Chapter 2

Basic Concepts and Terminology
2.1 Basic terms of inventory management

In this section we define and explain some basic terms of inventory management system:

(1) **Inventory**: Inventory is the stock of an item or a resource used in an organization for current and future requirements and smooth running of the organization.

(2) **Inventory system**: An inventory system is the set of rules or policies that observes levels of inventory and determine what levels should be maintained, when stock should be replenished and how large orders should be.

   → By convention, manufacturing inventory generally refers to items that contribute to or become part of a firm’s product output. Manufacturing inventory is typically classified into raw materials, finished products, component parts, supplies, and work-in-process.

   → In distribution, inventory is classified as in-transit, meaning that it is being moved in the system and warehouse, which is inventory in a warehouse or distribution center.

   → Retail sites carry inventory for immediate sale to customers. In services, inventory generally refers to the tangible goods to be sold and the supplies necessary to administer the service.

The inventory system consists of various parameters which are defined as follows:

(3) **Demand**: The number of units or quantity of a product required is known as Demand

   The demand of a product depends on many factors like price, time, inventory level etc or it can be uncertain or even fuzzy.

(4) **Order quantity**: The number of units or quantity of a product which is to be borrowed for an inventory is known as order quantity.

(5) **Supply**: The term supply is defined as the number of units or quantity of a product which is received into an inventory. It is also known as replenishment. Supply or Replenishment is the amount of stock to be added into an inventory and it can be uniform or instantaneous.
(6) **Lead time:** The time duration between placing an order for an inventory and its arrival. If the demand is stochastic the lead time is a major factor in inventory management.

(7) **Cost:** The amount of money required to manage the inventory is known as Inventory cost or simply cost. It can be of various types described as follows:

(a) **Purchase or Product cost:** The cost of purchasing a unit of the product for the inventory is called the purchase cost \( c \). If the cost is independent of the amount \( q \) ordered then the total cost is \( cq \). Alternatively, the product cost may be a decreasing function of the amount ordered.

(b) **Ordering Cost:** The cost of placing an order to an outside supplier or releasing a production order to a manufacturing shop is called ordering cost. If the amount ordered is \( q \) then the ordering cost is a function of \( q \) say, \( c(q) \) and it is often nonlinear.

(c) **Setup cost:** Generally an ordering cost is partly constant and partly variable that depends on the amount ordered. The fixed cost is called the setup cost \( K \).

(d) **Holding or Carrying Cost:** The cost of carrying or holding an item in the inventory for a given unit of time is called holding or carrying cost \( h \). If \( c \) is the unit cost of the product, this component of the cost is \( ca \), where \( a \) is the discount or interest rate. The holding cost may also include the cost of storage, insurance and other factors that are proportional to the amount stored in inventory.

(e) **Shortage or Stock-Out Cost or Penalty Cost:** When a customer wants to buy the product and finds that it is unavailable in the inventory then either the demand of the customer remains unfulfilled or will be satisfied later when the product becomes available. The former case is called a lost sale and the latter is called a back order. In each of these case there is a cost associated to it and is called shortage cost.

(f) **Opportunity Cost:** It is the cost due to lost sale, which is defined by Opportunity cost = gross profit margin + loss of goodwill

(8) **Types of Inventory Model:** Depending on the nature of the parameters (like
demand, order, time etc.) of an inventory system it can be classified in the following types of inventory models:

(a) **Deterministic Inventory Model:** If all the parameters of the inventory are well defined and determined by a rule then such a model is known as deterministic or crisp inventory model.

(b) **Stochastic Inventory Model:** If one or more parameters of the inventory are uncertain or random then we apply some probability distribution to this parameter using past record of the that random variable, then such a model is called probabilistic or stochastic inventory model.

(c) **Fuzzy Inventory Model:** If some parameter of an inventory is imprecise or vague then we consider that parameter as a fuzzy variable and develop an inventory model. Such a model is called a fuzzy inventory model.

### 2.2 Basic calculus for optimization

For optimization of inventory problems the role of calculus is very important. So this section is dedicated to extreme values of function two variables and other concepts.

#### 2.2.1 Extreme value

We shall consider profit function of our inventory model as a function of two independent parameters namely order (q) and price (p) of inventory items. So here we need the concept of extreme values of functions of two variables, which is defined as follows.

**Definitions:**

Let $f : E \subset \mathbb{R}^2 \rightarrow \mathbb{R}$ be a function of two variables $x$ and $y$. Let there be a $\delta$ neighborhood $N$ of a point $(a, b) \in E$ with $N \subset E$.

(i) **Local maximum:** If $f(x, y) \leq f(a, b)$, $\forall (x, y) \in N$ then $f$ said to have local maximum at the point $(a, b)$ and the value of $f(a, b)$ is called local maximum value of the function $f$ at the point $(a, b)$.

(ii) **Local minimum:** If $f(a, b) \leq f(x, y)$, $\forall (x, y) \in N$ then $f$ said to have local minimum at the point $(a, b)$ and the value of $f(a, b)$ is called local minimum value of the function $f$ at the point $(a, b)$. 

(iii) **Saddle point**: If the function \( f(x, y) \) has neither maximum nor minimum at \((a, b)\) then \((a, b)\) is said to be a saddle point of \( f \).

**Theorem**: [Sufficient conditions for extreme values]
Let \( f : E \subset \mathbb{R}^2 \to \mathbb{R} \) be a function of two variables \( x \) and \( y \). Suppose \( f \) has continuous partial derivatives w.r.t. \( x \) and \( y \) up to order 2. Let \((a, b) \in E\). Put \( A = f_{xx}(a, b) \), \( B = f_{xy}(a, b) \) and \( C = f_{yy}(a, b) \).

(A) If \( f_x(a, b) = 0 \) and \( f_y(a, b) = 0 \) and
(B) If, (i) \( AC - B^2 > 0 \) and \( A < 0 \) then \( f \) has local maximum at \((a, b)\).
(ii) \( AC - B^2 > 0 \) and \( A > 0 \) then \( f \) has local minimum at \((a, b)\).
(iii) \( AC - B^2 < 0 \) then \( f \) has a saddle point at \((a, b)\).
(iv) \( AC - B^2 = 0 \) then the test fails.

- The points at which a function \( f(x, y) \) of two variables has local maximum or local minimum are also known as **stationary points** of \( f \).
- Now \( z = f(x, y) \) represents a surface in \( \mathbb{R}^3 \) and the figure 2.1 explains the above concepts geometrically for the surface \( z = f(x, y) = x^3 - y^3 - 3x + 3y \) as an illustration.

![Figure 2.1: Geometrical interpretation of stationary points](image-url)
2.2.2 Leibnitz’s Rule

If \( F(y) = \int_{x=g(y)}^{x=h(y)} f(x, y) \, dx \) then the derivative of \( F \) w.r.t. \( y \) is given by,

\[
F'(y) = \int_{x=g(y)}^{x=h(y)} \frac{\partial}{\partial y} (f(x, y)) \, dx + f(h(y), y) \frac{dh}{dy} - f(g(y), y) \frac{dg}{dy}. \tag{2.1}
\]

2.3 Basic statistical concepts

In this section we recall some concepts of statistics and define probability distributions which are useful in sensitivity analysis of the exchange rate risk models which are developed in the thesis.

2.3.1 Probability

**Random experiment:** An experiment whose all possible outcomes or results are known in advance but which outcome occurs can be determined only after the experiment is performed, is known as a random experiment.

**Sample space:** The set \( S \) of all possible outcomes of a random experiment is called a sample space. e.g. (i) For the random experiment, of tossing a coin the possible outcomes are ‘H’ and ‘T’. So the sample space is \( S = \{H, T\} \).

(ii) For the random experiment, of rolling a die all possible outcomes are positive integers from 1 to 6 on its top face. So the sample space is \( S = \{1, 2, 3, 4, 5, 6\} \).

**Probability function:** Let \( S \) be the sample of a random experiment and \( \Omega \) be the set all subsets of \( S \) i.e. power set of \( S \), whose members are called ‘events’. A function \( P : \Omega \to [0, 1] \) which satisfies the following conditions is called a probability function.

(i) \( P(\phi) = 0 \) and \( P(S) = 1 \).

(ii) If \( A \) and \( B \) are two mutually exclusive events i.e. \( A \cap B = \phi \) then \( P(A \cup B) = P(A) + P(B) \).
Note that the structure containing the sample space $S$, the universal space of events $\Omega$ and the probability function $P$ is called the probability space associated with a random experiment and it is denoted by $(S,\Omega,P)$. Also the probability function can not be defined by some expression but we can only estimate its values. Next we define another way of describing all outcomes of a random experiment as a function called random variable.

**Random variable:** Let $S$ be the sample associated with a random experiment. A function $X : S \rightarrow \mathbb{R}$ is called a random variable.

- If $X$ assumes only discrete values then it is called **Discrete Random Variable** and if it takes all real values in some interval or on $\mathbb{R}$ then it is called **Continuous Random Variable**.

  e.g. One can define a random variable for the experiment of tossing a coin thrice, whose sample space is $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$ as $X : S \rightarrow \mathbb{R}$ as $X(\omega) = \text{number of } H \text{ in } \omega$. , which is a discrete random variable since it can take values from $\{0, 1, 2, 3\}$.

- The advantage of defining a random variable is that we can solve problems like, ‘what is the profit if demand is less than the order?’ or ‘How many times a boxing match can be played until one of the player wins?’; ‘what is the probability that the sum of the numbers on the top faces of two dice is less greater than 6 when they are rolled?’, etc.

### 2.3.2 CDF and PDF of continuous random variable

**Definition:** Let $X$ be a continuous random variable associated with a probability space $(S,\Omega,P)$. A function $F : \mathbb{R} \rightarrow [0,1]$ defined as $F(x) = P(X \leq x)$ is said to be a “Cumulative Distribution Function” (CDF) or simply “Distribution Function” if it satisfies the following conditions:

(i) $x_1 < x_2 \Rightarrow F(x_1) \leq F(x_2), \forall x_1, x_2 \in \mathbb{R}$.

(ii) $F$ is right continuous for $\forall x \in \mathbb{R}$ i.e. $F(x^+) = F(x)$.

(iii) $\lim_{x \to -\infty} F(x) = 0$ and $\lim_{x \to +\infty} F(x) = 1$

- In simple language the value of $F(x)$ is the probability $P(A)$, of an event $A = \{X(\omega) \leq x : \omega \in S\}$.

- The CDF $F(x)$ is also denoted by $F_X(x)$.
**Definition:** Let X be a random variable associated with a probability space \((S, \Omega, P)\) and F be the CDF of X. Then the “probability density function” (PDF) of the random variable X is denoted by \(f(x)\) and it is defined as \(f(x) = \frac{d}{dx} F(x)\) OR \(F(x) = \int_{-\infty}^{x} f(t)dt\).

- The pdf \(f(x)\) of a random variable has the following properties:
  1. \(f(x) \geq 0, \forall x \in \mathbb{R}\)
  2. \(\int_{-\infty}^{+\infty} f(t)dt = 1\).
- The probability density function of a discrete random variable is known as “probability mass function” (pmf) and it is denoted by \(p(x)\).

**Definition:** Let X be a continuous random variable with a probability density function \(f(x)\). Then the “expected value” of the random variable X is denoted by \(E(X)\) or \(\mu\) and it is defined as, \(E(X) = \int_{-\infty}^{+\infty} xf(x)dx\).

- The expected value of a random variable, as the name suggests, gives the approximate average value that the variable X assumes.

**Definition:** Let X be a continuous random variable with a probability density function \(f(x)\) and suppose \(g(X)\) is a function of the random variable X. Then the “expected value” that the function \(g\) assumes, is denoted by \(E(g(X))\) and it is defined as, \(E(g(X)) = \int_{-\infty}^{+\infty} g(x)f(x)dx\).

### 2.3.3 Probability distributions

In this section we consider some important probability density functions which are useful to measure the error or fluctuation in a random variable. We also list its moments and discuss its truncated distributions.

**(A) Uniform distribution:**
The uniform probability density function of a random variable X is defined as,

\[
f(x) = \begin{cases} 
\frac{1}{b-a}, & \text{if } a \leq x \leq b \\
0, & \text{otherwise}
\end{cases} \tag{2.2}
\]
The CDF of uniform probability density function is given by,

\[ F(x) = \begin{cases} 
0, & \text{if } x < a \\
\frac{x-a}{b-a}, & \text{if } a \leq x \leq b \\
1, & \text{if } x > b
\end{cases} \]  

(2.3)

- **Mean and standard deviation of the uniform distribution:**

  (i) Mean: \( \mu = \frac{a + b}{2} \)

  (ii) Standard deviation: \( \sigma = \sqrt{\frac{(b - a)^2}{12}} \)

The graphical representation of uniform pdf (fig-2.2) and cdf(fig-2.3) is as follows:

![pdf of uniform distribution](image)

Figure 2.2: pdf of uniform distribution
(B) Normal distribution:
The normal probability density function of a random variable $X$ with mean $\mu$ and standard deviation $\sigma$ is defined as,

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x-\mu}{\sigma}\right)^2}$$  \hspace{1cm} (2.4)$$

The CDF of normal probability density function, which is also known as Gaussian distribution, is given by,

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{t-\mu}{\sigma}\right)^2} dt$$  \hspace{1cm} (2.5)$$

• The standard normal distribution:
The normal distribution with $\mu = 0$ and $\sigma = 1$ is called the “standard normal distribution” and its PDF and CDF are respectively given as:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$  \hspace{1cm} (2.6)$$

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$  \hspace{1cm} (2.7)$$
The graphical representation are given for normal pdf (fig-2.4) and for normal cdf (fig-2.5) for $\mu = 0, \sigma = 1$ below:

Figure 2.4: pdf of normal distribution

Figure 2.5: cdf of normal distribution
(C) **Beta distribution:** The general beta probability density function of a random variable X with parameter $\alpha > 0$ and $\beta > 0$ over $[a,b]$ is defined as,

$$f(y/a,b,\alpha,\beta) = \frac{(y-a)^{\alpha-1}(b-y)^{\beta-1}}{B(\alpha,\beta)(b-a)^{\alpha+\beta-1}}$$  \hspace{1cm} (2.8)

Taking $a=0, b=1$ we get the **standard beta density function** with parameter $\alpha > 0$ and $\beta > 0$ as below:

$$f(x/0,1,\alpha,\beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$$  \hspace{1cm} (2.9)

$\rightarrow$ Where the beta function is $B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt$.

The CDF of the standard beta density function is given by,

$$F(x) = \int_0^x \frac{t^{\alpha-1}(1-t)^{\beta-1}}{B(\alpha,\beta)}dt$$  \hspace{1cm} (2.10)

The graphical representation of the pdf (fig-2.6) and the cdf (fig-2.7) of beta distribution is as follows:

![Graph](image.png)
Mean and standard deviation of the standard beta distribution:

(i) Mean: \( \mu = \frac{\alpha}{\alpha + \beta} \)

(ii) Standard deviation: \( \sigma = \sqrt{\frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}} \)

(D) Exponential distribution:

The exponential probability density function of a random variable \( X \) which assumes only positive values with parameter \( \alpha > 0 \) is defined as,

\[
f(x) = \frac{1}{\alpha} e^{-\frac{x}{\alpha}} \tag{2.11}
\]

The CDF of uniform probability density function is given as,

\[
F(x) = \int_0^x f(t)dt = 1 - e^{-\frac{x}{\alpha}} \tag{2.12}
\]
Chapter 2. Basic Concepts and Terminology

2.4 Basic terminology of exchange rate risk models

In this section we explain the basic terms used in the core part of the thesis. We first define some of the terms used in the news vendor model which is the base of the research work.

2.4.1 News vendor terminology

(1) **Order:** An order quantity is the number of units of the product that is to be borrowed and kept in the inventory. It is denoted by $Q$ or $q$. An order can be the number units in the safety stock or target inventory or capacity. In the news vendor problem an order is to be placed only once and before the selling season begins. So it is a decision variable and the vendor needs to find its optimal value for the maximum profit.

(2) **Supply:** Supply indicates the number of units of the product which is delivered to the vendor or buyer by the manufacturer. In the NVP we assume that the order is equal to supply.

(3) **Demand:** The number of units of the product that customer wants to buy is known as the demand. It is denoted by $D$. Demand can be a function of price or time or other parameters. If the demand depends upon the price of the product then it must be a decreasing function of the price. It may be discrete or continuous. Generally demand is not known and it is uncertain, it can be taken as a random variable.

(4) **Selling price:** The amount of money for one unit of the product at which the vendor or retailer wants to sell, is known as selling price or simply price. It is denoted by $p$.

(5) **Cost:** The amount of money to be paid for keeping or borrowing one unit of the product in an inventory is called the cost. It is denoted by $c$. The cost may or may not depend on the order quantity ($q$).

(6) **Salvage value:** If the vendor orders more units of the product than the actual demand then he has overstocking of the items. So he sells some of the leftover at a very low price which is called salvage revenue and disposes the remaining items
which contributes cost of disposal. The **salvage value** is defined as \( s = (\text{salvage revenue}) - (\text{cost of disposal}) \) per unit of the product.

(7) **Overage cost:** The overage cost is denoted by \( c_o \) and is defined as \( c_o = (\text{the cost per unit}) - (\text{the salvage value of the item per unit}) = c - s \) for a single period. The overage cost occurs only if the vendor orders more units of the product than the actual demand i.e. when he overstocks the items.

(8) **Underage cost:** The underage cost is denoted by \( c_u \) and is defined as \( c_u = (\text{the price per unit}) - (\text{the cost of the item per unit}) = p - c \), for a single period. It is also called **shortage cost** or **penalty cost**. The overage cost occurs only if the vendor orders less units of the product than the actual demand i.e. due to understocking of the items.

(9) **Profit:** The profit is the revenue obtained from the items of the inventory less the expenses incurred for all the items in the inventory. It is denoted by \( \Pi \).

### 2.4.2 Exchange rate risk model basic terminology

Next we explain the concept of exchange rate error and risk hedging of our main foreign exchange transaction exposure model:

(1) **Demand error:**

Consider a retailer who wants to order \( q \) units from a foreign manufacturer of a certain product. The demand \( (D) \) which is uncertain remains partly fixed and partly random. The random or uncertain part of the demand is known as **demand error** or **demand fluctuation**. It is denoted by \( \varepsilon \) and estimated using some PDF.

- The demand error does not depend upon the price and can be modelled either in an additive or a multiplicative fashion. Specifically, demand is defined as \( D(p, \varepsilon) = y(p) + \varepsilon \) in the additive case Mills [72] and \( D(p, \varepsilon) = y(p) \cdot \varepsilon \) in the multiplicative case Karlin & Carr [55], where \( y(p) \) is a decreasing function that captures the dependency between demand and price, and \( \varepsilon \) is a random variable with its range \([A, B]\). The function \( y(p) \) is the deterministic demand and it is formulated as a **linear demand** function \( y(p) = a - bp \) in additive case and as **iso-elastic** demand \( y(p) = ap^{-b} \) in multiplicative case.
(2) Exchange rate:
   An exchange rate is the rate of 1 unit of the currency of one country in terms of the currency of the other country. It is denoted by \( r \) in the exchange rate model.

(3) Exchange rate error:
   In an international business of supply-chain there is always a crucial factor namely the change in the exchange rate of the currencies of the two countries involved. The exchange rate is not constant but it depends upon unanticipated market changes and it is always uncertain or random. Also as in case of demand, the exchange rate \( r \) can be partly constant and partly random. The randomness in the exchange rate \( r \) is called the exchange rate error (or fluctuation). It is denoted by \( \varepsilon_r \) and it is also a random variable.

(4) Future exchange rate: If the amount for buying one unit of a product is to be paid after some fixed time interval in an international business then we consider the conversion rate between the currencies of two countries as \( r + \varepsilon_r \) (additive form of exchange rate error) or \( r + r \varepsilon_r \) (multiplicative form of exchange rate error), which is known as Future Exchange Rate (F.E.R.)
   - The F.E.R. is the actual exchange rate that a retailer is going to pay or the manufacturer is receiving.
   - We shall consider only the multiplicative form of the F.E.R. in the exchange rate risk model and estimate it using various distributions-uniform, normal and beta.

(5) Exchange Rate Risk: Foreign exchange risk (also known as FX risk, exchange rate risk or currency risk) is a financial risk that exists when a financial transaction is denominated in a currency other than that of the base currency of the company.