vol% SiCp/2124Al (Nieh et al, 1988), 30 vol% SiCp/6061Al (Park et al, 1990), 20 vol% SiCp/Al (Pandey et al, 1992), 26 vol% Al₂O₃/Al–5%Mg (Dragone and Nix, 1992) and 30 vol% SiCp/Al (Cadek et al, 1994b) composites and concluded that the relationship between \( \dot{\epsilon}^{1/n} \) and \( \sigma \) in these composites could be linearly described by assuming \( n = 5 \). They also observed that the linear fitting of creep data obtained for SiCp/Al by Pandey et al (1992) could be achieved satisfactorily for various values of stress exponent \( (n = 3, 5 \text{ and } 8) \). It is due to relatively narrow interval of experimental creep rates, which are less than four orders of magnitude.
Pandey et al. (1996) investigated the creep behaviour of 10 vol% SiCp/Al-4% Mg composite and observed that the values of stress exponent \( n = 8 \) and \( Q \approx 224 \text{kJ/mol} \) for this composite at higher stress levels. The activation energy observed is higher than the activation energy for lattice self-diffusion. Li and Langdon (1997b) during re-appraisal of the same data noticed that \( n \approx 3 \) and \( Q \approx 125 \text{kJ/mol} \), which is consistent with the creep controlled by a viscous glide process and the dragging of magnesium atom atmosphere. It is well supported by the fact that the activation energy for diffusion of Mg in an aluminum lattice is \( \sim 130 \text{kJ/mol} \). The marked difference noticed between these two analyses is attributed to the difficulties associated with determining the best value of \( n \) for experimental creep data, which span over a very limited range of strain rates (Li and Langdon, 1999b). The reported fact is in agreement with the earlier work carried out by Cadek and Sustek (1994), which suggests that the creep data of MMCs should extend over at least five orders of magnitude of strain rate to enable an unambiguous determination of the value of true stress exponent \( (n) \).

The creep behaviour of 6061Al and 7005Al matrix composites reinforced with 20 vol% of irregularly shaped Al\(_2\)O\(_3\) indicate that the true stress exponent is close to 3 for Al\(_2\)O\(_3\)/6061Al composite after taking threshold stress into account (Li and Langdon, 1997a, 1998a). In addition, the true activation energy obtained is close to that observed for diffusion of magnesium in aluminum matrix (Li and Langdon, 1997a). The results indicate that the creep in 6061 Al matrix composites is controlled by viscous glide process of dislocations. However, for Al\(_2\)O\(_3\)/7005Al composite, the true stress exponent appears to be \( \sim 4.4 \) and the creep is controlled by a dislocation climb process. Therefore, the creep in metal matrix composites is controlled by creep of matrix alloys and the creep behaviour
of composites could be divided into different two classes: (i) class M (metal-type) with \( n \approx 3 \) and (ii) class A (alloy-type) with \( n \approx 5 \), as observed in solid solutions (Li and Langdon, 1998b). Similar observations are also reported by Li and Langdon (1999a) during reanalysis of the available creep data of 20 vol\% SiCp/2024Al composite (Gonzalez and Sherby, 1993).

It is quite evident from the literature consulted so far that though, some of the researchers (Mishra and Pandey, 1990; Pandey et al, 1992; Gonzalez and Sherby, 1993; Pandey et al, 1994) used a true stress exponent of 8 to describe steady state creep in Al-SiCp,w (subscript ‘p’ for particle and ‘w’ for whisker) composites but a number of other researchers (Park et al, 1990; Mohamed et al, 1992; Park and Mohamed, 1995; Cadek et al, 1995; Yoshioka et al, 1998; Li and Mohamed, 1997; Li and Langdon, 1997a, 1999a) observed that a stress exponent of either ~3 or ~5 rather than 8 provides a better description of steady state creep data observed for discontinuously reinforced Al-SiC composites.

2.4 FUNCTIONALLY GRADED MATERIALS

Functionally graded materials (FGMs) are a new generation of engineered materials that are gaining interest in recent years. FGMs were initially designed as thermal barrier materials for aerospace structural applications and fusion reactors (Hirai and Chen, 1999; Chan, 2001; Uemura, 2003). FGMs also find applications in structural components operating under extremely high-temperature environments (Noda et al, 1998; Librescu and Song 2005). FGMs are the composite materials in which the content of reinforcement is gradually varied in some direction to achieve gradient in properties. Due to graded variation in the content of constituent materials, the properties of FGMs undergo smooth and continuous change from one surface to another, thus eliminating interface
problems and diminishing thermal stress concentrations. As an example FGMs based on ceramic reinforcement in metal matrix, are able to withstand high-temperature environments due to better thermal resistance of ceramic constituents, while the metal constituents enhance their mechanical performance and reduce the possibility of catastrophic fracture. The application of concept of FGMs to Metals Matrix Composites (MMCs) has led to the development of components designed with the purpose of employing selective reinforcement in certain regions where enhanced properties like increased modulus, strength and/or wear resistance are required (Jolly, 1990; Hirai, 1996; Takezono et al, 1996; Koizumi, 1995, 1997; Akira and Watabane, 1997; Pattnayak et al, 2001; Zhu et al, 2001; Kieback et al, 2003).

2.4.1 Applications of Functionally Graded Materials

Functionally graded materials (FGMs) have a high potential for diversified area of applications. Some of the salient applications are reported in Table 2.2 below:

<table>
<thead>
<tr>
<th>Application Areas</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smart structures, where piezoelectric effect could be used to suppress the dynamic or static response, while grading may result in reduced stresses and deformations</td>
<td>Ootao and Tangiwara (2000); Lim and He (2001); Wang and Noda (2001);</td>
</tr>
</tbody>
</table>
2.4.2 Fabrication of FGMs

The various techniques have been proposed for manufacturing of FGMs such as electrophoretic deposition (Put et al, 2003; Vanmeensel et al, 2005), chemical vapor deposition (Kim et al, 2005), spark plasma sintering (Shen and
Nygren, 2002; Tokita, 2003) and centrifugal casting (Biesheuvel and Verweij, 2000; Velhinto, 2003). These methods are used to manufacture FGMs with the properties varying in the thickness direction. To achieve in-plane variation of properties in FGMs ultraviolet irradiation method is used (Lambros et al, 1999).

The reduction of gas consumption and weight of a car are the motivating forces for growth and innovation in the automotive industry. A significant reduction in weight can be achieved by producing cylinder liners in Al matrix composite by employing centrifugal casting technique. The wear resistance of this composite under proper working conditions is superior to cast iron, which is commonly used for the production of liners (Bonollo et al, 2004).

Discontinuously reinforced composites viz. ceramic particles reinforced in aluminum alloy matrix produced by centrifugal casting can be considered as FGMs as they possess varying distribution of reinforcement in the radial direction owing to centrifugal force effect. The different densities of ceramic particles and aluminum alloy matrix lead to a centrifugal separation where the higher density constituent moves towards the outer zones and vice versa. The concentration profile of ceramic particles in the radial direction can be controlled and optimized by adjusting the parameters such as mould rotation speed, mould temperature, content and size of ceramic particles and temperature of molten aluminum. The hardness profile of such FGM is directly proportional to the amount of hard ceramic particles (Kang and Rohatgi, 1996; Liu et al, 1996; Gao and Wang, 2000).

Bonollo et al (2004) used centrifugal casting technique to manufacture cylinder liners in Al matrix to produce functionally Graded (FG) composite (Al-
SiC; Al-Al₂O₃). They also made an attempt to analyze the role of process parameters in centrifugal casting to optimize the reinforcement distribution at the inner surface of the liner. The ideal reinforcement distribution was achieved for some combination of main process parameters, including casting temperature, mould temperature and content of reinforcement.

Kiran et al (2009) used Al/17wt%Si alloy to fabricate and characterize a FG Al/Si in situ material. A specially designed centrifugal casting process was employed for the purpose of fabricating the FGM. The, microstructural characteristic and hardness profile of the fabricated FGMs was evaluated.

2.4.3 Residual Stress in FGMs

When a FGM part is cooled down from its processing or heat treatment temperature to room temperature, residual stresses are generated in the composite due to the mismatch in coefficient of thermal expansion of its constituents, which significantly deteriorates the performance of structural components made of FGMs (Delfosse et al, 1997). The residual stress thus induced is responsible for different yield strength of the composite material in tension and compression.

Povirk et al (1991) investigated the effect of thermal residual stress on mechanical properties and ductility of Al-SiC composites. The behaviour of composite under uniaxial loading was obtained with and without the presence of residual stress. The study reveals that the residual stress has a small effect on the predicted ductility of composite. The residual stresses redistribute as interfacial failure is approached. A close end-to-end fiber spacing gives greater flow strength in compression than in tension and the residual stress induced during thermo-mechanical processing tends to enhance this effect.
Williamson et al (1995) performed FEM studies to investigate the effects of interlayer and creep on the residual stress in nickel-alumina metal ceramic joints. The study revealed that creep strain significantly influences residual stress and joint behaviour above 700 $K$.

Ravichandran (1995) presented one-dimensional computation scheme for estimating thermal residual stresses, which induced during fabrication of a FGM system. The FGM system investigated consists of ceramic ($\text{Al}_2\text{O}_3$) and metal (Ni) phases having variation in only one direction. Several functional forms of gradation of constituents have been examined and it is observed that the linear variation in composition from fully ceramic to fully metal region show the least residual stress. The residual stresses are found to increase when fully ceramic and/or fully metal regions are included in the structure, adjoining the graded zone.

Liew et al (2003) analyzed thermal stresses in FGM hollow cylindrical shells by subdividing the shell into a number of homogeneous sublayers. The stresses and displacements within each sublayer were determined from the homogeneous solution for the layer subjected to the continuity and boundary conditions at the layer interfaces and at the surface of the shell.

Fernandez et al (2004) conducted an experimental and theoretical study of the strength differential effect (SDE) observed during uniaxial tensile/compressive testing of discontinuously reinforced metal matrix composites. The predictions were compared with the experimental data obtained for 6061Al–15 vol% SiCw composites. The analysis of residual stress state was carried out by using an extended Eshelby equivalent inclusion methodology. The study indicates that the
residual stress and the mechanism of load transfer are the origin of SDE in discontinuously reinforced metal matrix composites.

Anne et al (2006) estimated the strength and residual stresses of functionally graded Al₂O₃/ZrO₂ discs by conducting biaxial strength tests. The study indicates that the strength of such discs, is almost double than the disc made of pure Al₂O₃. The increase in strength is attributed to the compressive residual thermal stresses in the Al₂O₃ surface layer, originating due to composition gradient.

Zhang et al (2006) developed analytical models to predict the distribution of thermal residual stresses caused by differential thermal contraction within the multilayer coatings with graded properties and compositions. The study reveals that the residual stress distribution within the functionally graded coating could be adjusted by controlling the compositional gradient or selecting a proper gradient exponent of the coating.

Bouchafa et al (2010) developed an analytical model for estimation of thermal residual stresses, originated due to fabrication of exponential functionally graded material (E-FGM) materials. They assumed that the thermo-mechanical properties of FG layers vary exponentially through the thickness. The study revealed that the magnitude of residual stresses increases when fully ceramic and/or fully metal regions are included in the structure, adjoining the graded zone. However, the effects of temperature dependent elastic modulii and thermal expansion characteristics of constituents on residual stress are small.
2.4.4 Creep in FGMs

In the past few years, the elastic stresses in FGM subjected to thermo-mechanical loading have been analyzed by many researchers (Arai et al, 1990; Erdogan and Wu, 1993; Fukui and Yamanaka, 1992; Hirano and Teraki, 1993; Obata and Noda, 1994; Tanigawa, 1995; Yang, 1998; You et al, 2007). However, the studies pertaining to creep in FGMs are rather scant.

Zhu and Miller (1999) examined creep behaviour of FGM, provided with a thermal barrier coating of Zirconium. In order to examine creep behaviour of this FGM, the thermal gradient was produced by heating the ceramic surface with laser. The ceramic layer was observed to have primary creep. The time, temperature and stress dependent deformation results in coating shrinkage in the loading direction and leads to stress relaxation.

Sadananda et al (1999) investigate compression creep properties of five layered MoSi2-Si3N4 composite at 1200 °C. The five layers consist of 0, 20, 40, 60 and 80 wt% of Si3N4 with the corresponding decrease in MoSi2 content. Each layer is 2 mm thick and has uniform distribution of Si3N4. The study indicates that creep rates in a single layer decrease with increasing content of Si3N4.

Yang (2000) proposed an analytical solution to obtain stresses in a FG layered cylinder by taking into account the elastic and creep behaviour of the material.

Zhai et al (2005) investigated the creep phenomenon of FGMs subjected to high temperature by using computational micromechanical method (CMM). Based on the real microstructure of FG interlayer with different volume fractions,
the emulation experiment was implemented for the creep test numerically and the creep parameters were estimated. The numerical results indicate that the creep phenomenon is obvious not only for the metal-rich interlayers but also for the ceramic-rich interlayers. The creep strain rate of the ceramic/metal interlayer is larger than the corresponding one of pure metal under the same load when the modulus of ceramic phase is lower than that of the metal phase.

2.5 CYLINDER

A cylinder is a commonly used component in various structural and engineering applications. In most of the applications, such as pressure vessel for industrial gases, transportation of high-pressurized fluids and piping of nuclear reactors, the cylinder has to operate under severe mechanical and thermal loads, resulting in significant creep and thereby decreasing its service life (Gupta and Pathak, 2001; Tachibana and Iyoku, 2004; Hagihara and Miyazaki, 2008). As an example, in the high temperature engineering test reactor the temperature reaches of the order of 900 °C (Tachibana and Iyoku, 2004). The piping of reactor cooling system are subjected to high temperature and pressure and may be damaged due to high heat generated in the reactor core (Hagihara and Miyazaki, 2008). Therefore, the prediction of long-term creep behaviour is extremely important for these applications.

2.5.1 Creep in Cylinder made of Monolithic Material

Creep analysis of thick-walled cylinder made of isotropic monolithic material subjected to internal pressure has been presented by Weir (1957) and King and Mackie (1967). Davis and Connelly (1959) studied plastic deformation of a thick-walled hollow cylinder and suggested that for a perfectly plastic material, the solution obtained corresponds to steady state creep condition.
Attia et al (1954) investigated residual stresses produced in cast iron hollow cylinders by the creep relaxation of thermal stresses and the work is related with failures of combustion chamber parts in large internal combustion engines. The study considered thick hollow cylinders subjected to a radial flow of heat due to heating of the bore while the outer diameter was water-cooling for a chosen period of time, during which the thermal stresses got relaxed by creep. The study indicates that the creep strains are confined to the inner layers of the wall and results in a steep residual stress gradient within those layers. The maximum residual stress at the bore is 4.5 times that at the outside diameter.

Rabotnov (1969) studied non-steady creep in rotating solid and hollow cylinders. The analyses assumed that the material of the cylinders is isotropic and remains so during creep.

Rimrott (1959) used generally accepted assumptions of constant density, zero axial strain and distortion energy theory and derived equations for creep rate, creep strains and creep stresses in a thick-walled, closed end, circular hollow cylinder subjected to internal pressure. The cylinder was assumed to be made of a homogeneous isotropic material and the strains were considered large. It has been shown that by considering finite strains, the creep rate in a thick-walled cylinder subjected to internal pressure increases, although, the creep rate in the same material subjected to constant true stress in simple tension is constant. The assumption of infinitesimal strains leads to a prediction of wall thickness that is on unsafe side.

Dev (1961) obtained the solution for a rotating hollow cylinder considering infinitesimal strains, isotropic yield criterion, and creep-strain law while assuming incompressible material.
Finnie (1960) developed equations for prediction of steady state creep stresses and strain rates in a thick-walled tube under combined internal pressure and axial load. The numerical solutions of these equations were obtained for a number of tubes, creep laws and loading conditions. Simple approximate solutions, which apply for the cases in which the additional axial load is small or large relative to the axial load due to internal pressure, are also provided in the study.

Johnson *et al* (1961) presented the analysis of stresses and strains in a thick-walled cylinder subjected to both internal and external pressures. The change in distribution of stress and strain was analyzed with respect to time. The theoretical relations between stress, strain rate and time were derived and applied to a number of metallic alloys at temperatures within their practical working range.

Pai (1967) solved the problem for orthotropic cylinders by using piecewise linear model. In this study it is assumed that the strains are infinitesimal and the deformation is referred with respect to original dimensions of the cylinder.

Sim and Penny (1971) analyzed a number of thick-walled tubes operating under creep conditions for different loadings such as internal pressure, external surface loading and inertia loading.

Simonian (1979) obtained the solution for a long thick-walled cylinder operating in a temperature field and subjected to a centrifugal force, axial force and internal-external pressure. It is observed that the material of the cylinder deforms according to the non-linear theory of heredity.

Ishikawa and Hata (1980) theoretically investigated the stresses in an infinite thick-walled tube subjected to rapid heating at the inner-surface. The
material of the tube was assumed to have temperature dependent properties and characterized by the Ramberg-Osgood’s stress strain relation and Norton’s law for secondary creep.


Mishra and Samanta (1981) investigated finite creep in orthotropic thick-walled cylindrical shells operating at high pressure and temperature. It is observed that the temperature variation has a significant effect on the strain as well as the strain-rate, particularly when the molecular anisotropy of the material is taken into account. Cederbaum and Heller (1989) used theory of thick orthotropic shells to analyze cylinder subjected to dynamic loads. The formulation includes shear deformation and rotating inertia effects similarly to the first-order, shear deformation, laminated plate theory.

Shukla (1997) obtained the expressions for elastic-plastic transitional stresses and axial strain in a compressible cylinder subjected to internal pressure by using Seth’s transition theory. The study indicates that the presence of compressibility having lesser value at the internal surface of the cylinder reduces the axial contraction and the stresses, increases the pressure required for the initial yielding and decreases its value for fully plastic state.

Gupta and Pathak (1999) estimated elastic-plastic stresses in a rotating non-homogeneous cylinder using Seth’s transition theory. The effect of angular speed on non-homogenity was discussed and depicted graphically. The study
indicates that a rotating cylinder made of non-homogeneous material with non-homogeneity increasing radially requires a high percentage increase in angular speed to become fully plastic than to its initial yielding in comparison to a homogeneous cylinder. The non-homogeneous cylinder is much safer compared to a homogeneous cylinder.

Gupta et al (2000) analyzed creep stresses and strain rates in a rotating non-homogeneous thick-walled cylinder by using Seth's transition theory. The study indicates that for a rotating non-homogeneous cylinder, with compressibility increasing radially, the circumferential stress is maximum at the external surface at lesser angular speed but at some higher angular speed it becomes maximum at the internal surface. The compressive value of axial stress, observed at the external surface, increases with an increase in angular speed.

Gupta and Pathak (2001) also used Seth’s transition theory to estimate creep stresses and strain rates in a thick-walled circular cylinder made of compressible/incompressible materials and subjected to internal pressure. It is observed that the maximum circumferential stress at external surface of the cylinder made of incompressible material are relatively higher than those observed in a cylinder made of compressible material. The introduction of thermal effects reduces the stresses at the external surface of cylinder when compared to effect of pressure alone. Strain rates are observed to be maximum at the internal surface of a cylinder made of compressible material, which tend to decrease with radius. In the presence of thermal effects, the creep rates are found to increase more at the internal surface than at the external surface, when the results are compared with cylinder subjected to pressure alone.
Nayebi and Abdi (2002) developed a numerical method to investigate the steady state creep behaviour of thick-walled cylindrical pressure vessels subjected to cyclic pressure and/or temperature.

Shi et al (2002) used finite element analysis to present a method for assessment of creep life of steam turbine parts. The method takes into consideration both steady and unsteady creep stresses of rotors, pipes, casings and valve housings. They used models with stepwise change in thickness to obtain steady creep stresses in rotors and discs while pipes and casings were modeled as thick-wall cylinders. The study presented quantitative calculation along with practical examples.

Filho et al (2004) derived a lower bound to the creep rupture time of internally pressurized cylinders. They described material behaviour by a phenomenological creep rupture theory, which accounts for creep of all the phases and full coupling between the deformation and damage process.

Altenbach et al (2008) analyzed steady state creep in a pressurized thick cylinder made of alloy steel and operating at 600 °C for linear and power law ranges. They presented an approximate solution to illustrate stress redistributions in cylinder as a result of creep process under plain strain condition. The analysis shows that for a certain range of internal pressure both linear and power law creep must be taken into account. The results estimated corresponding to extended constitutive model differs from the classical ones.

Hagihara and Miyazaki (2008) developed a method to predict high temperature short-term creep deformation of piping of the reactor cooling system. They applied creep constitutive equation including tertiary creep based on isotropic damage mechanics to nuclear-grade cold-drawn stainless steel (SUS
Pipe. Finite element analysis was employed for estimation of local creep deformation of the coolant piping under isothermal condition and circumferential temperature gradient. The study indicates that the pipe failure can be predicted by integrating the damage variable into the creep constitutive equation, and failure occurred from the outside of the pipe wall.

Sharma (2009) estimated creep stresses in a non-homogeneous thick-walled cylinder under the combined effect of angular speed and temperature. The study concludes that a cylinder made of less compressible material at the internal surface and highly compressible material at the outer surface is on the safer side of the design for different angular speeds and temperature, as compared to cylinder having highly compressible material at the internal surface and less compressible material at the outer surface.

Sharma et al (2009) used Seth’s transition theory to obtain the elastic-plastic stresses in a transversely isotropic thick-walled rotating cylinder subjected to internal pressure. The study revealed that the yielding occurs at the internal surface of isotropic cylinder at a higher pressure than that observed for transversely isotropic cylinder. Therefore, circular cylinder made of transversely isotropic material is on the safer side of design as compared to cylinder made of isotropic material.

2.5.1.1 Creep in the Presence of Anisotropy

Some materials possess directionally dependent properties and are known as anisotropic materials e.g. wood, rolled sheets of metal etc. Further, on the basis of experimental work conducted by various researchers, it is concluded that initially isotropic material becomes anisotropic during creep deformations. The difference between the predictions based on isotropic and anisotropic theories of
creep goes on escalating as the creep deformation progresses. In this direction, various studies concerning creep deformations of cylinders considered anisotropy of the material.

Davis (1959) analyzed creep in an isotropic thick-walled cylinder subjected to internal pressure. The results obtained were compared with the experimental results. The discrepancy observed between the analytical and experimental results is attributed to the development of anisotropy during creep.

Bhatnagar and Gupta (1969) obtained the solution for thick-walled cylinder made of an orthotropic material and subjected to internal pressure using constitutive equations of anisotropy creep and Norton’s creep law. The stresses and creep rates were estimated under the assumptions of plane strain, generalized plane strain and plane stress conditions. The results estimated for plane strain case were compared with those obtained by Pai (1967). The study indicates that anisotropy has significant effect on the creep behaviour of cylinder.

Bhatnagar and Arya (1974) used finite strain theory to analyze creep behaviour of pressurized (internal, external and both internal as well as external) thick-walled cylinder made of anisotropic material under large strains. It is observed that anisotropy exhibits significant effect on the axial stress, strain and strain rate.

Arya and Bhatnagar (1976) presented creep analysis of thick-walled anisotropic cylinder subjected to combined internal and external pressures by considering elastic strains. The time hardening law was used to obtain the fundamental equations of creep in an orthotropic cylinder. The equations were solved for a particular case of planar anisotropy. A numerical example pertaining to internally pressurized cylinder was worked out and the results obtained for the
anisotropic cylinder were compared with those estimated for isotropic cylinder. It is revealed that the radial and tangential strain rates in an anisotropic cylinder are less than those noticed in an isotropic material at all radial locations and at all times. The strain rates are observed to decrease with increase in material anisotropy. The study also reveals that the difference in values of strain rates at the inner radius is more pronounced, which diminishes with increasing radius.

Bhatnagar et al (1980) analyzed the stress and strain rate distributions in a hollow thick circular cylinder rotating about its own axis with a constant angular speed. The cylinder was assumed to be made of a homogeneous and orthotropic material and the steady state creep was described by Norton’s law. The effect of anisotropy and stress exponent \(n\) was investigated on the creep behaviour of cylinder. The study concludes that the stress and strain-rate distributions in the cylinder are significantly affected by anisotropy of the material and the value of exponent. The effective strain rate observed in anisotropic cylinder is much lower than those obtained for isotropic cylinder. The study suggests that anisotropic material may be beneficial for the manufacture of cylinders because of longer life and ability to sustain larger forces without a risk of failure under creep.

Arya et al (1983) investigated non-steady state creep in a thin circular cylindrical shell made of homogeneous, incompressible and orthotropic material. The results indicate that the presence of anisotropy could lead to a better shell design with lower strains and longer service life.

Bhatnagar et al (1986a) analyzed creep in a rotating thick-walled orthotropic cylinder and also confirmed that the material anisotropy may have beneficial effect on the distributions of stress and strain rate in the cylinder. The results obtained using small strain theory are found to be on unsafe side as
compared to those estimated using finite strain theory. Bhatnagar et al (1986b) further presented the analysis for orthotropic thick-walled cylinder undergoing creep under the combined action of internal and external pressures and rotary inertia. The study indicates that the material with more strength in the radial direction may be beneficial for the design of cylinder due to lower value of effective stress.

2.5.2 Cylinder made of Composite Materials

Under severe thermo-mechanical loads, cylinder made of monolithic materials may not perform well. The excellent mechanical properties like high specific strength and stiffness, and high temperature stability offered by metal matrix composites (MMCs), such as aluminum and aluminum alloy matrix composites reinforced with ceramics like silicon carbide, make them an appropriate material for applications involving high pressure and high operating temperature (Nieh, 1984; Roy and Tsai, 1988; Fukui et al, 1993; Salzar et al, 1996; Gupta et al, 2004). Numerous investigators carried out analysis of stress and strain in composite cylinders. Some of these salient contributions are discussed as below.

2.5.2.1 Analysis of Elastic/Plastic Stresses in Composite Cylinder

Sherrer (1967) conducted a sensitivity analysis of the resin properties on the failure of a multilayered composite cylinder. He obtained the optimum lay-ups in two layers based on equal fiber stress.

Tauchert (1981) studied composite cylinders to obtain an optimal fiber distribution through the thickness to maximize failure pressure and to minimize radial displacement. In this analysis the fibers are assumed to confine only in the hoop direction.
Spencer (1986) conducted experimental studies to characterize thick-walled composite structures and to characterize the stiffness and strain of an anisotropic thick-ring subjected to internal pressure.

Hose and Kitching (1987) analyzed strain in thick mixed walled glass-reinforced plastic pipes subjected separately to internal pressure and bending.

Roy and Tsai (1988) proposed a simple and effective design method for thick composite cylinders. They integrated micro and macro mechanics approach by simple relations, which were adopted to enable the designer to instantly study the sensitivity of the micromechanical variables on the final design. The stress analysis was based on 3-dimentional elasticity and the assumption of generalized plane strain condition. The failure of cylinder was predicted by using a 3-dimentional quadratic failure criterion. In addition, the design parameters and material use efficiency of multilayer closed cylinder subjected to internal pressure was also been studied.

Fukui and Yamanaka (1992) investigated the effects of composition gradient on the strength and deformation of FG thick-walled tubes under internal pressure based on plain strain condition. The work was further extended by Fukui et al. (1993) to include the effect of uniform thermal loading on the performance of a FG thick-walled tube. In this work, the effect of graded components on residual stresses was investigated, in order to achieve the optimum composition gradient generated by compressive circumferential stress at the inner surface.

Wang and Lin (1993) analyzed stresses in composite cylindrical shells rotating at a constant speed about its longitudinal axis. The circumferential stress, motivated by the conventional thin shell theory, was assumed to vary linearly through the thickness of the layer. The radial stress was determined in terms of
circumferential stress through the equilibrium condition by using an average compatibility condition through the thickness of thin layer. The numerical results obtained through the analysis show nearly perfect agreement with the exact solution reported for homogeneous and isotropic cylinders.

Obata and Noda (1994) presented a solution scheme to estimate steady state thermal stresses in a FG hollow circular cylinder to obtain the optimal distribution of reinforcement under different thermal loads.

Salzar (1995) evaluated the effects of material property and fiber grading on the overall mechanical response of metal matrix composite tubes subjected to mechanical loadings by developing a fully elastic-plastic axisymmetric generalized plane strain tube model. Micromechanics algorithm was used to obtain elastic-plastic response of a heterogeneous fiber-reinforced composite cylinder. The study indicates that a tube having 40% fiber distributed uniformly undergoes plastic yielding at the inner radius. However, by grading the fiber content, the same tube behaves elastically under the same pressure loading. The grading also results in 60% weight saving of the tube.

Salzar et al (1996a) presented an exact elastic-plastic analytical solution for an arbitrary layered tube made of MMC and subjected to axisymmetric thermo-mechanical and torsional loadings. The exact solution was developed for transversely isotropic and off-axis elasto-plastic cylindrical shells. The micromechanics method of cells was employed to calculate the effective elastic-plastic properties of the individual layers used in determining the elements of the local and thus global stiffness matrix. The resulting system of equations was solved using Mendelron’s interactive method of successive elastic solution. The
solution strategy was later on (Salzar et al, 1996b) validated by comparing with the available closed-form solutions and FE results.

Tanigawa et al (1997) investigated one-dimensional transient thermal stress problem in a FG hollow cylinder. Horgon and Chan (1999) studied the effects of material inhomogeneity on the response of linearly elastic cylinders. They also addressed the issue of optimizing the material coefficients to minimize the stress in FGM cylinders. Tutuncu and Ozturk (2001) provided closed-form solutions for stresses and displacements in FG cylindrical vessels subjected to internal pressure by using the infinitesimal theory of elasticity.

Tarn (2001) obtained the exact solution for temperature distribution, thermo-elastic distribution and stress fields in inhomogeneous hollow and solid cylinders subjected to axial force, torque at the ends and axially varying surface loads. The material of the cylinder was assumed to be anisotropic.

Jabbari et al (2002) developed a general analysis for one-dimensional steady-state thermal stresses in a FG thick hollow cylinder. The temperature distribution was assumed to vary in the radial direction, with thermal and mechanical boundary conditions applied at the inner and the outer surfaces. The direct method was employed to solve the heat conduction and Navier’s equations by assuming the material properties, except Poisson’s ratio, to be function of radial distance (r).

Chen et al (2002) obtained analytical solution of axisymmetric thermo-elastic problem of uniformly heated, FG isotropic hollow cylinder. The exact solution was obtained by considering modulus of elasticity and coefficient of thermal expansion as a power law function of radial distance. The study indicates that for a homogeneous and isotropic cylinder, no stress occurs when it is
subjected to uniform temperature. The numerical calculations indicate that by choosing a proper value of inhomogeneity parameter, it is possible to design a cylinder to meet some special requirements.


Shao (2005) used a multi-layered approach, based on the theory of laminated composites, to obtain the solutions of temperature, displacements and thermal/mechanical stresses in a FG hollow circular cylinder having finite length and subjected to axisymmetric thermal and mechanical loads. The material properties were assumed to be temperature-independent and radially varying, but homogeneous in each layer. The numerical results for a mullite/molybdenum FG hollow circular cylinder were also presented.

Eraslan and Akis (2006) obtained the plane strain analytical solutions, based on Tresca’s yield criterion, for FG elastic and elastic–plastic pressurized tube problems using small deformation theory. The Young’s modulus and the uniaxial yield limit of the tube material were assumed to vary radially according to two-parametric parabolic forms. The study also investigated various types of stress states i.e. elastic, partially plastic and fully plastic. It is observed that the elasto-plastic response of the FG pressurized tube is significantly affected by the material nonhomogeneity. The non-homogeneous elasto-plastic solution is
observed to reduce to that obtained for homogeneous one, when the material parameters are appropriately selected.

Batra (2008) found closed-form solutions for axisymmetric plane strain radial deformations of a FG hollow cylinder with internal and external hydrostatic pressures. The cylinder was assumed to be made of isotropic and incompressible linear elastic materials with the shear modulus as a general function of radial distance. It is reported that the optimal value of tangential stress in the FG cylinder remains constant and occurs for the linear variation shear modulus with radial distance.

Tutuncu and Temel (2009) estimated axisymmetric displacements and elastic stresses in FG hollow cylinders subjected to uniform internal pressure by using plane elasticity theory and complementary functions method. The material properties such as Young’s modulus and Poisson’s ratio were assumed to be arbitrary functions of radial distance. The radial deformations and stresses in the cylinders were estimated by using several material models reported in the literature.

Nezad and Rahimi (2009) used infinitesimal theory of elasticity to obtain closed form solutions for one-dimensional steady state thermal stresses in a rotating FG pressurized thick-walled hollow circular cylinder by using generalized plane strain and plane stress assumptions. The material properties were assumed to vary non-linearly with the radial distance while Poisson’s ratio was kept constant. The temperature distribution in the cylinder was assumed to be a function of radius, with general thermal and mechanical boundary conditions at the inner and outer surfaces of the cylinder. The direct method was employed to solve the heat conduction and Navier’s equations. The steady state temperature,
displacements and stresses were estimated in FG cylinder depending on an inhomogeneity constant and were compared with those obtained for homogeneous cylinder.

Khoshgoftar et al (2009) studied the thermo-piezoelectric behaviour of a FG thick-walled cylinder subjected to internal and external pressures and operating under a temperature gradient. The mechanical, thermal and piezoelectric properties, except the Poisson ratio, were expressed as a power function of radial distance. The study concluded that by applying a proper distribution of mechanical and thermal properties in the solid structures made of FG piezoelectric material (FGPM), the distribution of stresses, electric potential and electric field can be controlled, which means that FGPM are suitable for applications in sensors or actuators.

Li and Peng (2009) presented elastic analysis of an internally pressurized FG tube with arbitrarily varying material properties. The study indicates that by changing the gradient in FG tube, the radial stress changes a little, while there is a substantial change in the tangential stress. The tangential stress reaches a maximum value at the outer surface when the tube is internally pressurized.

2.5.2.2 Analysis of Creep in Composite Cylinders

Creep behaviour of composites with tailored distribution of reinforcement is of importance in view of their applications at high temperature. In recent years, the problem of creep in cylinders made of FGMs and operating under high pressure and temperature has attracted the interest of many researchers.

Hulsurkar (1981) applied Seth’s transition theory of elastic-plastic and creep deformations to solve the problem of creep in composite cylinders subjected to uniform internal pressure. The generalized expressions for creep transition
stresses were obtained, which, in a special case reduce to those derived by assuming the creep laws.

Hyde et al (1996) presented analytical solutions using creep conditions for the deformations and stresses in thick cylinder made of two-materials and having two-bar structure. The results obtained were compared with those obtained corresponding to single-material solutions and were used to assess the applicability of reference stress and other simple design concepts, established for single-material structures, to two-material components and structures.

Loghman and Wahab (1996) developed a model to estimate creep damages in a thick-walled tube subjected to internal pressure and thermal gradient. The study predicted the changes in creep damage rates during life cycle of the tube due to variation in stresses with time and through-thickness variations. The $\theta$ projection concept was used to predict the long-term creep properties up to rupture and the creep rupture data.

Tzeng (1999) investigated creep and stress relaxation behaviour of fiber-reinforced composite overwrapped cylinders based on anisotropic viscoelasticity. The analysis accounts for ply-by-ply variations of material properties, ply orientations, and temperature gradients through the thickness of cylinders. The study of thermal and mechanical stress relaxation shows that the visco-elastic characteristics of composite cylinders are quite different from the isotropic cylinders. It is also revealed that the Poisson effects resulting from the creep behaviour in the transverse properties of the composite can result in a drastic change of stress and strain state of the cylinder.
Yang (1999) presented an analytical solution to calculate stresses in a jointed FG cylinder by considering the creep behaviour of materials. The analysis is useful for investigating the time and temperature dependence of the stresses.

Yang (2000) proposed an analytical solution to obtain stresses in a FG layered cylinder by taking into account the elastic and creep behaviour of the material. The study considered a two layered cylinder with inner layer made of homogeneous material showing elastic behaviour while the outer layer was assumed to be made of an FGM having creeping material. It is reported that for elastic material the solution is exact, whereas for creeping material the solution is asymptotic. The FEM calculations indicate that the fifth-order asymptotic solution may be used to calculate stresses for a long time creep. However, for a short time creep, the first order asymptotic solution yields the approximate values of stresses.

Tzeng (2002) presented a visco-elastic analysis to investigate the creep and stress relaxation in a rotating thick-walled multilayer composite cylinder. The analysis accounts for layer-by-layer variation of material properties, fiber orientation, temperature and density gradients through the thickness of cylinder. A closed form solution based on the corresponding elastic problem was obtained for a generalized plane strain state.

Muliana and Ali (2006) analyzed creep behaviour and collapse of thick-section and layered composite structures by using a nonlinear visco-elastic and multi-scale modeling framework. The creep analysis of axially compressed laminated cylinder under surface pressure was presented. It is shown that the compressive loading ratio, along with the residual stiffness of the structure after buckling can affect the creep behaviour and the magnitude of critical time for
initiation of unstable response. The proposed models can be used to assess the service life of structures.

Chen et al (2007) studied the creep behaviour of thick-walled cylinders made of FGM and subjected to both internal and external pressures. The asymptotic solutions were obtained on the basis of Taylor expansion series and were compared with the results estimated by using Finite Element analysis (FEA) through ABAQUS software.

You et al (2007) analyzed steady state creep in thick-walled cylinders made of arbitrary FGM and subjected to internal pressure. The stresses and strain rates were calculated by using Norton’s creep law. The impact of radial variations of material parameters was investigated on stresses in the cylinder.

Abrinia et al (2008) obtained analytical solution to obtain radial and circumferential stresses in a FG thick cylindrical vessel under the influence of internal pressure and temperature. The effect of non-homogeneity in FG cylinder was analyzed in the context of achieving the lowest stress levels in the cylinder.

### 2.6 PROBLEM FORMULATION

The cylinder has been receiving considerable attention due to its wide use in pressure vessels, accumulator shells, emergency breathing cylinders, cylinder for aerospace industries, nuclear reactors, military applications and civil structures etc. In some of these applications like pressure vessels for industrial gases or a media transportation of high pressurized fluids and piping of nuclear reactors, it has to operate under severe mechanical and thermal loads. As a consequence, the cylinder undergoes significant creep deformations, thereby, reducing its service life. The excellent mechanical properties like high specific strength and stiffness and high temperature
stability offered by aluminum or aluminum alloy matrix composites containing ceramic reinforcement such as silicon carbide (particles, whiskers or fibers) make them an appropriate material for use in cylinder applications. Keeping this in view, it has been decided to investigate the creep behaviour of a thick-walled cylinder made of aluminum and aluminum alloy matrix composites reinforced with SiC. Since the steady state creep deformation is very large as compared to primary and tertiary creep deformations and covers around 30-40% life of the component. Therefore, the steady state creep behaviour of cylinder made of either Al-SiC or 6061Al-SiC composites subjected to high pressure and temperature have been investigated in the present study.

The literature consulted so far reveals that the problem of determination of elastic and elasto-plastic stresses and deformations in a cylinder made of monolithic material has been solved by several investigators. However, the literature pertaining to analysis of creep in a cylinder that too of composite materials, are rather scant. Therefore, it is felt that a study must be undertaken to investigate creep behaviour of the composite cylinder subjected to high pressure and temperature.

Functionally Graded Materials (FGMs) have great potential for applications involving severe mechanical and thermal loadings due to their unique performance owing to spatial tailoring of constituent phases. Stresses in the FG cylinder have been extensively analyzed with regard to the elastic material behaviour. A very limited number of studies deal with the creep behaviour of cylinders made of FGM. In this light, it becomes imperative to study the creep behaviour of composite cylinder with tailored distribution of reinforcement.
Deformation processing of composites by processes such as forging, rolling or extrusion often results in alignment of reinforcement. The orientation of reinforcement such as whiskers or fibers contributes to reinforcing the composite to different degree depending on their alignment in the direction of application of load. Therefore, a part of the study will also aim to investigate the consequence of anisotropy on the creep behaviour of composite cylinder made of FGM.

The processing of FGMs involves cooling from a high temperature to room temperature, which leads to the development of residual stresses in the composite. The presence of residual stresses is responsible for different yield strength of FGMs in tension and compression. In order to investigate the influence of residual stress on the creep behaviour of composite material, a study must be undertaken.

For the purpose of modeling creep behaviour of the composite cylinder, a threshold stress based law, Eqn. 2.22, with a stress exponent of 5 will be used. This law has been widely adopted for describing steady state creep in aluminum/aluminum alloy based composite materials.

2.7 OBJECTIVES OF THE STUDY

The following objectives have been set for the present study:

(i) To develop a mathematical model of steady state creep in a cylinder made of isotropic uniform composite and to investigate the effect of reinforcement size, reinforcement content and temperature on the creep behaviour of cylinder.

(ii) To analyze steady state creep in an isotropic FG cylinder with different kinds of linear gradient of reinforcement.
To analyze steady state creep in transversely isotropic FG cylinder and to investigate the effect of anisotropy on the creep response of FG cylinder.

To analyzed steady state creep in a FG cylinder in the presence of thermal residual stress.

2.8 RESEARCH METHODOLOGY

In the proposed plan of work, mathematical models will be developed to calculate the distributions of stresses and strain rates in the composite cylinder. The constitutive equations for the composite materials will be developed using different yield criteria. In order to estimate steady state creep stresses and strain rates in the composite cylinder, the equilibrium equation of cylinder will be solved along with the constitutive equations for multiaxial creep.

The entire work shall consist of the following sequence:

(i) Development of constitutive equations for steady state creep under multiaxial stress on the basis of different yield criteria.

(ii) Estimation of material properties depending on the distribution of reinforcement.

(iii) Estimation of material parameters, appearing in creep law, on the basis of available experimental results and regression analysis.

(iv) Development of computer code to compute steady state creep stresses and strain rates in the composite cylinder.

(v) Calculation of creep stresses and strain rates in the composite cylinders from the developed computer code.

(vi) Analysis of the results obtained and drawing appropriate conclusions.