CHAPTER IV: ANOTHER ILLUSTRATIVE MODEL II.

1. In the present chapter, we continue with illustrating our method, with a different kind of short-period adjustment behaviour. In the last chapter we considered the unit period adjustment as taking place through changes in capacity utilisation and the price level, the latter affecting all prices and all categories of income uniformly. A different, and perhaps somewhat more interesting unit period adjustment in prices has been considered in the so-called Cambridge literature on growth and distribution, namely, a change in the income shares by categories of income resulting from price changes.\(^1\) In the present chapter, we consider the long-period path where the adjustment in the unit period occurs through changes both in the degree of capacity utilisation and the income shares.

2. The dynamics within the short period is generated, as in the last chapter, by a certain amount of investment expenditure at the beginning of the period, generating the matching amount of saving at the end of the period. But the saving is assumed to be brought into line with investment, now, through a change in the income shares and utilisation.

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Accordingly, we write the saving function in a way to capture the effect of income share change on it. In particular,

\[ S = s_p P + s_w W \]

\[ = s_p P + s_w (Y - P) \]

\[ = \left[ s_p \frac{P}{Y} + s_w (1 - \frac{P}{Y}) \right] Y \]

\[ = [s_p D + s_w (1-D)] Y. \]

Where \( D = \frac{P}{Y} \), the ratio of profits to income both at current prices.

Expressing \( Y \) as \( \frac{K}{v} \cdot u \), where \( K \) is the capital stock (at current prices), \( v \) is the capital to full-capacity output ratio (at constant price) and \( u \) is degree of utilisation as defined in the last chapter, we finally have:

\[ S = [s_p D + s_w (1-D)] \frac{K}{v} u. \]

By fixing the time subscripts appropriately, and maintaining our convention that capital stock installed up to the beginning of a period is productive in that period, we have:

\[ S_t = [s_p D_t + s_w (1-D_t)] \frac{K_{t-1}}{v} u_t. \]

By rearrangement:

\[ S_t = \left[ (s_p - s_w) \cdot D_t + s_w \right] \frac{K_{t-1}}{v} u_t \] .............................. (1)

This is the saving function we will use for our analysis.
The investment behaviour of the last chapter is retained for the time being\(^1\), so that following the same notations as there, we write:

\[
\frac{I_t}{K_t} = \frac{I_{t-1}}{K_{t-1}} + f(u_t) \hspace{1cm} (2)
\]

and \(f'(u_t) > 0\) \hspace{1cm} (2)(a)

\(f(1) = 0\) \hspace{1cm} (2)(b)

The saving-investment equality, serving as the unit period equilibrium condition is (as in the last chapter):

\[I_t = S_{t+1}\]

which, is now rewritten, in view of the equation (1) above, as:

\[I_t = [(s_p - s_w)D_{t+1} + s_w] \frac{K_t}{v} u_{t+1} \hspace{1cm} (3)\]

Equation (3) yields:

\[\frac{I_t}{K_t} = [(s_p - s_w)D_{t+1} + s_w] \frac{u_{t+1}}{v} \hspace{1cm} (4)\]

Substituting (4) in (2), we have:

\[
\frac{(s_p - s_w)}{v} \left[ D_{t+1} u_{t+1} - D_t u_t \right] + \frac{s_w}{v} (u_{t+1} - u_t) = f(u_t) \hspace{1cm} (5)
\]

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1. Later in this chapter we will alter the specification about investment behaviour to study the implications of different investment behaviour on the long-period path.

2. Investment and capital stocks now are however taken to be measured at current prices. This does not create any difference in the dimensionality of \(f(u_t)\) as compared to the same function used in chapter 3 above, because of our use of the proportions \(I/K\) in the formulation.
This is a single difference equation in two variables 

u and D, and the system can be closed by providing some 

other relation between D and u.

3. In the last chapter, we had left the short-period 

relation between price-level and utilisation (or between 

their short period adjustment speeds) open, and studied 

its impact on the long-period path through a suitably defined 

parameter m. Here also we could proceed along similar 

lines and study the nature of the family of paths under 

various values of some suitably defined parameter 

describing the relation between u and D.

In the present case, however, we can make use of some 
hypothesis regarding the relation between u and D available 
in the literature and make our model slightly more 
definitive.

We propose to use the aggregative relation between 
the rate of profit and utilisation as suggested for example 
by Steindl\(^1\) or by Baran and Sweezy\(^2\), which utilises the 

notions of a fixed component of cost, fixed at all 
utilisation levels and a variable component which varies 

with the level of utilisation. Steindl's own formulation, 
upon some simplification to fit our context, proceeds as 
follows:

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1. See J.Steindl: Maturity and Stagnation in American 
Capitlism: Chapter IX, especially, p.110.

2. See P.A.Baran and P.M.Sweezy: Monopoly Capital 
\[ P = Y - (hY + j), \quad h > 0; \quad j > 0. \]

where \( hY + j \) is the cost of producing the output \( Y \). Here \( hY \) is the variable component in total cost, whereas \( j \) is the fixed part.\(^1\)

Accordingly, the rate of profit, \( r \) can be written as:

\[ r = \frac{P}{K} = \frac{X(1-h)}{K} - \frac{j}{K} = (1-h) \frac{u}{V} - \frac{j}{K}. \]

Steindl takes \( j/K \) as constant for any given level of capital stock \( K \), so that the rate of profit can be written as:

\[ r = au - q, \quad a > 0; \quad q > 0. \quad \text{...................... (6)} \]

i.e. as a linear function of the utilisation rate, given the capital stock.

For our purpose, however, it seems appropriate to slightly modify the relation between the rate of profit and utilisation. In our model, where the question of the long-run stability is left open, one possible outcome of the long-period path may be an exploding behaviour of utilisation rate period after period. It seems reasonable

\(^1\) In describing the saving function we have taken profits and wages as the only income categories. As a result, when we describe profits as \( P = Y - (hY + j) \), the whole of \( (hY + j) \) i.e. the cost of production has to be attributed to wages. We can interpret this by saying that a part of wage payment is in the nature of fixed commitment irrespective of output level, and the other part varies with actual production. The analysis can be made more realistic by, having, say, an income category like rentier-income, which could account for a part of \( j \), and including rentiers' saving in the saving function. We avoid this in order to keep the analysis formally simple.
to suppose that at some high levels of utilisation, a further rise in utilisation rate may reduce the profit margin - an idea that would go well with the usual description of average cost curves as U-shaped.

Accordingly, in the relation: Total Cost = hY+j, we now replace the constant h with a function h(Y),

\[ r = \frac{P}{K} = \frac{Y-h(Y)}{K} = \frac{Y}{K} - h(Y) - \frac{1}{K}. \]

Since \( Y = \frac{K}{V} u \), we can therefore write \( r \) as a function of utilisation with the following characteristics:

\[ r = r(u) \]

\[ \frac{dr}{du} > 0, \text{ for } u < u^\sigma \]

\[ \frac{dr}{du} = 0, \text{ for } u = u^\sigma \]

\[ \frac{dr}{du} < 0, \text{ for } u > u^\sigma \]

where \( u^\sigma \) is the level of utilisation at which the rate of profit is at its maximum, given the capital stock.\(^1\)

The relation (7) can be converted into one between the income shares \( D \), and utilisation level \( u \) as follows:

\[ D = \frac{P}{Y} = \frac{P}{K} \cdot \frac{K}{Y} = r \cdot \frac{Y}{u}. \]

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1. The utilisation level \( u^\sigma \) should not be confused with the long-period desired utilisation rate, taken as unit by definition. The profit rate - maximising utilisation level \( u^\sigma \) depends on the existing capital stock and may in general change from one period to another. The long-run desired rate \( u^\sigma \), on the other hand, is independent of the amount of existing capital stock. The latter is a subjectively held desired level, as opposed to the former which is determined by technology.
Therefore
\[ \frac{du}{v} = r = r(u) \] ................................. (8)

We can use (8) in our equation (5) to obtain:
\[ \left( s_p - s_w \right) \left[ v \cdot r(u_{t+1}) - v \cdot r(u_t) \right] + \frac{s_w}{v} (u_{t+1} - u_t) = f(u_t) \]  or
\[ \left( s_p - s_w \right) [r(u_{t+1}) - r(u_t)] + \frac{s_w}{v} (u_{t+1} - u_t) = f(u_t) \]  ............ (9)

4. Qualitative properties of equation (9) can be noted. Firstly, putting \( u_t = u_{t+1} = \bar{u} \), we have from (9), \( 0 = f(\bar{u}) \).

which solves as \( \bar{u} = 1 \).

This means that the path has a steady solution that coincides with the long-period desired utilisation level.

Secondly differentiating (9) with respect to \( u_t \), we have:
\[ \left( s_p - s_w \right) [r' \cdot \frac{du_{t+1}}{du_t} - r'] + \frac{s_w}{v} \left( \frac{du_{t+1}}{du_t} - 1 \right) = f' \]

which simplifies to:
\[ \frac{du_{t+1}}{du_t} = 1 + \frac{f'}{\left( s_p - s_w \right) r' + \frac{s_w}{v}} \]  ................................. (10)

The stability properties of the model is directly related to the value of the expression \( \frac{du_{t+1}}{du_t} \). To examine such properties, we may first examine the expression \( \left( s_p - s_w \right) r' + \frac{s_w}{v} \) that appears in the right hand side of (10).

The sign of this expression can be ascertained by referring back to our discussion of the stability of the short period adjustment process. The equilibrium condition for the short
period adjustment process was written as: (please refer to

equation (3) above, in this chapter):

\[ I_t = [(s_p - s_w) D_{t+1} + s_w] \frac{K}{v} \cdot u_{t+1} \]

This can be rewritten as:

\[ I_t = [(s_p - s_w) r(u_{t+1}) + \frac{s_w}{v} u_{t+1}] K_t \]

Our suggestion about the nature of short-period adjust-

ment was that if the investment level rises, the utilisation

level also rises to ultimately generate the requisite

amount of savings. Therefore, a necessary condition for

the stability of the short period adjustment process is

that the right hand side of (11) should rise when the rate

of utilisation rises. In formal terms this means that the

short period process is stable, if

\[ \frac{d}{du} [(s_p - s_w) r(u) + \frac{s_w}{v} u] > 0. \]

i.e., if: \((s_p - s_w) r' + \frac{s_w}{v} > 0 \) .................ll (a)

Since our analysis of the long period path depends

on the stability of each of the underlying short-period

processes, the condition ll(a) is satisfied, by assumption,

everywhere on the long-period path.

If we now look back at equation (10), we find that,
in view of ll(a) and that \( f' > 0, \frac{du_{t+1}}{du_t} > 1, \) everywhere

on the long-period path, and at the steady state in

particular. This means that the path is unstable both at

the steady state as also at points away from it.
5. We can now look at the methodological implications of the model. We have assumed the convergence of the short period process; the long period process has been kept completely separate from it. The convergence of the long-period path to a steady state crucially hinges on the postulated nature of the investment decision. We can easily check that if we replace our specification 2(a) by:

\[ f' < 0 \]

then the long period path stabilises at the steady rate \( u = 1 \).

We should also note that condition 11(a), assuring the stability of the short-period process is neither very restricting nor very novel. It is just a variant of the condition

\[ \frac{dS}{dy} > \frac{dI}{dy}, \]

which is necessarily used in the postulation of the short-period adjustment processes involving a variable aggregate saving propensity.\[ ^{1} \] In our model, by construction, \( \frac{dI}{dy} = 0 \), within the short period. On the other hand,

\[ S = (s_p - s_w)P + s_wY. \]

Therefore,

\[ \frac{dS}{dy} = (s_p - s_w) \frac{dP}{dy} + s_w = (s_p - s_w) \frac{d}{dy} (rE) + s_w. \]

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\[= (s_p - s_w) K r' \cdot \frac{du}{dy} + s_w = (s_p - s_w) K r' \cdot \frac{v}{K} + s_w \]

\[= (s_p - s_w) v r' + s_w. \]

therefore,

\[(s_p - s_w) r' + \frac{s_w}{v} > 0 \quad \text{.................11(a)}\]

implies, \((s_p - s_w) v r' + s_w > 0.\]

implies, \(\frac{dS}{dy} > \frac{dI}{dy}.\]

This means that in this particular case, the stability of the short-period adjustment is not a very demanding requirement; but by separating out the short-period and the long-period processes, it has been possible to isolate the factors that may be more important in contributing to the instability of the system in the long-run (e.g. nature of investment decisions). We should note one curious point here. Since \(r\) first rises and then falls with \(u\), it is possible that for very large values of \(u\), \(r'\) becomes negative. If the negativity is so pronounced as to make the inequality 11(a) invalid, then the short-period adjustment also breaks down. Thus on a path where \(u\) is increasing every period, it is possible that after a point, our analysis based on a stable short period adjustment will break down.

6. At the level of substance, the method of taking the short-period process as stimulated by a given investment level to be stable, and then looking at the long-period path as the succession of such processes stimulated by the
successive investment decisions, clearly reflects the view that the nature of the long-period path is principally shaped by the nature of investment behaviour. We should emphasize that our exercises are basically aimed at looking at the methodological implications of such a viewpoint. One important implication that seems to have already emerged is a separation between the conditions assuring stability of the short-period process and that of the long-period path. The stability of the long period path, in these models, seem to be entirely dependent on the nature of the postulated investment behaviour - i.e. on whether the feed backs of the relevant short-period adjustment variables, on the ex-ante growth rate follow a converging sequence in time. In the usual literature on growth theory the preoccupation with the full employment path has resulted in the neglect of an analysis of the long-period stability of this sort, and the emphasis has been solely on analysing the conditions for the stability of the short-run process. Therefore, we propose to examine this aspect further by using a different investment behaviour hypothesis in the framework of the above model.

1. Since in our framework, the single periods are generated by successive investment decisions, clearly the long-period stability will hinge on the nature of the investment function. In models where the investment level is taken exogenously by full-employment consideration, or by "animal spirits", it is natural to focus on the question of the short-period stability.
7. To illustrate the nature of influence of alternative investment hypotheses on the long-period path, we will now replace the investment behaviour as written in equation (2) above, by the hypothesis that current investment is influenced by the level of consumption demand in the last period. If we make a simplifying assumption that the propensity to save out of wage income is nil, then in terms of the notation used in this chapter, the consumption demand at the end of the t-th period can be written as:

\[ C_t = Y_t(1-D_t \cdot s_p) = \frac{K_{t-1}}{v} \cdot u_t(1-D_t \cdot s_p). \]

Consumption demand as a proportion of full-capacity output at the end of period t, is then:

\[ \frac{C_t}{K_{t-1}/v} = u_t(1-D_t \cdot s_p). \]

We will use this expression to determine the amount of revision in the ex-ante growth rate. But we will make a simplifying assumption that the propensity to save out of wage income is nil, then in terms of the notation used in this chapter, the consumption demand at the end of the t-th period can be written as:

\[ S_{t+1} = s_p \cdot D_{t+1} \cdot \frac{K_t}{v} \cdot u_{t+1} \]  

Therefore the equilibrium relation:

\[ S_{t+1} = I_t \]

1. We should note that this expression is dimensionally similar to the rate of growth \( I/K \).
now gives
\[ \frac{I_t}{K_t} = \frac{s}{v} \cdot D_{t+1} \cdot u_{t+1} \].................................(14)

using (14) in (12) we have:
\[ \frac{s}{v} \left[ D_{t+1}u_{t+1} - D_t u_t \right] = f[u_t(1 - s_p D_t)] \].................................(15)
since by definition:
\[ r = \frac{MD}{V} \].................................(16)
equation (15) can be written as:
\[ s_p (r_{t+1} - r_t) = f[u_t - v \cdot s_p \cdot r_t] \].................................(17)

If for simplicity we now use a linear relation\(^1\) between
u and r of the form:
\[ r_t = mu_t - n, \ m > 0, \ n > 0, \ \text{for all} \ t \].................................(18)
then we can write (17) as:
\[ s_p \cdot m(u_{t+1} - u_t) = f[u_t - v \cdot s_p \cdot r_t] \].................................(19)

To solve equation (19) explicitly, we take again a
linear form for the function \(f\), i.e.
\[ f(x) = Ax + B, \ A > 0. \]
Since by hypothesis, \(f(u=1)=0\), we have
\[ A[1-v s_p (m-n)] + B = 0. \ i.e. \ B = A[v s_p (m-n)-1]. \]

On substituting this linear form for \(f\), and substituting
for \(r\) in terms of \(u\) on the right hand side, equation (19)
takes a particularly simple form:

\(^1\) See section 3 above, in the present chapter.
This is a linear first-order difference equation with constant coefficients and the solution can be written as:

\[ u_{t+1} = u_t \left[ 1 + A \left( \frac{1}{s_p \cdot m} - v \right) \right] - A \left( \frac{1}{s_p \cdot m} - v \right) \] ..................(20)

where \( u_0 \) is the utilisation level at \( t=0 \).

8. This path i.e. equation (21), has the same steady state solution \( u=1 \). But now the stability conditions for the path are very different. The condition for stability here is:

\[ \left| 1 + A \left( \frac{1}{s_p \cdot m} - v \right) \right| < 1 \] ......................(22)

We can easily check that our postulation that \( A > 0 \) (regarding investment behaviour) and that the short-period adjustment is stable does not unambiguously state whether the path is stable or unstable. The stability of the short period adjustment, in view of equation (14) requires that savings should increase with utilisation, as the latter increases in response to an increase in investment.

From equation (13),

\[ S = \frac{k}{v} \cdot s_p \cdot \Delta u \], omitting time subscripts.

Therefore, stability requires that

\[ \frac{ds}{du} = \frac{k}{s_p} \cdot \frac{\Delta u}{\Delta t} > 0 \], i.e. \( k \cdot s_p \cdot m > 0 \).

This condition is necessarily satisfied if \( s_p \) and \( m \) are separately both positive.
As a result, the short-run stability which only requires that \( s_{p \cdot m} \leq 0 \), leaves the magnitude \( \left( \frac{1}{s_{p \cdot m}} - v \right) \) open in respect of its sign. Since \( A \) is by hypothesis positive, a necessary condition for long-period stability is

\[
\left( \frac{1}{s_{p \cdot m}} - v \right) < 0.
\]

a condition which is neither unambiguously fulfilled nor violated by the stability of the short-period adjustment process.

The exact significance of the stability condition for the long-period path can be understood if we rewrite the condition (22) as:

\[
|s_{p \cdot m} + A(1-v \cdot s_{p \cdot m})| < s_{p \cdot m} \tag{23}
\]

by multiplying both sides of (22) with the positive quantity \( s_{p \cdot m} \).

We can easily check that the expression \( A(1-v \cdot s_{p \cdot m}) \) stands for the revision in the planned rate of growth due to an unit change in utilisation level in the last period.\(^1\) Also \( s_{p \cdot m} \) is the revision in growth rate that gives rise to a unit change in utilisation in course of the short period adjustment.\(^2\)

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1. This is obtained by differentiating the expression \( f[u_t(1-s_{p \cdot D_t})] \) with respect to \( u_t \). For \( f' \) we are using \( A \), as per our linear hypothesis.

2. This is so because, expected growth rate = \( S/K \) and \( \frac{dS}{du} = s_{p \cdot m} K \), so that \( \frac{dS}{du} = s_{p \cdot m} \).
Thus condition (23) essentially means that if there is a change in growth rate \( (a_{p,m}) \) in the current period, then the change in growth rate necessary in the next period (due to utilisation change that the earlier revision of growth rate) should be smaller in magnitude. This is how the long-period condition guarantees the tapering off of the successive changes in growth rate, to ultimately stabilise at a steady state:

It is clear that this condition essentially refers to the feed-back of the last period's decision about growth-rate on the current period's, and it thus refers more or less completely to the nature of the investment function. This feed-back, of course, also depends on how much utilisation would change in response to a given investment, and therefore on the saving function. But what is important is that the stability of the long-period path depends on a set of conditions very distinct from those ensuring the stability of the short-period process.