ABSTRACT

Theory of special functions has a long and varied history with immense literature due to their applications in solving various problems arising in physical, biological and engineering sciences. Special functions have an origin in the solution of partial differential equations satisfying certain prescribed conditions. At present special functions are defined in several ways notably by power series, generating functions, infinite products, integrals, difference equations, trigonometric or orthogonal function series. Eminent mathematicians notably Euler, Legendre, Gauss, Jacobi, Weierstrass, Kummer, Riemann, Ramanujan worked hard to develop special functions like Bessel functions, Whittaker functions, Gauss hypergeometric function and the polynomials that go by the names of Jacobi, Legendre, Laguerre, Hermite etc. The Gaussian hypergeometric function $\frac{p}{q}$ and its special cases are commonly used in applied mathematics and mathematical physics. Since $\frac{p}{q}$ diverges for $p > q + 1$, in an attempt to give meaning to it in this case, MacRobrtand Meijer introduced the E-function and the G-function respectively.

The main object of this thesis is to obtain numerous applications of fractional derivative operator concerning special functions by introducing new classes and deriving new properties. Our finding will provide interesting new results and indicate extensions of a number of known results. In this thesis we investigate a wide class of problems. The third chapter is concerned to the following points:(i)Recurrence relations of the generalized M-series. (ii)Integral representation and fractional calculus of the generalized K-function.(iii)The functions $\Omega(c,\nu, p, q, x)$ and $\Omega(c,\mu, p, q, x)$ within conditions of the generalized K-function and its properties using fractional integral and differential. we derive recurrence relation of the generalized M-series. The outcome is represented in the structure of a theorem. we consider solutions which are derived as $a^{\alpha}\beta^{\beta}$ $p M_{q}(z)$ is the generalized M-series. There are solutions to the problems and equations to be considered for the generalized Mittag- Leffler functions. The fourth chapter devoted to the use of the generalized M-Series process that will expand the
application of the process to linear / nonlinear differential equations with fractional order. A
innovative solution is built in the form of power series. The fractional derivatives provided the
outcomes show that the procedure introduced at this time is extremely efficient and practical to
solve fractional order linear differential equations. Some examples that express the presentation
and effectiveness of the comprehensive process for solving linear and nonlinear differential
equations through fractional generalized M-series. We will provide the information how to
explain a number of differential equations through fractional level by means of the generalized
M-series. The generalized M-series process suggests that the linear term $y(x)$ is decaying by the
countless sequence. In fifth chapter we deals with the study of the K-function, is regulated its
various properties, including Laplace transform, Beta transform, Mellin transform, Whittaker
transform, generalized hypergeometric function, integral representation of Mellin-Barnes and its
relation with Fox’s H function and the Wright’s hypergeometric function. The relationship
between this function and the fractional integral Riemann-Liouville operators is also derived.