Hybrid Quantum-inspired Evolutionary Algorithms

6.1 Introduction

The hybrid constraint handling technique (HCT) presented in chapter 5 is a better alternative as compared to Feasibility Rules in handling constraints for solving Constraint Optimization Problems (COPs). In this chapter, the Adaptive Real-coded Quantum-inspired Evolutionary Algorithm (ARQEA) developed in chapter 4, is integrated with HCT, and is known as Hybrid Adaptive Real-coded Quantum-inspired Evolutionary Algorithm (HARQEA). The proposed algorithm has been tested on a standard set of six benchmark engineering design optimization problems that have been attempted by several researchers. These problems are quite diverse in nature.

6.2 Proposed Algorithm

The proposed algorithm has two populations A and B as it is employing HCT for handling constraints. Feasibility Rules are used for handling constraints in population A whereas Adaptive Penalty Factor method is employed for handling constraints in population B. In this implementation, population A has been given primary preference and is evolved in every iteration, whereas population B is evolved only when population A fails to improve its best solution in any iteration. This introduces diversity in the search process of population A only when required and also reduces the overall computational burden.

The pseudo code for the proposed algorithm is given in figure 6.2.1, which is explained as follows:

a) Initialization of the first set of qubits in quantum registers $QR_{1A}$ and $QR_{1B}$ is performed randomly. $QR_{1A}$ and $QR_{1B}$ store alphas ($\alpha$) corresponding to the
solution vectors scaled between \([0, 1]\) for both the populations. The population size is taken as 100 times the number of variables.

b) Computation of the fitness of each solution vector of \(QR_{1A}\) is performed by using Feasibility Rules, which is as follows [Deb2000]:

i. If the two solution vectors being compared are both feasible, the one with better objective function value \(f(x)\) is considered fitter.

ii. If one solution vector is feasible and the other is infeasible, the feasible one is considered fitter.

iii. If both the solution vectors are infeasible, the one with the lower level of constraint violation or degree of infeasibility is considered fitter.

c) \(QR_{2A}\) stores the ranked and scaled objective function values of the solution vectors in \(QR_{1A}\). The ranked and scaled objective function value of the \(i^{th}\)
solution vector \((QR_{2AI})\) is stored in \(QR_{2A}\) as amplitude \(\alpha_{2i}\) whose value \([0, 1]\). The fittest vector’s objective function value is assigned 1 and the worst vector is assigned value 0. The rest of the solution vectors’ objective function value is ranked and assigned a value between 0 and 1.

d) Adaptive Quantum Rotation based Crossover is performed by using all the three strategies R-I, R-II and R-III. R-I is used with 50% percent probability, R-II with 25% probability and R-III with 75% probability to strike a proper balance between exploration and exploitation during the search, while increasing the efficiency by 50% when all the three rotation strategies are used with 100% probability.

e) The population for the next generation are selected by comparing individual parent’s fitness with their best child’s fitness and applying tournament selection i.e. the fitter one would make it to the next generation.

f) If there is no improvement in the best solution vector of Population A in any generation, then Adaptive Penalty Factor is computed as discussed in chapter 3. It is designed to make the objective function value of the fittest solution vector in population A and the modified objective function value (equation 6.2.2 given in (g)) of the Individual with the best objective function value in population B equal in the current generation. This automatically chooses a new value of the adaptive penalty factor for the generation to guide the search in entire solution domain. The equation for computing adaptive penalty factor ‘S’, is as follows:

\[
\text{If } (CV_{Bi} > 0) \quad \{ \\
\quad S = \frac{(f_{BAi}(x) - f_{BBi}(x))}{CV_{Bi}} \text{ for Min} \\
\quad S = \frac{(f_{BBi}(x) - f_{BAi}(x))}{CV_{Bi}} \text{ for Max } \\
\} \quad \text{(6.2.1)}
\]

\[
\text{If } (S <= 0) \\
\quad S = 0
\]
where \( f_{BA}(x) \) = Objective function value of the Best Individual of Population A in the \( i \)th generation, \( f_{BB}(x) \) = Objective function value of the Individual with best objective function value of Population B in the \( i \)th generation, \( CV_{Bi} \) = Constraint Violation of the Individual with best objective function value of Population B in the \( i \)th generation.

Adaptive penalty factor methods have been known to work better provided the penalty factor is adaptively made smaller if the feasible solutions are being found and made larger otherwise [Coe2002]. This is done so that there is an indirect pressure for finding feasible solutions. However, these methods do not provide any assistance in actually finding the feasible solutions. In the two population method, population A has a greater propensity towards feasible solutions because it utilizes the Feasibility Rules and population B ensures that the domain on search is not restricted.

g) Population B’s fitness is evaluated by using modified objective function value, which is determined by using adaptive penalty factor.

Objective function = \( f(x) \)

Constraint Violation = Degree of infeasibility = \( \{ \Sigma (g_{i}(x)) + \Sigma (h_{i}(x)) \} \).

Modified Objective function \( \Phi = f(x) + S \times \{ \Sigma (g_{i}(x)) + \Sigma (h_{i}(x)) \} \) \hspace{1cm} (6.2.2)

Where, \( g_{i}(x) = \max \{ 0, g_{i}(x) \} \) and \( h_{i}(x) = 0 \), if \( |h_{i}(x)| - \delta < 0 \) else \( |h_{i}(x)|, \delta = 10^{-10} \).

h) \( QR_{2B} \) stores ranked and scaled objective function values of the solution vectors in \( QR_{1B} \). The ranked and scaled objective function value of the \( i^{th} \) solution vector \( (QR_{1B}) \) is stored in \( QR_{2B} \) as amplitude \( \alpha_{2i} \) whose value \([0, 1]\). The fittest vector’s objective function value is assigned 1 and the worst vector is assigned value 0. The rest of the solution vectors’ objective function value is ranked and assigned a value in between 0 and 1.
i) Adaptive quantum rotation based crossover is performed only 50% time using R-III as this population is primarily used for exploration.

j) Same as in e).

k) Swap exchanges a part of population A and B using Greedy and Random strategies so that the search is guided towards global optima while maintaining the diversity in both the populations. Greedy strategy is implemented through the replacement of the least-fit individual of both the populations A and B by the fittest individual in the other population. Random strategy is implemented by selecting and exchanging part of the population randomly, which precludes individuals used in the Greedy strategy. The number of individuals being exchanged in the Random Strategy is a strategy parameter, named swap_per and is 10% of the population size in this implementation for all the problems.

6.3 Testing and Results

The proposed algorithm has been tested on six benchmark engineering constrained optimization problems, which have been widely used for testing similar optimization algorithms.

Thirty independent runs are performed for each problem using the proposed algorithm, which is implemented in 'C' programming language on an IBM Workstation with Pentium-Iv 2.4 GHz processor, 2 GB RAM and Windows XP platform. The testing has been performed for determining the stability of the proposed algorithm. The stability and robustness have been determined by analyzing statistically the quality of the solutions produced for each problem in thirty independent runs. The efficiency has been determined by the number of generations and the maximum number of generations is limited to 500 for all the problems.
The problems are arranged in the order of their increasing difficulty for the proposed algorithm. The experiments have been designed to gain better insight into working of the proposed algorithm by analyzing its Adaptive Quantum rotation based Crossover Operator (AQCO) in four different configurations. The first experiment is performed with the standard configuration (SC-I) as described in section 6.2 i.e. all three types of the rotation strategies have been used. The second configuration (SC-II) uses deterministic rotation instead of random rotation in the standard configuration to determine whether random rotation is a better design option than deterministic rotation. The third configuration (SC-III) uses R-I and R-II of the standard configuration but not R-III, i.e. random exploration is turned off to find the effectiveness of the exploration mechanism of AQCO. The fourth configuration (SC-IV) uses standard configuration but without Swap to validate the selection of the hybrid constraint handling technique.

**Problem P-1 : Himmelblaus Nonlinear Optimization Problem [Heq2008]**

The first problem (P-1) is originally proposed by Himmelblau with five design variables, six nonlinear inequality constraints and 10 boundary conditions.

Minimize \( f(\bar{x}) = 5.3578547x_3^2 + 0.8356891x_1x_5 + 37.293239x_1 - 40792.141 \)

Such that:

\[
\begin{align*}
g_1(\bar{x}) &= 85.334407 + 0.0056858x_2x_5 + 0.00026x_1x_4 - 0.0022053x_3x_5 \\
g_2(\bar{x}) &= 80.51249 + 0.0071317x_2x_5 + 0.002955x_1x_2 + 0.0021813x_3^2 \\
g_3(\bar{x}) &= 9.300961 + 0.0047026x_3x_5 + 0.0012547x_1x_3 + 0.0019085x_3x_4
\end{align*}
\]

where \( 0 \leq g_1(\bar{x}) \leq 92, \quad 90 \leq g_2(\bar{x}) \leq 110, \quad 20 \leq g_3(\bar{x}) \leq 25, \quad 78 \leq x_1 \leq 102, \quad 33 \leq x_2 \leq 45, \quad 27 \leq x_3, x_4, x_5 \leq 45. \)

The results for all the four configurations for P-1 are presented in Table 6.3.1. The performance of SC-I, SC-II and SC-IV is optimal in all the runs. This indicates that for certain class of problems even the deterministic rotation and
use of the Feasibility rules can be effective. SC-III reached optimal in its best run but is less robust than the other three configurations as indicated by the statistical parameters. This indicates that limiting exploration by turning off R-III adversely affects searching ability of the algorithm for this problem.

Table 6.3.1. Results for all the configurations in Problem P-1

<table>
<thead>
<tr>
<th></th>
<th>SC-I</th>
<th>SC-II</th>
<th>SC-III</th>
<th>SC-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best</td>
<td>-31025.56</td>
<td>-31025.56</td>
<td>-31025.56</td>
<td>-31025.56</td>
</tr>
<tr>
<td>Median</td>
<td>-31025.56</td>
<td>-31025.56</td>
<td>-31024.07</td>
<td>-31025.56</td>
</tr>
<tr>
<td>Mean</td>
<td>-31025.56</td>
<td>-31025.56</td>
<td>-31023.13</td>
<td>-31025.56</td>
</tr>
<tr>
<td>S.D.</td>
<td>3.25E-11</td>
<td>1.92E-11</td>
<td>2.91E+00</td>
<td>3.25E-11</td>
</tr>
<tr>
<td>Worst</td>
<td>-31025.56</td>
<td>-31025.56</td>
<td>-31016.95</td>
<td>-31025.56</td>
</tr>
</tbody>
</table>

Problem P2: Speed Reducer [Sha2007, Heq2008]

Gears are machine elements that transmit motion by means of successively engaging teeth. The gear teeth act like small levers. Gear trains consists of driving gears that are attached to the input shaft and driven gears that are attached to the output shaft for the purpose of transmitting motion from one axis to another. These gears works together by meshing their teeth and turning each other in a system to generate power and speed. Gear train reduces speed and increases torque. To create large gear ratio, gears are connected together to form gear trains. The most common of the gear train is the simple gear train with a gear pair connecting parallel shafts having axis, relative to the frame, for all gears comprising the train. Figure 6.3.1 shows a simple ordinary gear train in which two or more gears may rotate about a single axis. The design of a gear train is considered with the face width (b), module teeth (m), number of teeth on pinion (z), length of shaft 1 (d₁), and diameter of shaft 2 (d₂) as design variables x₁, x₂, ..........x₇, respectively. The constraints include limitations on the bending stress of gear teeth, surface stress, transverse deflections of shafts 1 and 2 due to
transmitted force, and stresses in shafts 1 and 2 due to transmitted force, and stresses in shafts 1 and 2.

![Figure 6.3.1: A gear pair in mesh](image)

The second problem (P-2) is designing of the speed reducer with face width, module of teeth, number of teeth on pinion, length of the first shaft between bearings, length of the second shaft between bearings, diameter of the first shaft, and diameter of the first shaft as variables (all of them are continuous except number of teeth on pinion, which is integer). The weight of the speed reducer is to be minimized subject to the constraints on bending stress of gear teeth, surface stress, transverse deflections of the shafts and stresses in the shaft. The variables $x_1, \ldots, x_7$ are the face width, module of teeth, number of teeth in the pinion, length of the first shaft between bearings, length of the second shaft between bearings and the diameter of first and second shafts as shown in figure 6.3.3.
Minimize \( f(x) = 0.7854x_1 x_2^2(3.3333x_3^3 + 14.9334x_3 - 43.0934) - 1.508x_1 (x_6^2 + x_7^2) \\
+ 7.4777(x_6^3 + x_7^3) + 0.7854(x_4 x_6^2 + x_5 x_7^2) \)

Subject to
\[
27x_1^{-1}x_2^{-2}x_3^{-1} \leq 1 \\
397.5x_1^{-1}x_2^{-2}x_3^{-2} \leq 1 \\
1.93x_1^{-1}x_2^{-1}x_4x_6^{-1} \leq 1 \\
1.93x_1^{-1}x_2^{-1}x_5x_7^{-1} \leq 1 \\
x_2x_3 \leq 40.0 \\
5 \leq \frac{x_1}{x_2} \leq 12.0 \\
(1.5x_6 + 1.9)x_4^{-1} \leq 1 \\
(1.1x_7 + 1.9)x_5^{-1} \leq 1 \\
\left[ \frac{(745x_4 x_2^{-1} x_3^{-1})^2 + 16.9 \times 10^6}{110.0x_6} \right]^{1/2} \leq 1 \\
\left[ \frac{(745x_2 x_2^{-1} x_3^{-1})^2 + 157.5 \times 10^6}{85.0x_6} \right]^{1/2} \leq 1 \\
\]

where \( 2.6 \leq x_1 \leq 3.6, \ 0.7 \leq x_2 \leq 0.8, \ 17 \leq x_3 \leq 28, \ 7.3 \leq x_4 \leq 8.3, \ 7.8 \leq x_5 \leq 8.3, \ 2.9 \leq x_6 \leq 3.9, \ 5.0 \leq x_7 \leq 5.5. \)

The results for all the four configurations for P-2 are presented in Table 6.3.2. The overall performance of SC-I is better than all the other configurations. SC-II and SC-III have reached optimal in their best run but are less robust than SC-I as indicated by other statistical parameters. This indicates the importance of
random rotation and exploration by R-III for robust performance in this problem. SC-II has performed better than SC-III, thus again, highlighting the importance of exploration by R-III in the search process for this problem. SC-IV has performed worst of all the four configurations, thus highlighting the importance of HCT for this problem, possibly due to the presence of an integer variable.

Table 6.3.2. Results for all the configurations in Problem P-2

<table>
<thead>
<tr>
<th></th>
<th>SC-I</th>
<th>SC-II</th>
<th>SC-III</th>
<th>SC-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best</td>
<td>2996.348</td>
<td>2996.348</td>
<td>2996.348</td>
<td>3000.604</td>
</tr>
<tr>
<td>Median</td>
<td>2996.348</td>
<td>2997.482</td>
<td>3002.919</td>
<td>3008.353</td>
</tr>
<tr>
<td>Mean</td>
<td>2996.348</td>
<td>3000.138</td>
<td>3005.807</td>
<td>3012.654</td>
</tr>
<tr>
<td>S.D.</td>
<td>1.10E-09</td>
<td>4.88</td>
<td>6.36</td>
<td>1.07E+01</td>
</tr>
<tr>
<td>Worst</td>
<td>2996.348</td>
<td>3007.472</td>
<td>3016.073</td>
<td>3035.650</td>
</tr>
</tbody>
</table>

Problem P-3: Design of a three bar truss [Gol1973]

The third problem (P-3) is minimization of the volume of material required for construction of a three bar truss while satisfying stress limitations under a load condition. The stress in each member is limited to 2KN/cm$^2$ and the load is limited to 2KN/cm$^2$. The volume of the truss structure is to be minimized subject to stress constraints. The three bar truss structure is shown in figure 6.3.3.

Figure 6.3.3: A three bar truss
The mathematical formulation is presented below:

\[
\text{Minimize } f(x) = \left(2(2)\right)^{\frac{1}{2}} x_1 + x_2 \right) \sqrt{L}
\]

Subject to:

\[
\sigma_1 = \frac{\left(2\right)^2 x_1 + x_2}{\left(2\right)^2 x_1 + 2x_1x_2} \leq 2
\]

\[
\sigma_2 = \frac{1}{x_1 + \left(2\right)^2 x_2} \leq 2
\]

\[
\sigma_3 = \frac{x_2}{\left(2\right)^2 x_2 + 2x_1x_2} \leq 2
\]

Where, \(0 \leq x_1 \leq 1\) and \(0 \leq x_2 \leq 1\). The other constants are \(l=100\) cm, \(P=2KN/cm^2\) and \(\sigma = 2KN/cm^2\).

The results for all the four configurations for P-3 are presented in Table 6.3.3. The overall performance of SC-I is better than all the configurations. SC-II and SC-IV have reached near optimal in their best run but are less robust than SC-I as indicated by other statistical parameters. This indicates that random rotation and HCT are important for robust performance of the proposed algorithm in this problem. SC-III has performed worst of all the configurations, thus highlighting the importance of exploration by R-III in the search process for this problem.

<table>
<thead>
<tr>
<th></th>
<th>SC-I</th>
<th>SC-II</th>
<th>SC-III</th>
<th>SC-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best</td>
<td>263.8959</td>
<td>263.9410</td>
<td>264.2101</td>
<td>263.8959</td>
</tr>
<tr>
<td>Median</td>
<td>263.8966</td>
<td>264.0030</td>
<td>265.0087</td>
<td>269.6909</td>
</tr>
<tr>
<td>Mean</td>
<td>263.8974</td>
<td>265.7040</td>
<td>268.2368</td>
<td>271.4222</td>
</tr>
<tr>
<td>S.D.</td>
<td>1.46E-03</td>
<td>3.71</td>
<td>6.21</td>
<td>1.06E+01</td>
</tr>
<tr>
<td>Worst</td>
<td>263.9003</td>
<td>282.8453</td>
<td>283.0051</td>
<td>301.6447</td>
</tr>
</tbody>
</table>
Problem P-4 : Spring Design [Coe2002a]

The fourth problem (P-4) is minimizing the weight of a tension/compression spring, subject to the constraints of minimum deflection, shear stress, surge frequency, and limits on the outside diameter and on the design variables. The design variables \( x_1, x_2 \) and \( x_3 \) are the mean coil diameter, the wire diameter and the number of active coils respectively and are shown in figure 6.3.4. The mathematical formulation of the problem is as given below [Sha2007].

Minimize \( f(x) = (x_3 + 2)x_1x_2^2 \)

Subject to the:

- **deflection constraint**
  \[
  \left[1 - \frac{(x_1x_3)}{71785x_2^4}\right] \leq 0
  \]

- **shear stress constraint**
  \[
  \left[\frac{\left(4x_1^2 - x_1x_2\right)}{12566\left(x_1x_3^2 - x_2^4\right)} + \frac{1}{5108x_2^2} - 1\right] \leq 0
  \]

- **surge wave frequency constraint**
  \[
  \left[1 - \frac{140.54x_2}{x_1^2x_3}\right] \leq 0
  \]

- and the outer diameter constraint
  \[
  \left[\frac{x_1 + x_2}{1.5} - 1\right] \leq 0
  \]

The lower bounds and upper bounds on the design variables selected are as follows:

- \( 0.25 \leq x_1 \leq 1.3 \),
- \( 0.05 \leq x_2 \leq 2.0 \) and
- \( 2 \leq x_3 \leq 15 \)
The results for all the four configurations for P-4 are presented in Table 6.3.4. The overall performance of SC-I is better than all the configurations. SC-II, SC-III and SC-IV have reached near optimal in their best run but are less robust than SC-I as indicated by other statistical parameters. This indicates that random rotation, exploration by R-III and HCT are important for robust performance of the proposed algorithm in this problem. SC-II has performed better than SC-III and SC-IV. SC-IV has performed worst than all the configurations. The relative performance of SC-II, SC-III and SC-IV indicates the relative importance of random rotation, exploration by R-III and HCT for this problem i.e. HCT is the most important followed by exploration through R-III and random rotation.

The convergence graph of all the configurations on median run for this problem has been plotted in figure 6.3.5, which also shows the superiority and efficiency of SC-I as compared to all other configurations. SC-II and SC-III found it difficult to reach the optima, the initial performance of SC-II was better than SC-III. SC-IV was superior to SC-II and SC-III. It managed to reach near optimum, but was slower as compared to SC-I. This validates the design of HARQEA SC-I.
Table 6.3.4: Results for all the configurations in Problem P-4

<table>
<thead>
<tr>
<th></th>
<th>SC-I</th>
<th>SC-II</th>
<th>SC-III</th>
<th>SC-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best</td>
<td>0.012665</td>
<td>0.012665</td>
<td>0.012665</td>
<td>0.012665</td>
</tr>
<tr>
<td>Median</td>
<td>0.012665</td>
<td>0.012684</td>
<td>0.012712</td>
<td>0.012695</td>
</tr>
<tr>
<td>Mean</td>
<td>0.012666</td>
<td>0.012687</td>
<td>0.012699</td>
<td>0.012712</td>
</tr>
<tr>
<td>S.D.</td>
<td>3.10E-07</td>
<td>1.88E-05</td>
<td>2.11E-05</td>
<td>6.38E-05</td>
</tr>
<tr>
<td>Worst</td>
<td>0.012666</td>
<td>0.012717</td>
<td>0.012719</td>
<td>0.01290</td>
</tr>
</tbody>
</table>

**Figure 6.3.5 Convergence Graph for the Problem P-4 on Median run.**

Problem P-5: Welded Beam [Coe2002a]

The fifth problem (P-5) is designing of a welded beam for minimum cost, subject to some constraints as shown in figure 6.3.6. The objective is to find the minimum fabrication cost, considering four design variables: $x_1$, $x_2$, $x_3$ and $x_4$ and constraints of shear stress $\tau$, bending stress in the beam $\sigma$, buckling load on the
bar $P_c$, and end deflection on the beam $\delta$. The mathematical formulation is as follows:

Minimize: $f(\vec{x}) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0 + x_2)$

Subject to: $g_1(\vec{x}) = \tau(\vec{x}) - 13600 \leq 0$

$g_2(\vec{x}) = \sigma(\vec{x}) - 30000 \leq 0$

$g_3(\vec{x}) = x_1 - x_4 \leq 0$

$g_4(\vec{x}) = 0.10471x_1^2 + 0.04811x_3x_4(14.0 + x_2) - 5.0 \leq 0$

$g_5(\vec{x}) = 0.125 - x_1 \leq 0$

$g_6(\vec{x}) = \delta(\vec{x}) - 0.25 \leq 0$

$g_7(\vec{x}) = -P_c(\vec{x}) + 6000 \leq 0$

where:

$\tau(\vec{x}) = \sqrt{(\tau')^2 + (2\tau'' \frac{x_2}{2R} + (\tau'')^2}$

$\tau' = \frac{6.000}{\sqrt{2x_1x_2}}$

$\tau'' = \frac{MR}{J}$

$M = 6.000 \left(14 + \frac{x_2}{2}\right)$

$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}$

$J = 2 \left(x_1x_2\sqrt{2} \left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right)$

$\sigma(\vec{x}) = \frac{504,000}{x_1x_3^3}$

$\delta(\vec{x}) = \frac{65,856,000}{(30 \times 10^6)x_1x_3^3}$

$P_c(\vec{x}) = \frac{4.013(30 \times 10^6)\sqrt{\frac{x_2x_4x_6}{36}}}{196} \left(1 - \frac{x_3}{\sqrt{\frac{30x_110^6}{412\times 10^6}}\left(\frac{x_3}{28}\right)}\right)$

and $0.1 \leq x_1, x_4 \leq 2.0$ and $0.1 \leq x_2, x_3 \leq 10.0$.

The results for all the four configurations for P-5 are presented in Table 6.3.5. The overall performance of SC-I is better than all the configurations. This indicates that random rotation, exploration by R-III and HCT are important for robust performance of the proposed algorithm in this problem. SC-IV performed better than SC-II and SC-III. SC-III performed worst than all the configurations.
The relative performance of SC-II, SC-III and SC-IV indicates the relative importance of random rotation, exploration by R-III and HCT for this problem i.e. exploration by R-III is the most important followed by random rotation and HCT.

The convergence graph of all the configurations on median run for this problem has been plotted in figure 6.3.7, which also shows the superiority and efficiency of SC-I as compared to all other configurations. SC-II and SC-III found it difficult to reach the optimum, the initial performance of SC-III was better than SC-II. SC-IV was superior to SC-II and SC-III. It managed to reach near optimum, but was slower as compared to SC-I. This validates the design of HARQEA SC-I.

Table 6.3.5. Results for all the configurations in Problem P-5

<table>
<thead>
<tr>
<th></th>
<th>SC-I</th>
<th>SC-II</th>
<th>SC-III</th>
<th>SC-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best</td>
<td>1.724852</td>
<td>1.724935</td>
<td>1.725143</td>
<td>1.724855</td>
</tr>
<tr>
<td>Median</td>
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<td>1.749831</td>
<td>1.725752</td>
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<tr>
<td>Mean</td>
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<td>1.759435</td>
<td>1.763129</td>
<td>1.730454</td>
</tr>
<tr>
<td>S.D.</td>
<td>2.23E-05</td>
<td>3.20E-02</td>
<td>3.01E-02</td>
<td>1.09E-02</td>
</tr>
<tr>
<td>Worst</td>
<td>1.724922</td>
<td>1.814038</td>
<td>1.816909</td>
<td>1.774934</td>
</tr>
</tbody>
</table>
Problem P-6: Pressure Vessel [Coe2002a]

The sixth problem (P-6) is designing a compressed air storage tank with a working pressure of 3,000 psi and a minimum volume of 750 ft$^3$. A cylindrical vessel is capped at both ends by hemispherical heads as shown in figure 6.3.8. Using rolled steel plate, the shell is made in two halves that are joined by two longitudinal welds to form a cylinder. The objective is to minimize the total cost, including the cost of materials forming the welding. The design variables are: thickness $x_1$, thickness of head $x_2$, inner radius $x_3$ and length of cylindrical section of the vessel $x_4$. The variables $x_1$ and $x_2$ are discrete values, which are integer multiples of 0.0625 inch. The mathematical formulation of the optimization problem is as follows:

Minimize: $f(x) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3$
Subject to: \( g_1(\bar{x}) = -x_1 + 0.0193x_3 \leq 0 \)
\( g_2(\bar{x}) = -x_2 + 0.00954x_3 \leq 0 \)
\( g_3(\bar{x}) = -\pi x_3^2 x_4^2 - \frac{4}{3} \pi x_3^3 + 1296000 \leq 0 \)
\( g_4(\bar{x}) = x_4 - 240 \leq 0 \)
where \(1 \leq x_1, x_2 \leq \{99\}, 10.0 \leq x_3 \text{ and } x_4 \leq 200.0\)

Figure 6.3.8 Pressure Vessel

The results for all the four configurations for P-6 are presented in Table 6.3.6. The overall performance of SC-I is better than all the configurations. SC-II, SC-III and SC-IV reached near optimal in their best run but are less robust than SC-I as indicated by other statistical parameters. This indicates that random rotation, exploration by R-III and HCT are important for robust performance of the proposed algorithm. SC-IV performed better than SC-II and SC-III. SC-II performed worst than all the configurations. The relative performance of SC-II, SC-III and SC-IV indicates the relative importance of random rotation, exploration by R-III and HCT for this problem i.e. random rotation is the most important followed by exploration by R-III and HCT.

The convergence graph of all the configurations on median run for this problem has been plotted in figure 6.3.9, which also shows the superiority and efficiency of SC-I as compared to all other configurations. SC-II and SC-III found it difficult to reach the optima, the initial performance of SC-II was better than SC-III. SC-IV was superior to SC-II and SC-III. It managed to reach near optimum, but was slower as compared to SC-I. This validates the design of HARQEA SC-I.
Table 6.3.6. Results for all the configurations in Problem P-6

<table>
<thead>
<tr>
<th></th>
<th>SC-I</th>
<th>SC-II</th>
<th>SC-III</th>
<th>SC-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best</td>
<td>6059.714</td>
<td>6059.714</td>
<td>6059.715</td>
<td>6059.714</td>
</tr>
<tr>
<td>Median</td>
<td>6059.714</td>
<td>6211.798</td>
<td>6112.626</td>
<td>6061.611</td>
</tr>
<tr>
<td>Mean</td>
<td>6068.958</td>
<td>6213.686</td>
<td>6183.784</td>
<td>6113.621</td>
</tr>
<tr>
<td>S.D.</td>
<td>1.14E+01</td>
<td>1.32E+02</td>
<td>1.20E+02</td>
<td>1.10E+02</td>
</tr>
<tr>
<td>Worst</td>
<td>6090.526</td>
<td>6410.220</td>
<td>6400.949</td>
<td>6383.949</td>
</tr>
</tbody>
</table>

Figure 6.3.9: Convergence Graph for the Problem P-6.

SC-I has performed better than any other configuration in all the problems thus validating our design and selection of all the three rotation strategies and the constraint handling technique. The relative importance of random rotation, exploration by R-III and HCT has been highlighted through SC-II, SC-III and SC-IV in all the problems. However, the relative importance has varied across the problems, thus showing the diversity of the problems. This has further
established that the proposed rotation strategies along with HCT are capable of effectively searching for near optimal solutions in diverse constrained optimization problems.

Table 6.3.7 presents comparison of the proposed hybrid QEA (HARQEA) with the existing state of art algorithms like RQEA [Alf2007], ECPSO [Heq2008] and DTS [Coe2002a] on their best solution for problems P-4, P-5 and P-6. The results of column SC-I in Table 6.3.1 to Table 6.3.6 along with Table 6.3.7 show that the proposed algorithm named HARQEA is far better than the earlier proposed RQEA [Alf2007], which is a QEA utilizing probabilistic representation of qubit for maintaining diversity. It is also better than the traditional state of art EAs like ECPSO [Heq2008] and DTS [Coe2002a].

The first three problems P-1, P-2 and P-3 have not been considered in the comparative study as these have been comprehensively solved and so offer no further insights. However, it is important to use them along with other problems in testing for two reasons. Primarily, it has facilitated in a better understanding of the impact of design alternatives and the role of various rotation strategies used in the proposed algorithm. Further, it has also showed that the proposed algorithm is not only good for the problems, which appear difficult to other algorithms, but is equally good for the problems, which appear easy to other algorithms. As otherwise, it can be argued that the performance improvement over one set of the problems is made possible by sacrificing the same over the other set.

Table 6.3.7. Comparison of the proposed Algorithm (SC-I) with known Algorithms on their Best Solution

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>P-4</th>
<th>P-5</th>
<th>P-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>HARQEA (SC-I)</td>
<td>0.012665</td>
<td>1.724852</td>
<td>6059.714</td>
</tr>
<tr>
<td>RQEA</td>
<td>0.012680</td>
<td>1.751317</td>
<td>6088.568</td>
</tr>
<tr>
<td>ECPSO</td>
<td>0.012669</td>
<td>1.724860</td>
<td>6059.714</td>
</tr>
<tr>
<td>DTS</td>
<td>0.012681</td>
<td>1.728226</td>
<td>6059.946</td>
</tr>
</tbody>
</table>
6.4 Conclusions and Future Work

Constrained Optimization is an important problem in engineering domain for which a new hybrid adaptive quantum evolutionary algorithm is proposed. The algorithm hybridizes the quantum entanglement and superposition principles with a conventional crossover operator to adaptively evolve the population using three different rotation strategies viz., Rotation towards the best, Rotation away from the worse and Rotation towards the better. The degree of rotation in each strategy is adaptively determined by the feedback from the fitness space. Thus, there is no need for fine tuning parameters in the quantum rotation based crossover operator. It has been implemented by using two qubits representation instead of one, which enables utilization of the quantum entanglement and superposition principles hitherto not tapped.

The proposed algorithm uses the latest hybrid constraint handling technique based on the hybridization of Feasibility Rules and Adaptive Penalty Factor method. This technique leverages best characteristics from both the methods in a simple and effective way by using two populations, each evolving by employing one of the methods of handling constraints and swapping part of the population at the end of the generation. Further, it is free from fine-tuning of the penalty parameters as the adaptive penalty factor is calculated from the fitness information available about the best individuals in both the populations.

The proposed algorithm has been tested on a standard set of six benchmark engineering optimization problems for validating design decisions and performing comparative study with the state of art algorithms. The results showed that it performed better than the existing algorithms without even employing a mutation operator or a local heuristic for improving the solution quality.

Future work would involve more in-depth analysis to understand the working of the proposed algorithm. Moreover, an effort would be made to study
its applicability in other areas requiring optimization like parameter estimation, dynamic economic dispatch problems etc.