GENERAL INTRODUCTION
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Considerable progress has been made over the last three decades in understanding nuclear ground state and low excited properties within the self-consistent Hartree-Fock (HF) theory. This has been possible due to the development of phenomenological effective nucleon-nucleon (n-n) interactions, which are nothing but simple mathematical parameterisations of Brueckner-Bethe reaction matrix[1,2]. Amongst the various n-n effective interactions the Skyrme-type interactions[3-6] because of their simplicity in handling, have been used extensively for the purpose. The HF energy density for these interactions becomes a functional of local one-body densities only, and the HF equations take the form of simple Schrodinger-like equations with local mean fields.

However, the HF calculation with effective interactions does not allow a physical insight into the way how different nuclear properties are connected to each other and their explicit dependence on the various ingredients of the effective interactions used to obtain them. This and the time factor involved in certain situation (such as, global description of nuclear properties) have given rise to a strong motivation for developing semiclassical methods, which avoid the tedious task of computing the single particle wave functions for the nucleus. This semiclassical calculations, first initiated by Brueckner and
collaborators[7-9] under the banner of energy density formalism (EDF) has been extensively used as the starting point of many nuclear calculations in the subsequent years with necessary sophistications.

The energy density formalism is based on the theorem of Kohn and Coworkers[10] according to which the ground state energy of a system of fermions can always be expressed as a functional of the local densities and their derivatives. The energy densities used in nuclear calculations, therefore, has a non-gradient part and a gradient part having derivative terms of various orders.

The non-gradient part of the energy density which is mainly determined by the saturation property of the nuclear matter (NM), such as, binding energy per particle, equilibrium density and incompressibility can be obtained accurately for any given effective n-n interaction. On the other hand, the exact form of the gradient terms in the energy density is not correctly known and they have been approximated either in simple forms obtained under empirical reasoning[7,8,11] or in terms of a semiclassical \( \hbar \)-expansion developed by Wigner[12] and Kirkwood[13] and subsequently used by Bhaduri and Ross[14]. Latter on, Grammaticos and Voros developed another semiclassical \( \hbar \)-expansion[15] which involves the expansion of the density matrix in the powers of \( \hbar \).
Although, both the semiclassical methods of Bhadhuri & Ross and Grammaticos & Voros have been obtained from different considerations both of them give the same results.

In the absence of a clear cut knowledge about the form of the derivative terms in the energy density, most of the calculations in the EDF have resorted to the use of the semiclassical $\hbar$-expansion. If the terms upto $\hbar^2$ only are retained in this $\hbar$-expansion, then the variational equation (Euler–Lagrange(EL) equation) for the density turns out to be rather a simple second order differential equation which is relatively easy to handle. Because of this mathematical simplicity in handling the problem, most of the semiclassical calculations have used derivative terms upto second order. However, it has been shown that the energy densities containing only second order derivative terms fail to reproduce, simultaneously, the correct surface energy and the correct behavior of the density in the outer part of the nuclear surface[16] and it becomes necessary to include fourth order derivative terms in the energy density.

If terms upto order $\hbar^4$ are retained in the $\hbar$-expansion, then the resulting energy density becomes very much complicated involving highly non-linear fourth order derivative terms and the corresponding EL equations become highly non-linear fourth order
differential equations which are extremely difficult to solve. Apart from this, it gives a density profile which behaves unphysically as \( r^{-\sigma} \) in the outer part of the nuclear surface i.e., \( \rho(r) \xrightarrow{r \to \infty} r^{-\sigma} \)[17]. Moreover, the inclusion of higher order terms, \( h^n n \geq 6 \), of the \( h \)-expansion leads to a divergence of the series[18].

In view of the above mentioned facts about the nature of the gradient terms in the energy densities, the motivation of the present dissertation is to construct a suitable energy density containing both second and fourth order gradient terms so that it reproduces, simultaneously, the correct value of the surface energy and the correct density profile in the nuclear surface. Using this energy density various nuclear properties have been studied, where emphasis has been given, throughout the work, in obtaining various physical quantities of interest in simple analytical forms to show the explicit dependence of these quantities on the different ingredients of the \( n-n \) effective interaction and also on the nuclear matter bulk properties such as, binding energy per particle, equilibrium density and incompressibility.

Chapter-I of this dissertation, gives a brief review of the details of the nature of the energy densities used earlier in the
various nuclear calculations together with the energy densities obtained from the HF-Skyrme energy density in the so-called extended Thomas-Fermi(ETF) approximation and stress has been given on the different drawbacks of these energy densities.

In Chapter-II, an attempt has been made to construct an energy density starting with the Skyrme interaction using the semiclassical $\hbar$-expansion upto second order in the exact form. We have then added a simple fourth order gradient term to the energy density. The fourth order gradient term has been chosen in such a convenient form that it allows the same simplification as in the case of an energy density containing second order gradient terms only. Simple analytical relations for different quantities of interest in a semi-infinite nuclear medium (SINM) have been obtained. The necessity of the inclusion of fourth order gradient terms in the energy density for correct description of the surface energy and the shape of the density in the tail region has been shown explicitly. It has also been shown that a suitable choice of the fourth order gradient term provides a theoretical justification for parameterising the nuclear density in a generalised Fermi shape ($F-\nu$).

Such $F-\nu$ density distributions are characterised by three parameters, namely, the central density $\rho_0$, the skewness parameter
v and the surface diffuseness parameter a. In case of the nucleus, while a and v determine the surface thickness t (90\% 10\% fall-off distance), the nuclear mean square radii are determined mainly by the central density \( \rho_o \). In Chapter-III, an attempt has been made to study the dependence of the central density and the surface thickness on the size of the nucleus. This study has been carried out with the help of the leptodermous expansion which is a good approximation for not too small nuclei. For the purpose of this study, a simple approximation to the surface energy density has been used so that the EL equation in SINM limit gives a pure Fermi distribution (F-1) for the density. This approximation, as has been shown, has the advantage of describing the variation of central density and surface thickness with the size of the nucleus in a way quite independent of the parameters involved in the approximated surface energy density.

The non-gradient part of the energy density which is entirely determined by the binding energy per particle in nuclear matter and is expressed as a functional of the local density is usually characterised by an equilibrium saturation density \( \rho_{nm} \), binding energy per particle \( \delta(\rho_{nm}) \), incompressibility K and effective mass \( m^*/m \). The values of these four parameters, particularly, those for K and \( m^*/m \), as referred to in the literature for different sets of n-n effective interactions span over wide range of values.
In spite of this, several sets of effective n-n interactions are found to provide a fair description of the ground state properties of nuclei like binding energies and radii causing a confusion about the correct equation of state (EOS) of nuclear matter. In view of this, the motivation of Chapter-IV of the dissertation has been to make a systematic analysis of the EOS of cold nuclear matter and to find its appropriate relevance to the nuclear surface. This has been done by using a restricted type of EDF, so that the density profile in SINM limit as prescribed by the corresponding EL equation is determined entirely by the EOS of cold nuclear matter. This has the advantage of showing the explicit dependence of nuclear surface characteristics on the saturation density and incompressibility of nuclear matter. Moreover, this systematic analysis, as has been shown, leads to the correct EOS of cold nuclear matter and constrains the nuclear matter incompressibility within a value 310 Mev to 315 Mev.

Emphasis has also been given in this Chapter-IV to show the explicit dependence of nuclear properties which are independent of the size of the nucleus, on nuclear matter properties like binding energy per particle, equilibrium density and incompressibility. Such nuclear properties are surface tension, surface thickness, moments of the surface energy densities and interaction energy per unit area between two semi-infinite nuclear slabs facing each
other. The surface thickness and moments of the surface energy density are very important in obtaining the total energy of a nucleus in the leptodermous approximation. On the other hand, the interaction energy per unit area between two semi-infinite nuclear slabs facing each other is crucial in the appropriate determination of the force acting between two interacting nuclei and also in the determination of the interaction energy between them. Simple analytical relations have also been obtained for the distance of closest approach and for the interaction energy in the tail region (corresponding to the large separation between the two SINM slabs facing each other). The behavior of the interaction energy in the tail region is quite important for scattering and various types of reactions involving two heavy-ions(HI).

Chapter-V contains an extensive discussion and conclusions on the work presented in this dissertation.