Chapter 5

Exchange rate risk in news vendor format under additive demand error
5.1 Introduction

We have already discussed the news vendor problem extension with pricing in the chapter 4. We also have introduced an important application of the news vendor problem in operation management namely, “Foreign exchange transaction exposure” in section 1.5 and its basic terminology in section 2.4. Now we develop mathematical models of this application in a news vendor framework, in the current and the next two chapters, under various business environments. This is the core part of the research work.

Let there be one retailer and one manufacturer from two countries are involved in an international business and are facing the unanticipated changes in the market. So they are affected by the fluctuation in the exchange rate. Hence the problem of bearing the exchange rate risk by one of them is inevitable. In this chapter we see the problem from both of their point of view, and develop exchange rate risk model under linear demand with additive error. We derive the analytic expression for the expected profit and closed form solutions for optimal policies of the retailer and manufacturer respectively, for each of the case that who takes the risk. We shall discuss the same under isoelastic demand and the hybrid demand in the chapters 6 and 7 respectively.

5.2 Assumptions and Notations

(a) Assumptions: We impose the following assumptions in the foreign exchange transaction exposure model:

1. All the standard assumptions of the basic news vendor model are applied.

2. The global supply chain consists of single retailer- single manufacturer.

3. The manufacturer and retailer are from different countries having different currency.

4. The time period between time the financial obligation has been incurred and the time its due to be settled is fixed.

5. The exchange rate fluctuates during the time period and the due is paid only after the time duration.
(6) The error in demand is additive.

(7) Only one of the two-retailer or manufacturer- bears the exchange rate risk.

(b) Notations:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>the cost of purchasing one unit of the product</td>
</tr>
<tr>
<td>$p$</td>
<td>the selling price per unit of the product</td>
</tr>
<tr>
<td>$q$</td>
<td>the order quantity of the product</td>
</tr>
<tr>
<td>$D$</td>
<td>the demand of the product</td>
</tr>
<tr>
<td>$v$</td>
<td>the salvage value per unit</td>
</tr>
<tr>
<td>$s$</td>
<td>the penalty cost per unit</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>demand error</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>exchange rate error</td>
</tr>
<tr>
<td>$\Pi(p,q)$</td>
<td>retailer’s profit as function of price $p$ and order quantity $q$</td>
</tr>
<tr>
<td>$\Pi_m$</td>
<td>manufacturer’s profit</td>
</tr>
<tr>
<td>$E[\Pi(p,q)]$</td>
<td>expected profit function</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>expected leftovers</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>expected shortages</td>
</tr>
<tr>
<td>$f(D)$</td>
<td>probability density function (PDF) for continuous demand $D$</td>
</tr>
<tr>
<td>$F(D)$</td>
<td>cumulative distribution function (CDF) for continuous demand</td>
</tr>
<tr>
<td>$\mu$</td>
<td>mean of probability distribution</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>standard deviation of probability distribution</td>
</tr>
</tbody>
</table>

5.3 Model Formulation

Consider a global supply chain in which there are two firms (or individuals) from two different countries involved in a business. Suppose a retailer/buyer (say in India) wants to order $q$ units from a foreign manufacturer/seller (say in U.S) of a certain perishable product. The retailer does not know the demand ($D$) of the product, which is partially price dependent and partially uncertain or random. But but he has information about the demand distribution. We define the demand $D$ by constructing the demand model as $D(p, \varepsilon) = g(p) + \varepsilon$. Where $g(p)$ is the price dependent deterministic demand and $\varepsilon$ is the randomness in the demand. Here $g(p)$ is a decreasing function of the price and the
demand error $\varepsilon$ is a random variable which follows some distribution say, $f(\varepsilon)$ with mean $\mu$ and standard deviation $\sigma$ in some interval $[A,B]$.

In this model we consider linear price dependent demand function $g(p) = a - bp$, where $a, b > 0$ and $A > -a$. These conditions on $a$ & $b$ are for keeping the demand positive. So we have, $D(p, \varepsilon) = a - bp + \varepsilon$. Therefore this model is referred as the **LDAE MODEL** i.e. the Linear Demand with Additive Error model.

Since the demand $D$ is unknown and random it can be less than or equal to $q$ or greater than $q$. If $D \leq q$ i.e. if the retailer has overstocking of items then he will sell each of the leftover from $q - D$ units at the salvage value $v$ per unit. So $v$ is the income to the retailer for disposing off the leftovers. Note that the salvage value is considered as a revenue ($s$) and not as the cost ($h$) as in case of the NVP with pricing given in the equation 4.14 of the chapter-4. On the other hand if $D > q$ i.e. if there is a shortage of the items or understocking then he will have a loss of profit and it is called the penalty cost. The penalty cost is denoted by $s$ and it is defined as the loss of income per each item short, if he would have purchased it for the shortages $D - q$.

As we assume that the two firms or individuals are from two different countries, their business is naturally affected by the foreign exchange transaction exposure in the form of exchange rate fluctuations. Let the exchange rate be $r$ in the retailer’s currency when he places the order. (say e.g 1$ = r$ Rs.) Let $w$ be the cost of one unit of the product in the manufacturer’s currency. If the retailer pays immediately then he has to pay $wr$ per unit of the product in his currency. But we assume that the time period between, when the financial obligation has been incurred and its due to be settled is fixed and positive.

In other words the retailer pays the due only after a fixed time lag. Thus there exists transaction exposure risk, as the exchange rate ($r$) may get fluctuate or change during this period. This risk is known as **exchange rate risk**. So the buyer has to pay more or less according to the existing rate at the time of the arrival of the product.

Generally the fluctuation in the exchange rate $r$ is very small and uncertain. Also the fluctuation in $r$ is always some percentage of $r$, hence we define the **Future Exchange Rate** (F.E.R) as, $F.E.R=r + r \varepsilon_r = r(1+ \varepsilon_r)$. [ e.g. 1$ = r(1+ \varepsilon_r)$ Rs.] The F.E.R is the actual exchange rate at which the buyer is going to pay the due or seller is receiving.

Note that the fluctuation $\varepsilon_r$ is also a random variable together with the random demand. The fluctuation $\varepsilon_r$ is unknown but its distribution is known say, $\psi(\varepsilon_r)$. If the fluctuation $\varepsilon_r$ is positive buyer has to pay more and if it is negative seller will get less.
So the main question arises is that,

“Who will bear the exchange rate risk?”

The retailer OR The manufacturer?

So we consider two models as follows in news vendor settings.
(1) The retailer bears the exchange rate risk.
(2) The manufacturer bears the exchange rate risk.

We develop models for both the scenarios for linear demand with additive error in the next two sections. In each case, the retailer’s objective is to jointly determine the optimum order \(q\) and selling price \(p\) of the product so that his expected profit is maximum. At the same time we obtain the manufacturer’s optimal policies as well. This indicates that the models are based upon the news vendor extension model with pricing as discussed in the section 4.3 of chapter 4.

5.3.1 The additive model for the risk taker retailer

In this model we assume that the retailer bears the exchange rate risk and manufacturer does not. So the manufacturer will get \(w\) per unit at any point of time and the retailer will have to pay according to the existing exchange rate that is according to F.E.R. So the retailer will be paying \(wr(1+\varepsilon_r)\) per unit in his currency, on the settlement day or when he acquires the product. This amount in terms of the manufacturer currency is \(wr(1+\varepsilon_r)/r = w(1+\varepsilon_r) = w_r\) (say). Thus \(w_r\) is the purchase cost per unit to the retailer in the manufacturer’s currency. Now the retailer will choose the selling price \(p\) and the order quantity \(q\) so as to maximize his expected profit.

The profit function for the retailer is denoted by \(\Pi(p,q)\) and is given by,
\[
\Pi(p,q) = \begin{cases} 
[pD + v(q - D)] - [qw_r] & \text{if } D \leq q & \text{(overstocking)} \\
[pq] - [s(D - q) + qw_r] & \text{if } D > q & \text{(shortage)} 
\end{cases} \tag{5.1}
\]

Note that all the parameters \(p, v, s, w_r\) are taken in the manufacturer’s currency.
Now put $D = D(p, \varepsilon) = g(p) + \varepsilon$ in the equation (1). So we get,

$$\Pi(p, q) = \begin{cases} 
 [pD + v(q - D)] - [qw_r] & \text{if } D \leq q \\
 [pq] - [s(D - q) + qw_r] & \text{if } D > q 
 \end{cases}$$

$\Rightarrow \Pi(p, q) = \begin{cases} 
 [pg(p) + \varepsilon] + v\{q - (g(p) + \varepsilon)\} - [qw_r] & \text{if } D \leq q \\
 [pq] - [s\{(g(p) + \varepsilon) - q\} + qw_r] & \text{if } D > q 
 \end{cases}$

$\Rightarrow \Pi(p, q) = \begin{cases} 
 p\{g + \varepsilon\} + v(z - \varepsilon) - w_r(z + g) & \text{if } D \leq q \\
 p(z + g) - s(\varepsilon - z) - w_r(z + g) & \text{if } D > q 
 \end{cases}$

Now put $g(p) = g$ and define a new parameter $z = q - g(p) = q - g$ i.e. $q = z + g$, for the additive demand.

$$\Rightarrow \Pi(p, q) = \begin{cases} 
 p\{g + \varepsilon\} + v(z - \varepsilon) - w_r(z + g) & \text{if } D \leq q \\
 p(z + g) - s(\varepsilon - z) - w_r(z + g) & \text{if } D > q 
 \end{cases}$$

Now $D \leq q \iff g + \varepsilon \leq q \iff \varepsilon \leq q - g \iff \varepsilon \leq z$ and similarly $D > q \iff \varepsilon > z$. Thus we have,

$$\Pi(z, p) = \begin{cases} 
 p\{g + \varepsilon\} + v(z - \varepsilon) - w_r(z + g) & \text{if } \varepsilon \leq z \\
 p(z + g) - s(\varepsilon - z) - w_r(z + g) & \text{if } \varepsilon > z 
 \end{cases} \quad (5.2)$$

The equation (2) gives the profit function for the buyer in the manufacturer’s currency.

Note that the parameter $q$ is replaced by $z$.

Now the retailer wants to find the optimal order quantity $q$ say $q^*$ for maximizing his expected profit. In order to do this he first finds optimal values of the price $p$ and the parameter $z$, that is $p^*$ and $z^*$. Then he determines the optimal order $q^* = z^* + g(p^*)$.  

Note that the expected profit $\Pi$ is a function of the random variable $\varepsilon$ - the demand error. Thus the buyer’s expected profit is given by,

$$E[\Pi(z, p)] = \int_A^B \Pi(z, p)f(\varepsilon)d\varepsilon$$

$$\Rightarrow E[\Pi(z, p)] = \int_A^Z \Pi(z, p)f(\varepsilon)d\varepsilon + \int_Z^B \Pi(z, p)f(\varepsilon)d\varepsilon \text{ as } A \leq z \leq B$$

Take $\varepsilon = u$ for simplicity in (2) and then writing the expected profit we get,
Chapter 5. Exchange rate risk in news vendor format under additive demand error

\[
E[\Pi(z,p)] = \int_A^Z [p(g+u) + v(z-u) - w_r(z+g)]f(u)du + \int_z^B [p(z+g) - s(u-z) - w_r(z+g)]f(u)du
\]

\[\Rightarrow E[\Pi(z,p)] = \int_A^Z [p(g+u) + v(z-u)]f(u)du + \int_z^B [p(z+g) - s(u-z)]f(u)du
- (w_r(z+g))\left[\int_A^f(u)du + \int_z^B f(u)du\right]
\]

\[\Rightarrow E[\Pi(z,p)] = \int_A^Z [p(g+u) + v(z-u)]f(u)du + \int_z^B [p(z+g) - s(u-z)]f(u)du - w_r(z+g)
\]

(5.3)

since \(\int_A^B f(u)du = 1\).

Now we derive an equivalent expression for the equation (3) in the following preposition.

**Preposition-5.1:**

The expected profit of the retailer under linear demand with additive error is,

\[E[\Pi(z,p)] = [(p - w_r)(g + \mu)] - [(w_r - v)\Lambda + (p + s - w_r)\Phi]\]

where,

\[\Lambda = \int_A^z (z - u)f(u)du \quad \text{[expected leftovers]} \quad (5.4)\]

and

\[\Phi = \int_Z^B (u - z)f(u)du \quad \text{[expected shortages]} \quad (5.5)\]

Note that \(\Lambda\) and \(\Phi\) both are functions of \(z\). Also \(\mu = \int_A^B uf(u)du\) is the expected value of the demand error.

**Proof:**

We have from equation (5.3),
\[ E [\Pi(z, p)] = \int_A^Z [p(g + u) + v(z - u)] f(u) du + \int_z^B [p(z + g) - s(u - z)] f(u) du - w_r(z + g) \]

Let us take \( X = (p - w_r)(g + \mu) \) and \( Y = (w_r - v)\Lambda + (p + s - w_r)\Phi \) for the two parts in the right side of the required result. The first one \( X \) represents risk less profit as it does not contain the randomness term \( \in \) and the second, \( Y \) gives the loss function. We shall prove the theorem by showing \( E[\Pi(z, p)] = X - Y \) as follows.

Consider,
\[
X - Y \\
= [(p - w_r)(g + \mu)] - [(w_r - v)\Lambda + (p + s - w_r)\Phi] \\
= (p - w_r)(g + \mu) - (w_r - v)\Lambda - (p + s - w_r)\Phi \\
= (p - w_r)g + (p - w_r)\mu - (w_r - v)\Lambda - (p + s - w_r)\Phi \\
= (p - w_r)g \int_A^B f(u) du + (p - w_r) \int_A^B uf(u) du - (w_r - v) \int_A^B (z - u)f(u) du - (p + s - w_r) \int_A^B (u - z)f(u) du
\]
Chapter 5. Exchange rate risk in news vendor format under additive demand error

\[ \begin{align*}
\int_A^z [p(g + u) + v(z - u)] f(u) du + \int_B^z [p(g + z) - s(u - z)] f(u) du - w_r(g + z) \left\{ \int_A^z f(u) du + \int_B^z f(u) du \right\} \\
= \int_A^z [p(g + u) + v(z - u)] f(u) du + \int_B^z [p(g + z) - s(u - z)] f(u) du - w_r(g + z)
\end{align*} \]

Hence we have proved that, \( E[\Pi(z, p)] = X - Y \). That is,

\[ E[\Pi(z, p)] = [(p - w_r)(g + \mu)] - [(w_r - v)\Lambda + (p + s - w_r)\Phi] \]  

(5.6)

Next we derive the optimal policies for the retailer for maximizing his expected profit.

**Proposition-5.2:**

The optimal order quantity \( q^* \) for maximum profit of the retailer is given by

\[ q^* = g(p^*) + F^{-1} \left( \frac{p^* + s - w_r}{p^* + s - v} \right) \]

Where \( F^{-1} \) is the inverse cumulative distribution function.

**Proof:**

We have obtained the expected profit of the retailer in (6) as,

\[ E[\Pi(z, p)] = [(p - w_r)(g + \mu)] - [(w_r - v)\Lambda + (p + s - w_r)\Phi]. \]

Clearly E is a function of two parameters \( z \) and \( p \). Now to maximize the expected profit function we use the Whitin’s method, see the section-4.3.4 in chapter-4. According to this method we first keep \( p \) fixed in ‘E’ and then use the second order optimality conditions \( \frac{\partial E}{\partial z} = 0 \) and \( \frac{\partial^2 E}{\partial z^2} < 0 \) and determine the optimal solution \( z = z^* \) as a function of \( p \). Then we substitute this \( z^* \) back in \( E[\Pi(z, p)] \) and obtain E as function of single variable \( p \) only so that we can obtain the optimal solution for the price \( p = p^* \) using the second order derivative test for extreme values.

Differentiate \( E[\Pi(z, p)] \) partially w.r.t. \( z \),

\[ \frac{\partial E}{\partial z} = 0 - (w_r - v) \frac{\partial \Lambda}{\partial z} - (p + s - w_r) \frac{\partial \Phi}{\partial z} \]

But \( \Lambda(z) = \int_A^z (z - u) f(u) du \Rightarrow \frac{\partial \Lambda}{\partial z} = \int_A^z (1) f(u) du + 0 - 0 = \int_A^z f(u) du \)

and
\[ \Phi(z) = \int_z^B (u - z) f(u) du \Rightarrow \frac{\partial \Phi}{\partial z} = \int_z^B (u - z) f(u) du = - \int_z^B (u - z) f(u) du \]

\[ \Rightarrow \frac{\partial E}{\partial z} = -(w_r - v) \int_A^z f(u) du + (p + s - w_r) \int_z^B f(u) du \]

\[ \Rightarrow \frac{\partial E}{\partial z} = -(w_r - v) \left\{ \int_A^z f(u) du + \int_z^B f(u) du - \int_z^B f(u) du \right\} + \int_z^B (p + s - w_r) f(u) du \]

\[ \Rightarrow \frac{\partial E}{\partial z} = -(w_r - v) \int_A^z f(u) du + (w_r - v) \int_z^B f(u) du + \int_z^B (p + s - w_r) f(u) du \]

\[ \Rightarrow \frac{\partial E}{\partial z} = -(w_r - v) + \int_z^B \left[ w_r - v + p + s - w_r \right] f(u) du \]

\[ \Rightarrow \frac{\partial E}{\partial z} = -(w_r - v) + \int_z^B [p + s - v] f(u) du \]

\[ \Rightarrow \frac{\partial E}{\partial z} = -(w_r - v) + (p + s - v) \int_z^B f(u) du \]

Now if we use the CDF \( F(z) = \int_A^z f(u) du \) then we have \( 1 - F(z) = \int_z^B f(u) du \). Thus,

\[ \frac{\partial E}{\partial z} = -(w_r - v) + (p + s - v) [1 - F(z)] \quad (5.7) \]

Again differentiate the above equation w.r.t. \( z \) we get,

\[ \frac{\partial^2 E}{\partial z^2} = 0 + (p + s - v) \left[ -\frac{\partial F}{\partial z} \right] \]

But \( F(z) = \int_A^z f(u) du \Rightarrow \frac{\partial F}{\partial z} = \int_A^z \frac{\partial f(u)}{\partial z} du + f(z) \frac{\partial (z)}{\partial z} - f(A) \cdot 0 \Rightarrow \frac{\partial F}{\partial z} = f(z) \Rightarrow \frac{\partial F}{\partial z} = f(z) \).

\[ \frac{\partial^2 E}{\partial z^2} = -(p + s - v) f(z) \quad (5.8) \]
Chapter 5. Exchange rate risk in news vendor format under additive demand error

For the maximum expected profit we must have, \( \frac{\partial E}{\partial z} = 0 \) and \( \frac{\partial^2 E}{\partial z^2} < 0 \).

\[
\frac{\partial E}{\partial z} = 0 \Rightarrow -(w_r - v) + (p + s - v)[1 - F(z)] = 0 , \text{from (7)}.
\]

\[
\Rightarrow (p + s - v)[1 - F(z)] = (w_r - v)
\]

\[
\Rightarrow 1 - F(z) = \frac{w_r - v}{p + s - v}
\]

\[
\Rightarrow F(z) = 1 - \frac{w_r - v}{p + s - v}
\]

\[
\Rightarrow F(z) = \frac{p + s - w_r}{p + s - v}
\]

\[
\Rightarrow z = F^{-1}\left(\frac{p + s - w_r}{p + s - v}\right) = z^*(\text{say})
\]

Now from equation (8) it is clear that \( \frac{\partial^2 E}{\partial z^2} < 0 \) for any value of \( z \).

Thus the optimal value of \( z \) for maximum profit is,

\[
z^* = F^{-1}\left(\frac{p + s - w_r}{p + s - v}\right) \quad (5.9)
\]

Next we substitute this \( z^* \) in equation (6) and find the optimal value of \( p^* \) as discussed in Whitin’s method. So we finally get the optimal order quantity \( q^* \) for the maximum expected profit of the retailer as \( q^* = g(p^*) + z^* \). Thus,

\[
q^* = g(p^*) + F^{-1}\left(\frac{p + s - w_r}{p + s - v}\right) \quad (5.10)
\]

Where \( F^{-1} \) is the inverse cumulative distribution function. Hence the result is proved.

Finally, recall that for manufacturer his purchase cost is \( c \) and his selling price when the retailer bears the risk is \( w \). So the manufacturer’s profit when the retailer bears the exchange rate risk is defined as,

\[\Pi_m = [(\text{selling price of the manufacturer})-(\text{cost of purchase of the manufacturer})] \times \text{no. of units sold}.
\]

That is \( \Pi_m = (w - c)q^* \).

5.3.2 The additive model for the risk taker manufacturer

In this model we assume that the manufacturer bears the exchange rate risk and the retailer does not, under the linear demand with additive error. So in this case the retailer
will have to pay \( wr \) per unit at any point of time in his currency. So the purchase cost of the retailer will be \( w \) only at any time, in the manufacturer’s currency. But as the manufacturer bears the risk, he will get \( \frac{wr}{r(1 + \epsilon_r)} = w_m \) (say) per unit, in his currency according to the existing rate i.e. F.E.R., on the settlement day.

Now the retailer will choose the selling price \( p \) and the order quantity \( q \) so as to maximize his expected profit. The profit function for the retailer is denoted by \( \Pi(p, q) \) and is given by,

\[
\Pi(p, q) = \begin{cases} 
[pD + v(q - D)] - [qw] & \text{if } D \leq q \quad \text{(overstocking)} \\
[pq] - [s(D - q) + qw] & \text{if } D > q \quad \text{(shortage)}
\end{cases}
\]

Then it can be shown that the expected profit of the retailer when the manufacturer bears the risk is obtained just by replacing \( w_r \) by \( w \) in (6). Thus we have

\[
E[\Pi(z, p)] = (p - w)(g + \mu) - (w - v)\Lambda - (p + s - w)\Phi, \text{ Where } z = q - g(p)
\]

Further maximizing the expected profit we get the similar results as in the above model where the buyer bears the risk.

The optimal order of the retailer is,

\[
q^* = g(p^*) + F^{-1}\left(\frac{p + s - w}{p + s - v}\right).
\]

At the end, recollecting that for manufacturer, his purchase cost is \( c \) and his selling price when he is bearing the risk is \( w_m \), we have the manufacturer’s profit when the he bears the exchange rate risk, is given by,

\[
\Pi_m = [(\text{selling price of the manufacturer}) - (\text{cost of purchase of the manufacturer})] \times \text{no. of units sold.}
\]

That is,

\[
\Pi_m = (w_m - c)q^* = \left[\frac{wr}{r(1 + \epsilon_r)} - c\right] q^*
\]
5.4 Summary

We have elaborated the foreign exchange rate risk model under linear demand with additive error, when the buyer or the seller any one of them undertakes the exchange rate risk. We summarize all the key terms and the optimal policies of the two models in the table 5.1 given below. The model’s sensitivity analysis is carried out in the chapter 8 and 9 under various exchange rate error distributions like uniform, normal and beta.

<table>
<thead>
<tr>
<th>The demand model</th>
<th>Buyer bears the risk</th>
<th>Seller bears the risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buyer’s purchase cost</td>
<td>$w_r = wr(1 + \varepsilon_r)/r = w(1 + \varepsilon_r)$</td>
<td>$w$</td>
</tr>
<tr>
<td>Seller’s selling price</td>
<td>$w$</td>
<td>$w_m = \frac{wr}{r(1 + \varepsilon_r)} = \frac{w}{(1 + \varepsilon_r)}$</td>
</tr>
<tr>
<td>Expected profit of buyer</td>
<td>$(p - w_r)(g + \mu) - (w_r - v)\Lambda - (p + s - w_r)\Phi$</td>
<td>$(p - w)(g + \mu) - (w - v)\Lambda - (p + s - w)\Phi$</td>
</tr>
<tr>
<td>Optimum Order quantity $q^*$</td>
<td>$q^* = g(p^*) + F^{-1} \left( \frac{p + s - w_r}{p + s - v} \right)$</td>
<td>$q^* = g(p^*) + F^{-1} \left( \frac{p + s - w}{p + s - v} \right)$</td>
</tr>
<tr>
<td>Expected profit of seller $\Pi_m$</td>
<td>$(w - c)q^*$</td>
<td>$(w_m - c)q^*$</td>
</tr>
</tbody>
</table>

Table 5.1: LDAE model