CHAPTER-IX

VOLTAGE STABILITY ANALYSIS WITH UPFC - INDICES
METHOD

9.0 INTRODUCTION

The voltage stability in a power system is characterized by stability in a bus or in a line. The case study has been carried out on IEEE 5 bus test system [Stagg and El-Abiad 196884] for the critical bus bar identification using L-Index method and critical transmission line identification using Line Stability Indices - $L_{MN}$, FVSI, and LQP. These indices provide reliable information about proximity of voltage instability—the index value 0 indicates stable and no load position whereas value 1 indicates voltage collapse. Enhancement of voltage stability margin has been demonstrated with UPFC installed in the critical line at increased values of L-index and Line indices.

Section 9.1 develops the bus stability L-index formula and its algorithm. Section 9.2.1 derives the expression for line stability index- $L_{MN}$. Section 9.2.2 derives the formula for line stability index FVSI. Section 9.2.3 derives the expression for line stability index LQP.

Section 9.3 presents the simulation analysis and results of stability indices method with and without UPFC.

Section 9.4 summarizes on the indices method and its advantages.
9.1 BUS STABILITY L-INDEX FORMULATION.

A voltage stability index based on the solution of power flow equations is proposed by [Kessel P et. al., 1986]. The L index is a quantitative measure of estimation of distance of the actual state of the system to the stability limit. \( L_j \) is a local indicator that determines the bus bars from where collapse may originate. The L index varies in a range between 0 (no load) and 1 (voltage collapse). The distance of L closer to value 0 indicates a better stability margin. For stability, the index \( L_j \) magnitude limit should not exceed the maximum value 1, for any of the j load buses. A simple line model shown in Fig 10.1 is considered here for analysis.

![Fig 9.1 Simple 2 Bus System- for L-index](image)

Bus 1 is the generator node. Bus 2 is the load node.

\[
Y_{11}V_1 + Y_{12}V_2 = I_1 = \frac{S_1}{V_1^*} 
\]

(9.0)

Let \( V_0 = \frac{Y_{12}}{Y_{11}} V_2 = \frac{Y_{12}}{Y_{12} + Y_{10}} V_2 \)  

(9.1)
\[ V_i^2 + V_0 V_1^* = \frac{S_i^*}{Y_{11}} = a_i + jb_i \quad (9.2) \]

\[ V_i = \sqrt{\frac{V_{0}^2}{2} + a_i \pm \sqrt{\frac{V_{0}^4}{4} + a_i V_0^2 - b_i^2}} \quad (9.3) \]

\[ = \frac{S_i^{*}}{Y_{11}} \left( r \pm \sqrt{r^2 - 1} \right) \quad (9.4) \]

where \( r = \frac{V Y_{11}}{2S_i} \quad (9.5) \)

Equation 9.3 can be written as \( |S_i - Y_{11} V_i| = V_0 V_i Y_{11} \quad (9.6) \)

i.e. all states having a constant amplitude \( V_1 \) lie on circles in the complex \( S_1 \) plane with \( Y_{11} V_1^2 \) deciding the centre and \( Y_{11} V_0 V_1 \) decides the radius.

**Stability Criterion**

If \( V_1 \) is varied in the permissible region, union of the circles so formed forms the solution. The peripheral curve of these circles is the stability limit. At the border line, the two solutions of Equation 9.3 have to coincide.

\[ \pm \sqrt{\frac{V_{0}^4}{4} + a_i V_0^2 - b_i^2} = 0 \quad (9.7) \]

Therefore R.P of \( \frac{V_1}{V_0} = 0.5 \quad (9.8) \)

\[ L = \left| 1 + \frac{V_0}{V_1} \right| = \left| \frac{S_i}{Y_{11} V_1^2} \right| = r + \sqrt{r^2 - 1} \quad (9.9) \]
Thus a factor to determine the proximity of the actual system state to the stability limit is got. Solving above equation, $S_1$ curves of constant $L$ values have an elliptical shape. For any load bus $j$

$$V_j = \hat{a}_{jia} Z_{ji} I_i + \hat{a}_{jia} F_{ji} V_i$$  

$$= V_j^2 + V_{oj} V_j^* = \frac{S_j^*}{V_{jj}^2} \quad (9.11)$$

$$V_{oj} = - \hat{a}_{jia} F_{ji} V_i \quad (9.12)$$

The nodal voltage $V_j$ is affected by load bus power $S_j$ and bus powers of other load buses. The Voltage $V_{oj}$ varies only slightly since the generator voltages are almost constant for varying loads.

$L_j \leq 1$ is not to be violated for any load bus $j$.

$$L_j = \left| 1 + \frac{V_{oj}}{V_j} \right| = \left| \frac{S_j^*}{V_{jj} V_j^2} \right| \quad (9.13)$$

For a large interconnected network, the L-Index describing the stability of the complete system and is given by $L_{\text{Max}} (L_j); j \in \text{all load buses}$.

$$L_j = \text{Max}(j \in a) \left| 1 - \frac{\hat{a}_{jia} F_{ji} V_i}{V_j} \right| \quad (9.14)$$

$V_i, V_j =$ complex voltage of generator buses and load buses.

$F_{ji} =$ is obtained from the $Y$ bus matrix.
\[ F_{ij} = \frac{Y_{iG} V_{G} + Y_{ij} V_{j} + Y_{Lj} V_{L}}{Y_{Lj} V_{L}} \]  
\[ (9.15) \]

\[ I_L, I_G, V_L, V_G = \text{Complex Current and Voltage Vectors at the Generator Buses and Load Buses}. \]

\[ Y_{GG}, Y_{GL}, Y_{LG}, Y_{LL} = \text{are associated portions of Y bus matrix}. \]

\[ F_{LG} = Y_{LL}^{-1} Y_{LG} \]  
\[ (9.16) \]

\[ (9.17) \]

**9.2 LINE STABILITY INDICES FORMULATION**

**9.2.1 Line Stability Index- \( L_{MN} \)**

A line stability index based on a power transmission concept in a single line, by equating the discriminant of the roots of quadratic equation of voltage or power greater than zero is developed by [Moghavemmi et al., 1998]. When equated to less than zero the roots become imaginary leading to voltage instability. The value as it becomes closer to 1 is a measure of voltage collapse.

An interconnected system is reduced to a single line network and the overall system stability is then assessed. This concept is applied to each line of the network. Let us take a single line of an interconnected network as shown below. Any line of the interconnected network can be represented in this manner.
\[ \mathbf{V}_s = \mathbf{R} + j\mathbf{X} \]

\[ \mathbf{V}_r \]

\( S_s = P_s + jQ_s \quad \quad \quad \quad \quad \quad \quad \quad S_r = P_r + jQ_r \)

**Fig 10.2 Two Bus Equivalent - \( L_{MN} \)**

\( \mathbf{V}_s, \mathbf{V}_r \) - sending and receiving end complex voltage = \( V_s \mathbf{d}_1 \), \( V_r \mathbf{d}_2 \)

\( S_s, \ S_r \) - Complex power on the sending and receiving end.

\( P_s, \ Q_s, \ P_r, \ Q_r \) - active and reactive power on the sending and receiving end.

\( \delta_1 - \delta_2 = \delta \) - difference between sending and receiving end angle.

\( \mathbf{Z} = \mathbf{R} + j\mathbf{X} \) - Complex line impedance.

The complex power flow at the receiving and sending end is expressed as:

\[ S_s = V_s V_r (q- \ d_1 + d_2) - \frac{V_r^2}{Z} Dq \] \hspace{1cm} (9.18)

\[ S_r = \frac{V_r}{Z} Dq - \frac{V_s V_r}{Z} D(q + d_1 + d_2) \] \hspace{1cm} (9.19)

\[ P_r = \frac{V_s V_r}{Z} \cos(q - d_1 + d_2) - \frac{V_r^2}{Z} \cos q \] \hspace{1cm} (9.20)

\[ Q_r = \frac{V_s V_r}{Z} \sin(q - d_1 + d_2) - \frac{V_r^2}{Z} \sin q \] \hspace{1cm} (9.21)

Equating \( \delta_1 - \delta_2 = \delta \) in Eqn (9.19) and solving for \( V_r \):

\[ V_r = \frac{V_s \sin(q - d) \pm \sqrt{[V_s \sin(q - d)]^2 - 4ZQ_r \sin q}}{2 \sin q} \] \hspace{1cm} (9.22)
With $Z \sin \theta = x$;

$$V_R = \frac{V_s \sin(q-d) \pm \sqrt{[V_s \sin(q-d)]^2 - 4xQ_R}}{2\sin q}$$  \hspace{1cm} (9.23)

Real value of $V_R$ is obtained in terms of $Q_R$ with real roots. Hence

$$\{[V_s \sin(q-d)]^2 - 4xQ_s \sin q\}^3 = 0$$  \hspace{1cm} (9.24)

Or

$$\frac{4xQ_R}{[V_s \sin(q-d)]^2} = L_{MN} \leq 1.00$$  \hspace{1cm} (9.25)

$L_{MN}$ is called as the line stability index of the line. The stability index for each line of the interconnected grid is found. The system is said to be stable until the index $L_{MN}$ is less than one, 1. When the index approaches the value 1 the system start losing its stability and voltage collapse is initiated.

### 9.2.2 Line Stability Index – FVSI

Fast Voltage Stability Index to predict voltage collapse and contingency analysis caused by line outages is introduced by [Ismail Musrin et al., 2002]. Line with the highest index value close to 1.0 is regarded as the most critical line. At this operating point the active and reactive load connected to the bus is considered as the maximum permissible load. The bus is ranked in the order of maximum permissible load in ascending order. Smallest maximum permissible load is ranked on the top and identified as the most critical bus requiring reactive compensation.
Fast voltage Stability Index is determined as explained below. The two bus equivalent of the grid is shown in the Fig 10.3.

\[ S_S = P_S + jQ_S \]
\[ S_R = P_R + jQ_R \]

**Fig 10.3 Two Bus Equivalent - FVSI**

The current flowing in the line is given by

\[ I_{line} = \frac{V_S D_0 - V_R D_d}{R + jX} \]  \hspace{1cm} (9.26)

The complex power at receiving end bus is \( S_R = V_R I^*_{line} \)

Therefore

\[ I_{line} = \frac{S_R}{V_R} = P_R - jQ_R / V_R D_d \]  \hspace{1cm} (9.27)

Equating the two we get

\[ \frac{V_S D_0 - V_R D_d}{R + jX} = P_R - jQ_R / V_R D_d \]  \hspace{1cm} (9.28)

\[ V_S V_R D_d - V_R^2 D_d = (R+jX) (P_R+jQ_R) \]  \hspace{1cm} (9.29)

Equating the Real part and the Imaginary part,

\[ V_S V_R \cos \delta - V_R^2 = R P_R + X Q_R \]
\[ -V_S V_R \sin \delta = X P_R - R Q_R \]  \hspace{1cm} (9.30)

Therefore the quadratic equation of \( [V_R] \) is obtained as

\[ V_R^2 - (R/X \sin \delta + \cos \delta) V_S V_R + (X + R^2/X) Q_R = 0 \]  \hspace{1cm} (9.31)

The roots of the above quadratic equation is as follows

\[ V_R = \frac{(R/X \sin \delta + \cos \delta)V_S \pm \sqrt{(R/X \sin \delta + \cos \delta)V_S}^2 - 4(X + \frac{R^2}{X})Q_R}{2} \]  \hspace{1cm} (9.32)
The real roots of the above quadratic equation are got by equating the discriminant greater than or equal to zero.

\[
\left( \frac{R}{X} \sin d + \cos d \right)V_s^2 - 4 \left( X + \frac{R^2}{X} \right)Q_R^3 \geq 0
\]  

(9.33)

Or

\[
\frac{4Z^2Q_RX}{V_s^2 (R \sin d + X \cos d)^2} \leq 1 ; \delta \approx 0, \ R \sin \delta \approx 0, \ X \cos \delta = X.
\]  

(9.34)

\[
FVSI_{km} = \frac{4Z^2Q_m}{V_k^2X}
\]  

(9.36)

The FVSI is now represented as above for the sending bus k and receiving bus m as, with Z= line impedance, X= line reactance, \(Q_m\)= Reactive power at the receiving end, \(V_k\) = sending end voltage. The value of FVSI close to 1 indicates collapse.

**9.2.3 Line Stability Index-LQP**

A line stability factor based on a power transmission concept in a single line is derived by [Mohamed A et.al, 1989]. The single line representation is as figured in the above. Here the power equation is equated.

\[
\frac{X}{V_s^2} Q_s^2 - Q_s + \left[ \frac{X}{V_s^2} P_s^2 + Q_R \right]
\]  

(9.37)

The line stability factor LQP is obtained equating the discriminant of the reactive power roots at the sending end bus to be greater than or equal to zero.
LQP = \(4 \left[ \frac{X}{V_s^2} \right] \left[ \frac{X}{V_s^2} P_s^2 + Q_s \right]\)  

(9.38)

LQP value if less than 1 the stability is maintained.

### 9.3 INDICES SIMULATION PROCEDURE AND RESULTS

The stability analysis has been carried out on IEEE 5 bus test system. Initially a power flow is conducted for the base case without UPFC to obtain the power flow solution. Results obtained from the power flow solution are used to calculate the indices values for each line in the system.

Based on the ranking of indices shown in Table 9.1 bus 4 has been identified as the weakest bus. The load at bus 4 is given an increase in steps of per unit until peak load of 148%. The indices values have increased with load variation as shown in Table 9.1.

Table 9.2 shows the stressed conditions of the lines for the base load condition. The line that presents the largest index with respect to a bus is considered as the most critical line of that bus. Line 7 is noted to have the largest index value under base case. The line stability indices have shown an increase in the value when the load at bus is increased in steps to peak load 148%. The index value of critical line 7 computed by three methods for the peak load has increased significantly as shown in Table 9.3.

Hence the line 7 which connects bus 4 to bus 5 with largest line stability index at base case and peak load has been identified as the most critical line. Based on the stability indices of lines, voltage collapse can be accurately predicted. Any further increase on the
load will cause the line to have an index value greater than 1.0 resulting in the entire system instability.

Table 9.1 Bus Stability Indices - before UPFC Installed

<table>
<thead>
<tr>
<th>Load Bus No</th>
<th>Voltage Stability L –Index-Base Load</th>
<th>Voltage Stability L –Index-Peak Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.299</td>
<td>0.313</td>
</tr>
<tr>
<td>4</td>
<td>0.657</td>
<td>0.813</td>
</tr>
<tr>
<td>5</td>
<td>0.432</td>
<td>0.542</td>
</tr>
</tbody>
</table>

Table 9.2 Line Stability Indices – at Base Load- with UPFC

<table>
<thead>
<tr>
<th>Line</th>
<th>From Bus</th>
<th>To Bus</th>
<th>L_{mn}</th>
<th>FVSI</th>
<th>LQP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0.299</td>
<td>0.267</td>
<td>0.324</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
<td>0.543</td>
<td>0.512</td>
<td>0.590</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
<td>0.411</td>
<td>0.426</td>
<td>0.432</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>4</td>
<td>0.109</td>
<td>0.099</td>
<td>0.210</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>5</td>
<td>0.396</td>
<td>0.266</td>
<td>0.412</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>4</td>
<td>0.408</td>
<td>0.407</td>
<td>0.421</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>5</td>
<td>0.678</td>
<td>0.586</td>
<td>0.653</td>
</tr>
</tbody>
</table>
Table 9.3 Line Stability Indices – at Peak Load- without UPFC

<table>
<thead>
<tr>
<th>Line</th>
<th>From Bus</th>
<th>To Bus</th>
<th>L&lt;sub&gt;mn&lt;/sub&gt;</th>
<th>FVSI</th>
<th>LQP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0.317</td>
<td>0.280</td>
<td>0.347</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
<td>0.570</td>
<td>0.538</td>
<td>0.631</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
<td>0.451</td>
<td>0.447</td>
<td>0.462</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>4</td>
<td>0.201</td>
<td>0.118</td>
<td>0.305</td>
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<tr>
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<td>2</td>
<td>5</td>
<td>0.415</td>
<td>0.319</td>
<td>0.441</td>
</tr>
<tr>
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<td>3</td>
<td>4</td>
<td>0.497</td>
<td>0.489</td>
<td>0.505</td>
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<tr>
<td>7</td>
<td>4</td>
<td>5</td>
<td>0.843</td>
<td>0.698</td>
<td>0.803</td>
</tr>
</tbody>
</table>

Table 9.4 Bus Stability Indices- with UPFC

<table>
<thead>
<tr>
<th>Load Bus</th>
<th>Voltage Stability L -Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.154</td>
</tr>
<tr>
<td>4</td>
<td><strong>0.263</strong></td>
</tr>
<tr>
<td>5</td>
<td>0.327</td>
</tr>
</tbody>
</table>
9.4 SUMMARY AND DISCUSSION

The voltage stability analysis using indices method is aimed to determine the voltage stability condition, identify the weak bus and load ranking. Bus 4 has exhibited highest index value of 0.657 under base case and index value of 0.813 at peak load of 148%.and therefore determined as the most critical bus.

Line 7 has been found to exhibit the largest index value computed by all three indices methods and therefore identified as the most critical line. This line varying to any load change at bus 4 or bus 5 has shown significant increase in the index value from base load to peak load.

<table>
<thead>
<tr>
<th>Line</th>
<th>From Bus</th>
<th>To Bus</th>
<th>( L_{\text{max}} )</th>
<th>FVSI</th>
<th>LQP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0.196</td>
<td>0.267</td>
<td>0.324</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
<td>0.457</td>
<td>0.512</td>
<td>0.590</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
<td>0.378</td>
<td>0.426</td>
<td>0.432</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>4</td>
<td>0.100</td>
<td>0.099</td>
<td>0.210</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>5</td>
<td>0.253</td>
<td>0.266</td>
<td>0.412</td>
</tr>
<tr>
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<td>3</td>
<td>4</td>
<td>0.315</td>
<td>0.307</td>
<td>0.3411</td>
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<tr>
<td>7</td>
<td>4</td>
<td>5</td>
<td>0.289</td>
<td>0.275</td>
<td>0.284</td>
</tr>
</tbody>
</table>
Hence UPFC has been chosen to be placed in line 7. The bus stability index and line stability indices have shown remarkable decrease in the index value indicating enhancement of voltage stability margin with UPFC installed as depicted in Tables 9.4 and 9.5. The UPFC parameters are set at rated limits to achieve maximum power transfer in this line.