CHAPTER 1

1.1 Introduction

Fluid Mechanics is the science that deals with the behavior of fluids at rest or in motion. It has a number of sub-disciplines that incorporate fluid dynamics, hydrodynamics and aerodynamics. Some of the applications of fluid dynamics are moments on aircraft, calculating forces, predicting weather patterns and determining the mass flow rate of petroleum through pipeline. In fluid dynamics the research has been extended into the exotic regimes of hyper velocity flight and flow of electrically conducting fluids. It has introduced new fields of interest such as hypersonic flow and magneto fluid dynamics. Hence, it is necessary to combine knowledge of thermodynamics, electromagnetic theory, mass transfer, heat transfer and fluid mechanics to fully understand the physical phenomena involved.

The analysis of fluid dynamics offers a logical structure that underlines these practical discipline which embrace empirical and semi-empirical laws that are derived from flow measurement and are used to solve applied problems. The solution to a fluid dynamics process typically contains manipulating different properties of the fluid namely velocity, temperature, pressure and density as functions of space and time.

Two approaches were carried out to study the fluid motion in 19th and 20th centuries. The first one is theoretical and the second is experimental. Several researchers contributed to examine the fluid behavior by implementing both of these approaches. But now-a-days, due to the influence of latest digital computers, a third approach came into existence to study the characteristics of fluid dynamics: computational fluid dynamics, or CFD for short. Computational fluid dynamics is utilized in most of the latest industrial and technical process to employ the fluid flow pattern than either theory or experiment. The possible theoretical studies have already been done, and studies through experiments are in general expensive and difficult. In the present days, the expenses on computing process are reduced and hence, computational fluid dynamics is given more priority to analyze the fluid flows in Science and Engineering.
1.2 Heat Transfer

Heat transfer is the transfer of thermal energy or simply heat from a hotter object to cooler object caused by temperature variation. When an object or fluid is at different temperature than its surroundings or another object, transfer of thermal energy takes place in such a way that the body and the surroundings reach thermal equilibrium. Heat transfer always occurs from a higher temperature object to a cooler temperature as described by the second law of thermodynamics or the Clausius statement. Where there is a temperature difference between objects in proximity, heat transfer between them can never be stopped, it can only be slowed.

There are many applications of heat transfer in science and technology. Mostly all methods of power generation involve fluid flow and heat transfer as necessary processes. The power plants generate electricity by converting heat contained within a working fluid. Coal will be burnt to produce hot burning products and these hot gases cause heat transfer to water which serves as the working fluid. Hence the engineer must provide heat transfer wherever it is needed. Heat transfer is utilized as a by-product in engineering process. In some cases if heat is not reduced, it may cause damage to the apparatus. So it is necessary to remove the heat from where it is not needed. In order to maintain a comfortable environment in commercial buildings heat must be provided based on the requirement. For economy, it is desirable to slow down the rate at which heat is lost from buildings. Thus, the engineer/scientist is concerned with preventing loss of heat from the area where the heat should be kept. The metallurgical and chemical industries use apparatus such as condensers, heat exchangers, furnace, and reactors. The process of heat transfer is commonly the limiting factor in the design of electrical circuits and electronic machines. In the course of time, temperature differences in a body are reduced by heat flowing from regions of higher temperature to those of lower temperature.

All fields of engineering deal with problems of heat transfer and fluid flow. Aerospace engineers are concerned with heat transfer in high-speed flow. Chemical engineers attend for input or removal of heat, depending on whether processes are endothermic or exothermic. Civil engineers and architects must be disturbed with heat transfer in the design and construction of buildings. Electrical engineers are disturbed with proper operating temperatures of equipment such as computers and electronic
devices. Mechanical engineers are concerned with many heat transfer situations, such as the automobile engines. Nuclear engineers are concerned with dynamic heat removal problems in fusion reactors. Thus, heat transfer is a basic engineering science of general concern for the engineering professionals.

1.2.1 Basic modes of heat transfer

Heat transfer is the transmission of energy from one region to another region as a result of temperature gradient that takes place by the following three modes.

a. Conduction: Conduction is the process of heat transfer through direct contact of adjacent molecules of matter. Heat transfer by conduction occurs mainly by elastic impact as in fluids or by free electron dispersion as leading in metals. In other words, heat is transferred by conduction when adjacent particles vibrate against one another, or as electrons travel from atom to atom. Conduction is superior in solids, where atoms are in stable contact. In liquids and gases, the molecules are generally further apart, giving a lesser chance of atoms colliding and passing on thermal energy.

b. Convection: The convection mode of heat transfer is the bulk diffusion of molecules of heater parts with cooler parts of a liquid or gas. This mode of heat transfer also relates to the energy transfer between a solid surface and fluid. If the fluid flow takes place through a solid surface by applying force with the use of a fan, mechanic pump, or any apparatus, the resulting heat transfer is called as forced-convection. On the other hand heat transfer by convection is known as free or natural convection, when cooler or warmer fluid adjacent to the solid allows a transmission due to the density variation caused by temperature variations of the fluid particles. Some examples of convection are cooling of a car radiator when the air is being dispersed by a fan, hot cup of coffee being cooled by blowing over the surface, preparation of foods in a vessel being stirred, and so on.

c. Radiation: The process of radiation is a familiar process of heat transfer. When two objects are placed at different temperature apart from a finite distance in a perfect vacuum, a net energy transfer occurs from the higher temperature object to the lower temperature object, even though there is no medium between the two objects to
support heat transfer. This net energy transfer process is called thermal radiation or radiation. This is the mechanism whereby the Sun transmits heat to the earth.

1.3 Mass Transfer

Mass transfer is defined as the transfer of matter by virtue of species concentration difference in a system. The difference in concentration provides a driving force for the transfer of mass. The phenomena of mass transfer are very common in the theory of stellar structure and observable effects are detectable at least on the solar surface. The involvement and application of mass transfer process goes to greater lengths in numerous fields of science, engineering and technology. Mass transfer operations quite often occur in the fields of electric engineering, civil engineering, aeronautics, metallurgy, environmental engineering, refrigeration, air conditioning, biological and industrial process. The study of geophysics, astronomy, meteorology, agricultural oceanography and food processing demands the knowledge of heat and mass transfer. Mass transfer flows are highly significant for their varied practical importance. Many examples of mass transfer applications can be cited from the environment.

Mass transfer occurs in two mechanisms.

a. Diffusion mass transfer:
In diffusion mass transfer the transfer of matter occurs by the movement of molecules or species or particles of one component to another. Diffusion mass transfer may occur either due to concentration gradient or temperature gradient or pressure gradient.

b. Convective mass transfer:
Convective mass transfer is a mechanism in which mass is transferred between the fluid and the solid surfaces as a result of movement of matter from the fluid to the solid surface.

1.4 Basic properties of Fluid dynamics

a. Viscosity:
The viscosity of a fluid is a significant property in the study of liquid behavior and fluid motion in the vicinity of a solid boundary. It is the result of intermolecular forces exerted as layers of fluid attempt to side by one another. The relation between the shearing stress and the transverse velocity gradient is given by $\tau_{xy} = \mu \frac{\partial u}{\partial y}$. In this relation, the constant of proportionality $\mu$ refers to the coefficient of viscosity or the dynamic viscosity. Due to shearing stress, a viscous fluid generates resistance to the body passing through it as well as between the particles of the fluid itself. Water and air are treated as inviscid fluids whereas syrup and heavy oil are treated as viscous fluids.

b. Density:

It is defined as mass per unit volume and is denoted by $\rho$

$$\rho = \frac{m}{v}$$

Where $m = \text{mass}$, $v = \text{volume}$

c. Specific volume: The specific volume of a fluid is the volume by a unit mass and is clearly the reciprocal of the density. If a fluid requires a large variation in pressure to produce some appreciable variation in the density refers to incompressible fluid and the remaining fluids to compressible.

The specific volume is denoted by $V_s$ and is given by

$$V_s = \frac{1}{\rho}$$

d. Specific weight:

It is the weight per unit volume. It is also known as weight density. This is denoted by $W$ or $\gamma$. i.e., $\gamma = \rho g$ where $g$ is the acceleration due to gravity.

e. Pressure:

It is defined as the force exerted per unit area.
\[ P = \frac{F}{A} \text{ where } F = \text{force}, \ A = \text{area}. \]

**f. Kinematic viscosity:**

It is the ratio of dynamic viscosity to density.

\[ \nu = \frac{\mu}{\rho} \text{ where } \mu = \text{viscosity}, \ \rho = \text{density} \]

**g. Heat generation/absorption:**

In many practical applications, it is observed that the generation or absorption of heat takes place within the body itself. The examples of such phenomena are in nuclear reactors, I.C. engines especially in between cylinder and piston, in chemical process, in combustion process, in structural applications of drying and setting of concrete, in electrical conductors etc.,. In all above applications, the temperature distribution can be determined by analyzing heat generation/absorption within the systems.

**1.5 Hall effect**

Hall Effect refers to the production of a transverse electric field when a current carrying conductor is placed at right angles to applied magnetic field. Thus when a block of metal carrying a current of density \( \overrightarrow{J} \) parallel to the \( y \) axis is placed in a field of magnetic induction \( \overrightarrow{B} \) parallel to the \( z \) axis, a potential difference appears across the metal in the direction of \( x \) axis. The Hall effect supplies the information of the sign of charge carrier. The magnetic induction \( B \) exerts a force on the charged particles carrying the current, displacing them in the \( x \) direction. This sets up a non-uniform charge density which gives rise to an electric field in the \( x \) direction. In equilibrium, the force due to this field must just balance that due to the magnetic field so that the Lorentz force acting on an electron,

\[ E = \frac{\overrightarrow{J}}{ne} \times \overrightarrow{B} = -R_H (\overrightarrow{J} \times \overrightarrow{B}) \]
Where \( \bar{J} \) = current density and \( R_H = \frac{-1}{ne} \) = Hall coefficient.

The negative sign is introduced explicitly to emphasize that we would expect \( R_H \) to be negative for electrons of charge negative \( e \). Suppose \( V_H \) is the magnitude of the Hall voltage and \( d \) is the width of the metal slab, then the electric field intensity set up across the slab is given by \( E = \frac{V_H}{d} \).

Now-a-days Hall effect is used to determine the sign of the charge carrier in a material and to develop e. m. f. (electro motive force) and to do refrigeration. Thus we have Hall generators and Hall refrigerators based on Hall voltage.

1.6 Porous medium

Porous medium is a continuous solid phase with many pores in it. Examples are sponges, clothes, wicks, paper, sand, gravel, filters, concrete, bricks, plaster walls, many naturally occurring rocks, packed beds used for distillation, absorption etc. Most of the studies of flow in porous media assume the Darcy’s law is valid. However this law is known to be valid only for relatively slow flows through porous media. In general we must consider the effect of fluid inertia as well as of viscous diffusion at boundaries which may become significant for material with high porosities such as fibrous and foams.

The study on flows through porous media is of great interest in many scientific and engineering applications. A study on such type of flows is applied to the problems of movement of underground water resources and for filtration and water purification process. The petroleum industry has been showing a lot of interest in these problems in connection with the crude oil production from the underground reservoirs. These reservoirs contain many process materials like limestone and dolomite where oil is preserved. Oil can be obtained by drilling wells down into the reservoir and can be allowed the oil to flow through the porous regions of the well. Since the percentage of oil recovery is an important factor in the oil economy, it is necessary to apply to know how concerning the mechanics of oil production to increase the recovery percentage.
The textile technologist is interested in fluid flow through fibers; biologists are interested in water movement through plant roots of the cells of living systems.

1.7 Boundary layer flows:

The study on fluid flows at the boundary layer plays a significant role in fluid dynamics. A boundary layer is a thin layer closest to the boundary where maximum viscous dissipation takes place resulting in high velocity gradient and shear stresses with consequent drag on the surface. Boundary layer theory encompasses the study of velocity gradient, forces, shear stresses and energy loss in the boundary layer. On the other hand, the boundary layer concept is used to find the solution of viscous flow problems through application of Navier-Stokes equations to the complete flow field. Thus, the introduction of boundary layer concept marked the beginning of the modern era of fluid mechanics.

1.8 Magnetohydrodynamics:

Magnetohydrodynamics (MHD) is an important branch of fluid dynamics. It is concerned with the interaction of electrically conducting fluids and electromagnetic fluids. When a conducting fluid moves through a magnetic field, an electric field and consequently current may be induced and in turn the current interacts with the magnetic field to produce a body force.

According to Faraday, when a conductor carrying an electric current moves in a magnetic field, it experiences a force tending to move it at right angles to the electric field and conversely, when a conductor moves in a magnetic field, a current is induced in the conductor in a direction mutually at right angles to both the field and the direction of motion.

In the case when the conductor is either a liquid or gas, electromagnetic forces will be generated and may be of the same order of magnitude as the hydrodynamical and inertial forces. Thus the equations of motion will have taken these electromagnetic forces into account in addition to the other forces. The science which treats these phenomena is called Magnetohydrodynamics.

MHD interactions occur both in nature and in new man-made devices. MHD flow occurs in the sun, the earth interior, the ionosphere, and the stars and their
atmosphere, to mention a few. In the laboratory many new devices have been made which utilize the MHD interaction directly, such as propulsion units and power generators or which involve fluid-electromagnetic field interactions, such as electron beam dynamics, travelling wave tubes, electrical discharges and many others.

MHD has many applications in science and industry. To name a few, Astrophysical Geophysical and Cosmic physics. It is still very important, in the problem of fusion power. It has applications in the creation and containment of hot plasmas by electromagnetic forces, since material walls would be destroyed. Astrophysical includes solar structure especially in the outer layers, the solar wind bathing the earth and other planets, and interstellar magnetic fields.

1.9 Viscous fluids or Real fluids:

Real fluids are those fluids which are available in nature. These fluids possess the property such as viscosity, surface tension and compressibility and hence a certain amount of resistance is always offered by these fluids when they are set in motion. Viscous or real fluids are classified into following categories.

I. Newtonian fluids

II. Non Newtonian fluids

a. Newtonian fluids:

Fluids which obey the Newton’s law of viscosity are known as Newtonian fluids i.e., For Newtonian fluids, the relationship between shear stress and velocity gradient is linear.

Ex: water, air, gasoline, oils etc.

b. Non-Newtonian fluids:

Fluids which do not obey the Newton’s law of viscosity are called as non-Newtonian fluids i.e., for non-Newtonian fluids, the relationship between shear stress and velocity gradient is not linear. Kuvshinski fluid, Casson fluid, visco-elastic fluid, dusty fluid are some well-known non-Newtonian fluid models.

Ex: Milk, blood, liquid cement, concentrated solution of sugar etc.
1.10 Casson fluid

Casson fluid is one of the non-Newtonian fluid models introduced by Casson in 1995. It is based on the model structure and its behavior of both liquid and solid of a two-phase suspension that exhibits yield stress. Casson fluid is well known for shear thinning liquid which is formed to an infinite viscosity at zero, if the shear stress less than the yield stress is applied to the fluid; it’s like a solid, if the shear stress larger than yield stress is applied, and its starts to move. Examples of Casson fluid are as follows: Jelly, tomato sauce, honey, soup, concentrated fruit juices. Human blood also treated as Casson fluid.

1.11 Soret and Dufour effects

The driving potentials and fluxes are associated with each other in a moving fluid in the existence of thermal and solutal buoyancy. The studies related to this phenomenon experienced that the energy flux can be produced by concentration gradients and also temperature gradients. The energy flux caused by a concentration gradient is known as Dufour (diffusion-thermo) effect. In addition to this process, mass flux can be generated by temperature gradients and this defines the Soret (thermal-diffusion) effect.

1.12 Dimensionless parameters:

Dimensionless numbers are the numbers which are obtained by dividing one force acting on a fluid with another force or one kind of energy transfer with another kind of energy. These non-dimensional numbers are very important to understand the flow behavior of any fluid or its heat transfer characteristics. Some common non dimensional parameters are given below.

a. Reynolds number:

It is given by the ratio of inertia force to viscous force.

\[ R_e = \frac{F_i}{F_v} \]
If the value of Reynolds number is low then the viscous forces are predominant and the fluid flow is laminar. If the Reynolds number is more than the inertial forces are predominant and the flow becomes turbulent.

b. Grashof number:

The Grashof number $Gr$ is a dimensionless number in fluid dynamics and heat transfer which approximates the ratio of the buoyancy to viscous force acting on a fluid. It frequently arises in the study of situations involving natural convection. It is named after the German engineer Franz Grashof. It is written as

$$Gr = \frac{g \beta (T_s - T_\infty) L^3}{\nu^2}, \text{ for vertical flat plate}$$

where $L$ indicates the length scale basis for the Grashof Number.

$g$ = acceleration due to Earth's gravity  
$\beta$ = volumetric thermal expansion coefficient  
$T_s$ = surface temperature  
$T_\infty$ = bulk temperature  
$L$ = length  
$\nu$ = kinematic viscosity

c. Prandtl Number:

The Prandtl number $Pr$ is a dimensionless number approximating the ratio of momentum diffusivity (kinematic viscosity) and thermal diffusivity. It is named after the German physicist Ludwig Prandtl. It can be expressed as

$$Pr = \frac{\mu C_p}{k}$$

Where

$\mu$ = absolute or dynamic viscosity  
$C_p$ = specific heat capacity  
k = thermal conductivity

d. Eckert number:
It is a dimensionless number used in fluid dynamics. It expresses the relationship between a flow's kinetic energy and enthalpy, and is used to characterize dissipation. It is named after Ernst R. G. Eckert.

It is defined as

\[ Ec = \frac{V^2}{C_p \Delta T} = \frac{\text{Kinetic Energy}}{\text{Enthalpy}} \]

Where

- \( V \) is a characteristic velocity of the flow.
- \( C_p \) is the constant-pressure specific heat of the flow.
- \( \Delta T \) is a characteristic temperature difference of the flow.

**e. Pressure coefficient:**

It is the ratio of static pressure difference to dynamic pressure.

\[ C_p = \frac{(P - P_\infty)}{(1/2) \rho V^2} \]

**f. Schmidt number:**

Schmidt number (Sc) is a dimensionless number defined as the ratio of momentum diffusivity (viscosity) and mass diffusivity, and is used to characterize fluid flows in which there are simultaneous momentum and mass diffusion convection processes. It was named after the German engineer Ernst Heinrich Wilhelm Schmidt.

It is defined as

\[ Sc = \frac{\nu}{D} = \frac{\mu}{\rho D} = \frac{\text{Viscous diffusion rate}}{\text{Molecular (mass) diffusion rate}} \]

Where

- \( \nu \) = kinematic viscosity
- \( D \) = mass diffusivity
- \( \mu \) = dynamic viscosity of the fluid
- \( \rho \) = fluid density.
g. Skin friction:

It is a type of friction force which exists at the surface of a solid body immersed in a large volume of fluid which is in motion relative to the body.

Skin friction is defined as drag per unit area and is given by

\[ \tau_{yx} = \mu \frac{\partial \mu}{\partial y} \]

h. Nusselt number:

Nusselt number is a dimensionless parameter used in the calculation of heat transfer between a moving fluid and a solid body.

In heat transfer at a boundary (surface) within a fluid, the Nusselt number is the ratio of convective to conductive heat transfer across (normal to) the boundary. Named after Wilhelm Nusselt, it is a dimensionless number. The conductive component is measured under the same conditions as the heat convection but with a (hypothetically) stagnant (or motionless) fluid.

The convection and conduction heat flows are parallel to each other and to the surface normal of the boundary surface, and are all perpendicular to the mean fluid flow in the simple case.

It is defined as

\[ Nu = \frac{hL}{k} \]

Where

\[ L = \text{characteristic length} \]
\[ k = \text{thermal conductivity of the fluid} \]
\[ h = \text{convective heat transfer coefficient} \]

A Nusselt number of order unity would indicate a sluggish motion little more effective than pure fluid conduction: for example, laminar flow in a long pipe. A large Nusselt number means very efficient convection: For example, turbulent pipe flow yields Nu of order 100 to 1000.
i. Sherwood Number:

It is the ratio of overall mass diffusion to species diffusion. It is denoted by $Sh$ and is given by $Sh = \frac{V_L}{D}$ where $V_L$ is overall mass diffusion and $D$ indicates species diffusion.

1.13 Basic equations of fluid dynamics:

The analysis to be carried out on any fluid motion involves solving the governing equations in the form of non-linear partial differential equations. These equations are called as primary equations of fluid dynamics. The basic governing equations of any flow phenomena in vector form are stated as follows:

**Conservation of mass:**

The law of conservation of mass states that mass can neither be created nor be destroyed. The equation of conservation of mass or equation of continuity takes the following vector. (Yuan [149])

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{q}) = 0 \tag{1.1}$$

Equation (1.1) may be interpreted as follows: For any closed surface drawn in the fluid, the increase in the mass of fluid with in the surface in any time interval must be equal to the excess of mass that flows into the volume through the surface over the mass that flows out. In the case of an incompressible fluid the equation (1.1) reduces to $\nabla.\vec{q}=0$. Here, $\rho$ is density and $\vec{q}$ is the velocity vector of the fluid.

**Conservation of momentum:**

The entire force acting on a fluid mass enclosed in a random volume fixed in space is equal to the time rate of change of linear momentum. In vector form it is given as

$$\rho \left[ \frac{\partial \vec{q}}{\partial t} + \vec{q} \cdot \nabla \vec{q} \right] = \rho \nabla \vec{F} + \mu \nabla^2 \vec{q} \tag{1.2}$$
Here, $\mu$ and $p$ represent viscosity and pressure of the liquid. The momentum equation (1.2) is called as the **Navier-Stokes equation**.

**Conservation of energy:**

The energy added to a closed system increases the internal energy per unit mass of the fluid. In vector form it is given as

$$\rho \frac{\partial u}{\partial t} = -\nabla \cdot \overrightarrow{Q} - p \nabla \cdot \overrightarrow{q} + \phi \tag{1.3}$$

Here, $u$ is called internal energy, $\overrightarrow{Q} = -k \nabla T$ (Fourier’s law of heat conduction) is heat flux vector, $\phi = \nabla \cdot (\tau \overrightarrow{q}) - q \nabla \cdot \tau$ is the dissipation function.

**Mass diffusion:**

The rate of mass diffusion of a chemical species in a stagnant medium in a specified direction is proportional to the local concentration gradient in that direction. This linear relationship between the rate of diffusion and the concentration gradient proposed by Fick in 1985 is known as Fick’s law of diffusion and it can be expressed as

$$\text{Mass flux} = \text{constant of proportionality} \times \text{Concentration gradient}$$

Fick’s law can conveniently be expressed in vector form as $\overrightarrow{j} = -\rho D \nabla u$

Where $\rho$ is the density and $D$ is the mass diffusivity.

Based on the above equations we have formulated mathematically the basis for the specific problems analyzed in the thesis.

1.14 **Mathematical formulation**

In the present thesis, the simultaneous occurrence of heat and mass transfer flow of conducting fluid past an infinite porous plate is considered. In order to interpret and analyze the flow of heat and mass transfer, we use the basic equations which results from the conservation of mass (Continuity equation), conservation of
momentum (Navier-Stokes equation), conservation of energy (Energy equation) and conservation of concentration (Diffusion equation) are of the following form.

\[ \nabla \cdot \vec{q} = 0 \]  
(1.4)

\[ \rho \left[ \partial_{t} \vec{q} + \vec{q} \nabla \vec{q} \right] = \rho \vec{X} - \nabla p + \mu \nabla^{2} \vec{q} \]  
(1.5)

\[ \rho C_p \frac{\partial T}{\partial t} = \kappa \nabla^2 T - \nabla q_r \]  
(1.6)

\[ \frac{\partial C}{\partial t} = D \nabla^2 C \]  
(1.7)

Where \( \vec{q} \) is the velocity vector, \( \rho \) is the density of the fluid near the plate, \( p \) is the pressure, \( \mu \) is the coefficient of viscosity, \( C_p \) is the specific heat at constant pressure, \( T \) is the temperature of the fluid near the plate, \( \kappa \) is the thermal conductivity of the fluid, \( q_r \) is the radiative heat flux, \( C \) is the species concentration in the fluid near the plate and \( D \) is the species diffusion coefficient.

**a. In presence of gravitational body force**

When the body is under gravitational force, then in Cartesian co-ordinate system, the above governing equations can be stated as follows.

\[ \frac{\partial \vec{v}}{\partial y} = 0 \]  
(1.8)

\[ \rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} - \rho g + \mu \frac{\partial^2 u}{\partial y^2} \]  
(1.9)

\[ \rho C_p \frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \]  
(1.10)

\[ \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2} \]  
(1.11)

Here, we note that

\[ -\frac{\partial p}{\partial x} - \rho_{\infty} g = 0 \]  
(1.12)

Substituting equation (1.12) in equation (1.9), we obtain
\[ \rho \frac{\partial u}{\partial t} = -(\rho - \rho_\infty) g + \mu \frac{\partial^2 u}{\partial y^2} \]  

(1.13)

Where \( x \)-axis is taken in the upward direction along the plate opposite to the gravity and \( y \)-axis is taken perpendicular to it. Hence, all physical quantities will be in terms \( y \) and \( t \). For small temperature and concentration variations, the density \( \rho \) in the equation (1.10) can be considered constant except for the term \((\rho - \rho_\infty)\). Boussineq’s first introduced this approximation, since the differences in flow pattern caused by both temperature and concentration difference expressing the effect of buoyancy force through volumetric coefficients, the density differences can be stated as

\[
(\rho - \rho_\infty) = -\rho \left[ \beta (T - T_\infty) + \beta' (C - C_\infty) \right]
\]  

(1.14)

In view of equation (1.14), the equation (1.13) can be written as

\[
\frac{\partial u}{\partial t} = g \beta (T - T_\infty) + g \beta' (C - C_\infty) + \nu \frac{\partial^2 u}{\partial y^2}
\]  

(1.15)

Where \( \nu \) indicates kinematic viscosity of the fluid. The equations (1.8), (1.10), (1.11) and (1.15) represent the governing equations for the flow under consideration.

b. In presence of magnetic body force

Suppose the fluid is electrically conducting and it is assumed that a magnetic field of uniform strength \( B_0 \) is applied transversely and then the interaction between the motion and the magnetic field can be described by the following Maxwell’s equations

\[
\nabla \times \vec{B} = \mu_e \vec{J}
\]

\[
\nabla \cdot \vec{J} = 0
\]

\[
\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}
\]

(1.16)

\[
\nabla \cdot \vec{B} = 0
\]

Ohm’s law can be written in the form

\[
\vec{J} = \sigma \left( \vec{E} + \nabla \times \vec{B} \right)
\]

(1.17)
Where $\vec{B}$ indicates the magnetic induction intensity, $\vec{J}$ represents density of the electric current, $\vec{E}$ denotes the intensity of the electric field, $\mu_e$ stands for magnetic permeability and $\sigma$ is the Stefan-Boltzmann constant. In the equation of motion, the body force that present here indicates $\vec{J} \times \vec{B}$. This force corresponds to the combination of the fluid motion and magnetic field, which is known as Lorentz force. It is assumed that the magnetic Reynolds number is very small and hence the induced magnetic field is ignored. The effect of polarization is also negligible under the consideration that the conductivity is minor when the external field is not applied. It has been taken $\vec{E}=0$ i.e. in the absence of convection outside the boundary layer, $\vec{B} = B_0$ and

$$\nabla \times \vec{B} = \mu_e \vec{J} = 0.$$ Then equation (1.17) leads to $\vec{J} = \sigma (\nabla \times \vec{B})$.

Thus the Lorentz force becomes

$$\vec{J} \times \vec{B} = \sigma (\nabla \times \vec{B}) \times \vec{B}.$$ 

Hence, the Lorentz force can be written as $(\nabla \times B_0) \times B_0 = -\sigma B_0^2 \vec{q}$.

Now, equation (1.15) takes the form

$$\frac{\partial \vec{q}}{\partial t} = g \beta (T - T_\infty) + g \beta \gamma (C - C_\infty) + \nu \frac{\partial^2 \vec{q}}{\partial y^2} + \frac{\sigma B_0^2 \vec{q}}{\rho}. \quad (1.18)$$

The above equations (1.10), (1.11) and (1.18) represent the governing equations for the hydro magnetic case.