CHAPTER-V: SHORT-RUN EXPECTATIONS.

1. The models in the last two chapters illustrate the rather general point that on the assumption of a convergent process of attaining flow equilibrium corresponding to a given investment level, the succession of such convergent processes can be taken as a description for the long-period path. For such a method, it is not necessary to abstract from the question of expectations, as should be clear from our use of the long-period expected utilisation level explicitly as a parameter in the models. But what is necessary is to separate the long run expectations from the short run expectations, as, (we noted in the first chapter) was also Keynes' method (in General Theory) of handling the question of expectations. Short period expectations may play an important role in the short-period adjustment; and that itself may constitute an important object of study. But as long as they do not lead to instability of the short-period adjustment process, they can be ignored as a first approximation to the discussion of the long period adjustment. Indeed this is also what we have actually done in the last two chapters, by ignoring any discussion of the exact dynamics within the short period, except by way of assuming that it is a convergent process.

But we should now note that we have so far ignored the
question of short-run expectations altogether for simplicity. While it is legitimate within our framework to ignore the role of short-run expectations in the adjustment process within the short period as long as the short-run process is convergent, it is not so to ignore its role, if any, in the long-period adjustment. Short-run expectations may affect the investment level and thus may partly affect the long period path itself. We may make the point clearer by referring to the investment function that we have so far used, namely:

\[
\frac{I_t}{K_t} = \frac{I_{t-1}}{K_{t-1}} + f(u_t) \quad \text{......................... (1)}
\]

\[
f'(u) > 0 \quad \text{................................. 1(a)}
\]

\[
f(1) = 0 \quad \text{................................. 1(b)}.
\]

This function implies that there is a stable expectation about the long-run utilisation level \((u=1)\), and also a given and stable way of reacting to the currently realised level of utilisation. But if we realise that when taking an investment decision, an investor is planning his capacity for future periods, we should admit that the more appropriate formulation should be to relate investment not to current utilisation level but to what it is expected to be in the immediate future. In that case the more appropriate formulation would be:

\[
\frac{I_t}{K_t} = \frac{I_{t-1}}{K_{t-1}} + f(u_t^c) \quad \text{......................... (2)}
\]
\[ f'(u^\sigma) = 0 \quad \text{........................................} \quad 2(a) \]
\[ f(l) = 0 \quad \text{........................................} \quad 2(b) \]

where \( u^\sigma \) is the level of utilisation expected to prevail in the next period. In effect, our earlier formulation amounts to assuming that:

\[ u^\sigma = u, \text{ for all } t \quad \text{...........................} \quad (3) \]

This clearly is a hypothesis suggesting that the current utilisation level would prevail in the immediate next period. Thus, we have ignored the question of utilisation level in the immediate future, which is a kind of short-run expectation, but which surely affects investment and thus the long-period path.

On the other hand, the expectation about the utilisation rate in the next period may affect the producers' (as opposed to the investors') idea about profitability etc., and may thus also affect the adjustment process within the short period. Our point is to suggest that this latter effect of the short-run expectations can be ignored (if this does not lead to non-convergence of the process within the period, of course,) in the analysis of the long period path, and not the former effect.

In the next few sections, we will try to explore the nature of the long-period path when we consider the short-period expectations explicitly. The general method we will pursue will be to assume that even if the short-period expectations are not fulfilled, the state of long-
period expectation is not altered. 

2. We will retain the same short period adjustment hypothesis, namely:

\[ I_t = S_{t+1} \] ............................................(4)

But since we are introducing additional complications, we will simplify the analysis by assuming that the adjustment occurs through changes in utilisation alone. The level of prices or income shares remain unchanged. Accordingly, we use the following saving hypothesis:

\[ S_t = s.Y_t = s. \frac{Y_{t-t-1}}{K_{t-1}} = s. \frac{u_t}{V} K_{t-1} \] .................(5)

(We are maintaining our convention that output \( Y_t \) is produced from capital stock \( K_{t-1} \).)

The investment function is now written as:

\[ \frac{I_t}{K_t} = \frac{I_{t-1}}{K_{t-1}} + f(u_t) \] .........................(2)

to take explicit account of short run expectations. The model can be closed by now specifying a hypothesis about short-period expectation formation. Since we are interested in the analytical implications of our procedure rather than in the realism of the model, we choose (rather

1. This is a simplifying assumption. By complicating the analysis, it is possible to describe the situation where long period expectation gets continuously revised in view of the disappointment of short-run expectations. However it is difficult to provide a convincing enough formulation for such revisions to use in any actual model. For Keynes' classification of different situations involving short-run and long-run expectations, see, J.A.Kregel: Economic methodology in the face of uncertainty; Economic Journal, June 1976.
arbitrarily) a hypothesis which is in frequent use in the literature. Namely:

\[ u_t^G = u_t + L(u_t - u_{t-1}) \] ...........................................................(6)

This is the so-called extrapolative expectation hypothesis, which suggests that expectation for the coming period is formed by extrapolation from the last period's and the current period's observed values. We shall refer to the parameter \( L \) as the parameter of short-run expectations.

Now equations (4) and (5) give:

\[ \frac{I_t}{K_t} = \frac{s}{v} \cdot u_{t+1} \] ..................................................(7)

Using (7) in the investment function (2), we have:

\[ \frac{s}{v} (u_{t+1} - u_t) = f(u_t) \] ..................................................(8)

Substituting for \( u_t^G \) in (8) from (6), we have:

\[ \frac{s}{v} (u_{t+1} - u_t) = f \left( u_t + L(u_t - u_{t-1}) \right) \] ..................................................(9)

Using, as earlier, a linear form for the function \( f \), namely:

\[ f(u_t^G) = bu_t^G - b, \quad b > 0, \]

we have:

\[ \frac{s}{v} (u_{t+1} - u_t) = b \left( u_t + L(u_t - u_{t-1}) \right) + b = \left( -\frac{v}{s} \right) u_t - L \] ..................................................(10)

or \[ u_{t+1} - \left( 1 + \frac{bv}{s} + \frac{bvL}{s} \right) u_t + \frac{bvL}{s} u_{t-1} = \left( -\frac{v}{s} \right) \] ..................................................(11)

Equation (11) is a second-order difference equation path for actual utilization rate \( u_t \). It is interesting to
note that the nature of the path now depends on the parameters of long-run as well as the short-run expectations, both, i.e., \( b \) and \( L \) respectively. The following properties of the path are relevant for our purpose:

(i) Putting \( u_{t+1} = u_t = u_{t-1} = \bar{u} \), the steady state solution is obtained as:

\[
\bar{u} = \frac{-\left(\frac{vb}{S}\right)}{1-(1+\frac{vb}{S}+\frac{vb}{S}L)\frac{vbL}{S}} = 1.
\]

So that steady state solution once again coincides with the long-period norm expected.

(ii) The characteristic roots of (11) are:

\[
(1 + \frac{bv}{S} + \frac{bvL}{S}) \pm \sqrt{(1 + \frac{bv}{S} + \frac{bvL}{S})^2 - \frac{4bvL}{S}}
\]

\[\frac{2}{2}\]

We can easily see that if both \( b > 0 \) and \( L > 0 \), the absolute value of the dominant root is necessarily larger than unity, so that the long-period path does not converge to the steady state.

(iii) If \( b \) and \( L \) are allowed to be negative, interesting possibilities involving oscillations or steady convergence are opened up. The exact conditions for such possibilities in the present case are rather complicated expressions in the parameters, and we are omitting them since they do not convey any immediate interpretation. But this clearly shows the relevance of the short-run expectations for the long-period path. In particular we should note that given
any \( b > 0 \), it is possible to find out a negative \( L \), numerically large enough to assure convergence to the steady state. This implies that even a destabilising long-run behaviour can be reversed by a suitable type of short-run expectation.

3. We had earlier criticised Marshall's method on the grounds that in his formulation the long-run equilibrium configuration is known to the agents before hand. Consequently, much of his dynamics is a rather unnecessary story of how an industry gets into a state, which the agents confidently expect to prevail eventually. In the models that we have discussed so far, in this chapter and earlier, the steady state coincides with the state held as the long-run expected state. It appears at first sight that, therefore we have made an unnecessary methodological detour by replacing Marshall's notion of a long-run normal state by Keynes' notion of a long-run state of expectation. However we will demonstrate below that the steady state need not be always determined by the state of long-run expectations alone. If we allow for the role of short-run expectations in influencing the long-run path, then the determinants of the short-run expectations may also enter into the determination of the steady state configuration. This particular property did not surface in the model just considered above, because of the particular short-run expectation hypothesis used there. We will illustrate below that depending on how the short-run expectations are formed, the steady state
configuration and its stability properties may change quite dramatically.

4. For illustrating the point we will use various hypotheses regarding the formation of short-run expectations in the model considered above in this chapter. The rest of the model, i.e. the investment and savings function and the short-period equilibrium condition will be used unchanged.

Consider the short-run expectation hypothesis:

\[ u_t^\varphi = u_{t-1}^\varphi + M (u_t - u_{t-1}) \]  \hspace{1cm} (12)

i.e. a corrective expectation hypothesis, where each period the expectation is of the rate of last period corrected by adding a multiple (or submultiple) of the rate by which actual utilisation has changed in the last period.

Suppose we use this hypothesis together with our earlier descriptions of saving, investment and the short-period adjustment equation, namely:

\[ s_t = s \cdot \frac{u^c_t}{v} \cdot K_{t-1} \]  \hspace{1cm} (5)

\[ \frac{I_{t+1}}{K_{t+1}} = \frac{I_t}{K_t} + f(u^\varphi_{t+1}) \]  \hspace{1cm} (2)

and \[ I_t = s_{t+1} \]  \hspace{1cm} (4).

Then we have from (5), (2) and (4), as earlier:

\[ \frac{S_v}{v} (u^\varphi_{t+1} - u_t) = f(u^\varphi_t) \]  \hspace{1cm} (8).

The equation (8) can be converted into a difference equation in one variable \( u_t \), if we can substitute for \( u^\varphi_t \) in terms of \( u_t \). In view of the hypothesis (12), this is...
not possible immediately, because (12) expresses $u_t^\sigma$ as a function of $u_{t-1}^\sigma$, apart from $u_t$ and $u_{t-1}$. We can however express $u_t^\sigma$ in terms of $u$ alone from (12) as follows:

$$u_t^\sigma = u_{t-1}^\sigma + M(u_t-u_{t-1})$$

Therefore,

$$(u_t^\sigma-Mu_t)=(u_{t-1}^\sigma-Mu_{t-1})=(u_{t-2}^\sigma-Mu_{t-2})=\ldots \ldots = u_0^\sigma-Mu_0.$$ 

So that, $u_t^\sigma = Mu_t + (u_0^\sigma-Mu_0)$ .............(13)

Where $u_0$ and $u_0^\sigma$ are the actual and expected utilisation at the initial time period, $t=0$.

Substituting (13) now in (8) we have:

$$\frac{s}{v} (u_{t+1}^\sigma-u_t^\sigma) = f[Mu_t + (u_0^\sigma-Mu_0)].$$

Using now the linear form for the function $f$, namely:

$$f(u^\sigma)=bu^\sigma+b,$$

we have:

$$\frac{s}{v} (u_{t+1}^\sigma-u_t^\sigma) = b [Mu_t+(u_0^\sigma-Mu_0)]-b.$$ 

or

$$u_{t+1}^\sigma = (1+ \frac{bvN}{s}) u_t^\sigma + \frac{bv}{s} (u_0^\sigma-Mu_0)=\ldots \ldots$$

Equation (14) above is a first order linear difference equation in $u_t$. The solution for it can be written as:

$$u_t^\sigma = (1+ \frac{bvN}{s})^t \cdot \left( \frac{u_0^\sigma-1}{N} \right) + \left( \frac{Mu_0-u_0^\sigma+1}{H} \right)$$

The solution (15) is quite interesting because the steady state for $u$ is given by:

$$\frac{Mu_0-u_0^\sigma+1}{N}$$

.............(16)
Which is in general different from the long-period expected level $u=1$. The steady state configuration here is determined by the short-period state of expectations $u_0^s$ at the initial period, in addition to, the objective initial condition $u_0$ and the long-run expectation level $u=1$. This solution is also interesting because for the same model, the same state of long-run expectations, and the same objective initial conditions $u_0$, the economy may stabilise into different steady states if the initial short-period expectations ($u_0^s$) were different. Since $u_0^s$ is largely governed by happenings before $t=0$, those happenings are as summarised in $u_0^s$ become a major determinant of the steady state configuration.

The stability condition is:

$$\left| 1 + \frac{bM}{\alpha} \right| < 1$$

If $b$ and $M$ are both positive the model is necessarily unstable as earlier.

It is easy to see that cases involving $M<0$ or $b<0$ may sometimes lead to stable monotonic growth, or oscillations, stable or unstable. Stability now depends on the nature of both short-run and long-run expectatons.

5. By introducing different hypotheses about the formation of short-run expectations, various interesting possibilities about the long-run path can be generated, still remaining within the broad framework of our method.
and the assumption of a given state of long period expectations. We will produce one more illustration.

Suppose we use as a short-run expectation hypothesis the following relation:

\[ \frac{u_t^e - 1}{u_{t-1}^e - 1} = \frac{u_t - 1}{u_{t-1} - 1} \quad \text{(17)} \]

This form implies that the expectation about the deviation of utilisation from the long-run expected rate (=1) changes in the same proportion as the deviation of actual utilisation from this rate changes.\(^\dagger\)

This relation (17) can be used to express \( u_t^e \) in terms of \( u_t \) as follows:

\[ \frac{u_t^e - 1}{u_{t-1}^e - 1} = \frac{u_t - 1}{u_{t-1} - 1} ; \]

or

\[ \frac{u_t^e - 1}{u_t - 1} = \frac{u_{t-1}^e - 1}{u_{t-1} - 1} = \frac{u_{t-2}^e - 1}{u_{t-2} - 1} = \cdots = \frac{u_0^e - 1}{u_0 - 1} ; \]

so that,

\[ u_t^e = \frac{u_0^e - 1}{u_0 - 1} + u_t \cdot \frac{u_0^e - 1}{u_0 - 1} \quad \text{(18)} \]

We can now use (18) in the equation (8), namely:

\[ \frac{\Delta}{\Delta} (u_{t+1} - u_t) = f(u_t^e) \quad \text{(8)} \]

to get,

\[ \frac{\Delta}{\Delta} (u_{t+1} - u_t) = f \left( \frac{u_0^e - 1}{u_0 - 1} + u_t \frac{u_0^e - 1}{u_0 - 1} \right) \]

Using the linear form for \( f(u^e) \) as \( f(u^e) = bu^e - b \) as before, we have,

\[ \frac{\Delta}{\Delta} (u_{t+1} - u_t) = -b + b \left( \frac{u_0^e - 1}{u_0 - 1} \right) + u_t \left( \frac{u_0^e - 1}{u_0 - 1} \right) \]

\(^\dagger\) This corresponds to unitary elasticity of expectations in the continuous case.
This simplifies to yield:

\[ u_{t+1} = u_t \left( 1 + \frac{bv}{s} \left( \frac{u^g_0 - 1}{u^g_0} \right) \right) - \frac{bv}{s} \left( \frac{u^g_0 - 1}{u^g_0} \right) \] ..........................(19)

Which is a first order linear difference equation in \( u_t \), the actual utilisation rate. The solution for (19) can be written as:

\[ u_t = l + (u_0 - l) \left( 1 + \frac{bv}{s} \left( \frac{u^g_0 - 1}{u^g_0} \right) \right)^t \] ....................(20)

The steady state for the path is thus \( u=1 \), the long-run expected rate. But this time it is the stability property of the system which has become related with the initial subjective state of short-run expectations, \( u^g_0 \). The steady state is stable if

\[ \left| 1 + \frac{bv}{s} \left( \frac{u^g_0 - 1}{u^g_0} \right) \right| < 1. \]

Even with \( b > 0 \), it is now possible to conceive of situations where the model stabilises at \( u=1 \). The interesting feature of this model is that the stability or instability of the path depends (for given values of \( b \)) on the initial state of short-run expectations. Since \( u_0 \) is largely determined by the past history of the system before \( t=0 \), the model may be stable if we start from one calendar date as \( t=0 \), but unstable for another calendar date as starting point. Thus there is no "absolute" criterion of stability within the set of parameters of the model; it may be stable or
unstable in relation to a given initial condition.

6. The models in this chapter have been designed to illustrate the point, that steady state configuration or the conditions of convergence to the steady state become related to the initial conditions (both objective and subjective), if we allow certain types of short-run expectation behaviour and use the method of tracing the long period path by stringing together successive flow equilibria through time. Since a steady state configuration, or a stationary state is not the starting point for this method, the configuration of a steady state or the possibility of its eventual realisation becomes tied to the exact history of the system.

We should, however, note that these features, may equally well arise whenever the steady-state configuration is not pre-supposed. For example in a method belonging to what we have described as the Swedish tradition in chapter I, the same features may sometimes be observed if certain types of short-run expectation hypotheses are used. In a short appendix to this chapter, we will illustrate this by a two-sector model of accumulation set up in the Swedish tradition.
APPENDIX TO CHAPTER V

A TWO-SECTOR MODEL: AN ILLUSTRATION IN THE
SWEDISH TRADITION

We will use a two-sector framework to illustrate the point made in the text of this chapter in section 6.

Write the product market equilibrium condition equating investment and savings as:

\[ \frac{dM}{dt} (p.M) = S \] ................................. (1)

where \( p \) is the price of machines, \( M \) is the machine stock in use and \( S \) denotes nominal savings.

Equation (1) can be written as:

\[ \frac{dM}{dt} + \frac{dM}{p} = S \] ................................. (2)

Assuming profits alone are saved, and there is no savings, either from wages or from capital gains, we write:

\[ S = e_c \cdot r \cdot M \] ................................. (3)

where \( e_c \) is the propensity to save out of profits, which is realised at the rate \( r \) on the capital value \( pM \).

On substitution of (3) in (2) and some manipulation, we have:

\[ \frac{dM}{p} + \frac{dM}{M} = e_c \cdot r \] ................................. (4)

If we take \( \frac{dM}{M} \) on the left hand side to denote the planned rate of addition to capital stock, the equation (4) can be interpreted as a process of adjustment of capital
goods prices, given the planned rate of accumulation $\frac{\dot{M}}{M}$.

Given $\frac{\dot{M}}{M}$, the equation can be treated as a complete
description of price dynamics, if $r$ is expressed in terms
of $p$, the price of machines.

We can effect this by using the competitive price
formation equations (5) below:

\begin{align*}
1 &= l_c w + a_c \cdot r \cdot p, \\
p &= l_1 w + a_1 \cdot r \cdot p. \\
\end{align*} \tag{5}

where the consumption good price has been taken as the
numeraire. The symbols $l_c$ and $l_1$ denote the labour re-
quirement for producing unit amounts of consumption good and
machine respectively; while $a_c$ and $a_1$ denote the machine
units used for unit production of these two commodities
respectively. The symbol $w$ stands for the wage rate in
terms of consumption good. The assumption necessary to
use equations (5) in our framework is that the rate of
profit instantaneously adjusts to the price to reach the
competitive equilibrium rate.

Eliminating $w$ from equations (5) we have:

\[ r = \frac{l_1 - l_c \cdot p}{pZ} \] \tag{6}

where $Z = a_c l_1 - a_1 l_c$, is an index of the difference in capital
intensity between the two sectors.

Substituting (6) in equation (4), we have:

\[ \frac{\dot{p}}{p} + \frac{\dot{r}}{r} = s_c \left( \frac{l_1 - l_c \cdot p}{pZ} \right) \] \tag{7}
Given the short period adjustment equation (7), our method as developed so far is the following: (i) Take a given rate of planned growth of stock \( \dot{M} \). (ii) Allow the process captured by (7) to come to an end (given suitable conditions), so that \( p \) and \( r \) stabilise. (iii) Now redefine \( \dot{M} \) on the basis of these equilibrium values of \( p \) and \( r \), to pose the next short-period process.

Suppose now we use a different hypothesis appropriate for the Swedish tradition, so that as soon as there is a price change equating savings and investment in the manner of equation (7), new plans of investment is formulated so as to redefine \( \dot{M} \). To effect this in the model in a particularly simple way we make a few drastically simplifying assumptions. For the ex-ante real growth function, we hypothesize:

\[
\dot{M} = s_c \cdot r^g, \quad \text{................................. (8)}
\]

where \( r^g \) is the expected profit rate in the next temporary equilibrium period.\(^1\)

Secondly, we assume that the relation between \( r \) and \( p \) is known to the investors and accordingly:

\[
r^g = \left( \frac{1_i - l_o \cdot p^g}{p^g \cdot Z} \right) \quad \text{................................. (9)}
\]

\(^1\) This means that planned investment \( \dot{M} = s_c \cdot r \cdot M \), i.e. is equal to the expected savings out of profits. One way of rationalising this is to say that the firms invest by borrowing from the banks an amount which they expect to be able to pay back from their savings out of profits expected.
where \( p^\sigma \) is the expected price of the machine goods.

Finally we assume a constant elasticity hypothesis about formation of short-period expectations:

\[
\frac{d\sigma}{\sigma} = b \frac{d\sigma}{\sigma}, \quad b > 0 \quad \text{(10)}
\]

Equation (10) would imply:

\[
p^\sigma = B p^b
\]

where \( B \) is a constant of integration and can be evaluated by using initial values:

\[
p_0^\sigma = B p_0^b
\]

so that \( B = \frac{p_0^\sigma}{p_0^b} \quad \text{(11)}\)

If we substitute (8) and (9) in equation (7) we have:

\[
\frac{d\sigma}{\sigma} = \frac{s c_1}{Z} \left( \frac{1}{p} - \frac{1}{p^\sigma} \right) \quad \text{(12)}
\]

Using (10) in (12):

\[
\frac{d\sigma}{\sigma} = \frac{s c_1}{Z} \left( \frac{1}{p} - \frac{1}{B p} \right) \quad \text{(13)}
\]

or \( \frac{d\sigma}{\sigma} = \frac{s c_1}{Z} - \frac{s c_1}{B Z} \cdot p^{-b} \quad \text{(14)} \)

The stability of this path is globally assured if and only if:

\[
\frac{d\sigma}{\sigma} = 0, \quad \text{i.e. if} \quad \frac{s c_1}{Z} (1-b) p^{-b} = 0.
\]

Since \( s, c_1 \), and \( B \) are all positive, the condition boils down to:

\[
\frac{1-b}{Z} = 0.
\]
The stability condition thus depends not only on the structural parameter \( z \), but also on the parameter of the short-period expectation formation.

The steady state price corresponds to \( \dot{p} = 0 \), and by solving (14) for \( \dot{p} = 0 \), we get,

\[
p = (B) \frac{1}{1-B}, \quad \text{where} \quad B = \frac{p_o}{p_0}
\]

This means that given the stability requirements, the system would stabilise to a level of prices determined by the initial situation of the model, both subjective and objective.