CHAPTER 3

3.1. Formulation of the problem:

An unsteady free convective flow of a viscous, incompressible, electrically conducting, chemically reacting, optically thick radiating, heat absorbing as well as radiation absorbing fluid past an exponentially accelerated infinite vertical plate in conducting field in the presence of ramped temperature and Hall current is considered. A Cartesian coordinate system \((x^*, y^*, z^*)\) is chosen in such a way that \(x^*\)-axis is taken in the upward direction along the vertical plate whereas \(y^*\)-axis is considered normal to the plane of the plate which is directed into the fluid region and \(z^*\)-axis is normal to \(x^*y^*\)-plane. A uniform transverse magnetic field of strength \(B_0\) is applied perpendicular to the plate in a direction parallel to \(y^*\)-axis. Since the strength of the applied magnetic field is considerably high, therefore the influence of Hall current cannot be neglected. Also the presence of homogeneous chemical reaction is considered. Initially, i.e. at time \(t^* \leq 0\), both the plate and surrounding fluid are at rest and maintained at uniform temperature \(T^*_{\infty}\) and uniform concentration \(C^*_{\infty}\). At time \(t^*>0\), the plate accelerates exponentially along the \(x^*\) direction with a velocity \(U(t^*) = e^{a^* t^*}\) (\(a^*\) being arbitrary constant). At the same time the temperature of the plate raised to \(T^*_{w} + (T^*_{w} - T^*_{\infty}) \left(\frac{t^*}{t_0^*}\right)\) when \(0 < t^* \leq t_0^*\) and it is maintained at uniform temperature \(T^*_{w}\) when \(t^* > t_0^*\) (\(t_0^*\) being critical time for rampedness) and concentration levels are maintained at \(C^*_{w} + (C^*_{w} - C^*_{\infty}) At^*\), when \(t^* > 0\). Since the plate is extended infinitely along \(x^*\) and \(z^*\) directions, all physical measures except pressure depend on \(y^*\) and \(t^*\) only. The induced magnetic field generated by movement of the flow is negligible in comparison to applied one. This is reasonable as the magnetic Reynolds number is very minor for partially ionized fluids and liquid metals which are generally used in several industrial processes. The effect of polarization is neglected which corresponds to the case where no energy is added or extracted from the fluid by electrical means.
With the above assumptions made and followed by Seth et al. [96], taking Hall parameter into consideration the governing equations for the fluid flow problem, under Boussinesq approximation are given by

\[
\frac{\partial \hat{u}^*}{\partial \hat{t}^*} = \nu \frac{\partial^2 \hat{u}^*}{\partial \hat{y}^*} - \frac{\sigma B_0^2}{\rho(1 + m^2)}(\hat{u}^* + m \hat{w}^*) + g \beta (T^* - T_{\infty}^*) + g \beta (\hat{C}^* - C_{\infty}^*) - \frac{\nu}{k^*} \hat{u}^* \tag{3.1}
\]

\[
\frac{\partial \hat{w}^*}{\partial \hat{t}^*} = \nu \frac{\partial^2 \hat{w}^*}{\partial \hat{y}^*} + \frac{\sigma B_0^2}{\rho(1 + m^2)}(m \hat{u}^* - \hat{w}^*) - \frac{\nu}{k^*} \hat{w}^* \tag{3.2}
\]

\[
\frac{\partial \hat{T}^*}{\partial \hat{t}^*} = \frac{\kappa}{\rho c_p} \frac{\partial^2 \hat{T}^*}{\partial \hat{y}^*} - \frac{Q_0}{\rho c_p} (T^* - T_{\infty}^*) - \frac{1}{\rho c_p} \frac{\partial q_x}{\partial \hat{y}^*} + \frac{Q_1}{\rho c_p} (\hat{C}^* - C_{\infty}^*) \tag{3.3}
\]
\[
\frac{\partial C^*}{\partial t^*} = D \frac{\partial^2 C^*}{\partial y^*^2} - K^*_{r^*}(C^* - C_{\infty}^*)
\]  

(3.4)

Initial and boundary conditions to be satisfied are as follows

\(t^* \leq 0: \ u^* = 0, \ w^* = 0, \ T^* = T_{\infty}^*, \ C^* = C_{\infty}^*, \) for all \(y^* \geq 0\)  

(3.5)

\(t^* > 0: \ u^* = e^{\alpha t^*}, \ w^* = 0, \ T^* = \begin{cases} T_{\infty}^* + (T_w^* - T_{\infty}^*) \frac{t^*}{t_0} & \text{at } y^* = 0 \text{ when } 0 < t^* \leq t_0, \\ T_w^* & \text{at } y^* = 0 \text{ when } t^* > t_0. \end{cases} \)

(3.6)

when \(t^* > 0, \ C^* = C_{\infty}^* + (C_w^* - C_{\infty}^*) At^* \) at \(y^* = 0\)

\(t^* > 0: \ u^* \to 0, \ w^* \to 0, \ T^* \to T_{\infty}^*, \ C^* \to C_{\infty}^*, \) as \(y^* \to \infty\)  

(3.7)

For an optically low thick gray fluid, the radiative heat flux \(q_r\) is approximated by Roseland approximation which is given

\[
q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^*}{\partial y^*}
\]  

(3.8)

It is assumed that the temperature difference between fluid in the boundary layer region and free-stream is very small so that is \(T^*\) being expressed as a linear function of temperature \(T^*\). Expanding \(T^*\) in Taylor series about \(T_{\infty}^*\) and neglecting second and higher order terms, we get

\[
T^* = 4T_{\infty}^* T^* - 3T_{\infty}^*
\]  

(3.9)

Using equations (3.8) and (3.9) in (3.3) we get the following equation

\[
\frac{\partial T^*}{\partial t^*} = \frac{\kappa}{\rho c_p} \left(1 + \frac{16\sigma^* T_{\infty}^*}{3kk^*} \right) \frac{\partial^2 T^*}{\partial y^*^2} - \frac{Q_0}{\rho c_p} (T^* - T_{\infty}^*) + \frac{Q_1}{\rho c_p} (C^* - C_{\infty}^*)
\]  

(3.10)
The following non dimensional quantities and flow parameters are introduced to present (3.1), (3.2), (3.4) and (3.10) along with initial and boundary conditions (3.5)- (3.7) in non-dimensional form

\[
\begin{aligned}
&y = \frac{U_0 y^*}{v}, \quad t = \frac{U_0^{*2} t^*}{v}, \quad u = \frac{u^*}{U_0^{*}}, \quad w = \frac{w^*}{U_0^{*}}, \quad T = \frac{(T^* - T_w^*)}{(T_w^* - T_x^*)}, \\
&C = \frac{(C_w^* - C_o^*)}{(C_w^* - C_o^*)}, \quad t_1 = \frac{U_0^{*2} I_0}{v}, \quad A = \frac{U_0^{*2}}{v}
\end{aligned}
\]  

(3.11)

Making use of equation (3.11), equations (3.1), (3.2), (3.4) and (3.10), in non-dimensional form, reduce to

\[
\frac{\partial F}{\partial t} + \frac{M^2 (1 - i m^2)}{1 + m^2} F = \frac{\partial^2 F}{\partial y^2} + GrT + GmC - \frac{1}{K} F
\]  

(3.12)

\[
\frac{\partial T}{\partial t} = \frac{(1 + Nr)}{Pr} \frac{\partial^2 T}{\partial y^2} - \zeta T + \chi C
\]  

(3.13)

\[
\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} - \Gamma C
\]  

(3.14)

Where \( F(y, t) = u(y, t) + iw(y, t) \).

Initial and boundary conditions (3.5) to (3.7), in non-dimensional form, are given by

\[
t \leq 0 : F = 0, \quad T = 0, \quad C = 0 \quad \text{for all } y \geq 0
\]  

(3.15)

\[
t > 0 : F = e^{at}, \quad T = \begin{cases} \frac{t}{t_1} & \text{at } y = 0 \text{ when } 0 < t \leq t_1, \\ 1 & \text{at } y = 0 \text{ when } t > t_1 \end{cases}
\]

\[
\text{when } t > 0, \quad C = t \quad \text{at } y = 0
\]  

(3.16)
$t > 0$: $F \rightarrow 0$, $T \rightarrow 0$, $C \rightarrow 0$ as $y \rightarrow \infty$  

(3.17)

Where $a = \frac{a^* \beta}{U_0^3}$ is non-dimensional constant.

$$Gr = \frac{\nu g \beta (T^*_w - T^*_\infty)}{U_0^3}$$

is the thermal Grashof number.

$$N_r = \frac{16 \sigma^* T^*_\infty}{3 k^*}$$

is the radiation parameter.

$$M^2 = \frac{\sigma B_0^* \nu}{\rho U_0^2}$$

is the magnetic parameter.

$$Pr = \frac{\nu \rho c_p}{\kappa}$$

is the Prandtl number.

$$\zeta = \frac{Q_0 \nu}{\rho c_p U_0^2}$$

is the heat generation parameter.

$$\Gamma = \frac{K^* \nu}{U_0^2}$$

is the chemical reaction parameter.

$$\chi = \frac{Q_0^* \nu (C_w^* - C_\infty^*)}{\rho c_p U_0^2 (T_w^* - T_\infty^*)}$$

is the radiation absorption parameter.

$$Sc = \frac{\nu U_0^2}{D U_0^4}$$

is the Schmidt number.

$$K = \frac{k^* U_0^2}{\nu^2}$$

is the permeability parameter.

$$Gm = \frac{\nu g \beta^* (C_w^* - C_\infty^*)}{U_0^3}$$

is the solutal Grashof number.
3.2 Solution of the problem

Using Laplace transform technique, the non-dimensional governing equations 3.12-3.14 subject to the initial and boundary conditions (3.15) to (3.17) are solved for velocity, temperature and concentration. First the solution for fluid concentration $C(y,t)$ is obtained by solving equation (3.14), secondly the solution for fluid temperature $T(y,t)$ is obtained by solving equation (3.13) and then using these solutions in equation (3.12), the solution for fluid velocity $F(y,t)$ is obtained. The exact solutions for fluid temperature $T(y,t)$, fluid concentration $C(y,t)$ and fluid velocity $F(y,t)$ are expressed in the following simplified form:

$$C(y,t) = \left( \frac{t}{2} - \frac{y\sqrt{s}}{4\sqrt{\Gamma}} \right) e^{-\sqrt{s}/\sqrt{\Gamma}} \text{erfc} \left( \frac{y\sqrt{s}}{2\sqrt{\Gamma}} - \sqrt{\Gamma} t \right) + \left( \frac{t}{2} + \frac{y\sqrt{s}}{4\sqrt{\Gamma}} \right) e^{\sqrt{s}/\sqrt{\Gamma}} \text{erfc} \left( \frac{y\sqrt{s}}{2\sqrt{\Gamma}} + \sqrt{\Gamma} t \right)$$

(3.18)

$$T(y,t) = \frac{1}{t_{1}} (T_{1}(y,t) - H(t-t_{1})T_{1}(y,t-t_{1})) - A_{2}T_{1}(y,t) + A_{4}T_{3}(y,t) + A_{6}T_{5}(y,t) - A_{2}T_{6}(y,t) - A_{6}T_{7}(y,t) + A_{3}C(y,t)$$

(3.19)

$$F(y,t) = F_{1}(y,t) - A_{38}F_{2}(y,t) - A_{39}F_{3}(y,t) - A_{40} \left[ F_{4}(y,t) - F_{5}(y,t) \right] - A_{44}F_{6}(y,t) - A_{45}F_{7}(y,t) + A_{46}H(t-t_{1})F_{1}(y,t-t_{1}) + A_{47}H(t-t_{1})F_{3}(y,t-t_{1}) + A_{48}F_{4}(y,t) + A_{49}F_{5}(y,t) - A_{42}H(t-t_{1})F_{1}(y,t-t_{1}) - A_{43}H(t-t_{1})F_{3}(y,t-t_{1}) + A_{44}F_{6}(y,t) + A_{45}F_{7}(y,t) + A_{46}F_{10}(y,t) - A_{47}H(t-t_{1})F_{1}(y,t-t_{1}) - A_{48}H(t-t_{1})F_{3}(y,t-t_{1}) + A_{49}F_{11}(y,t) + A_{50}F_{12}(y,t) + A_{32}F_{14}(y,t)$$

(3.20)

Where

$$T_{1}(y,t) = \left( \frac{t}{2} - \alpha_{1} \right) e^{-\alpha_{1}} \text{erfc} \left( \alpha_{2} - \alpha_{3} \right) + \left( \frac{t}{2} + \alpha_{1} \right) e^{\alpha_{1}} \text{erfc} \left( \alpha_{2} + \alpha_{3} \right)$$

$$T_{3}(y,t) = (1/2) \left\{ e^{-\alpha_{1}} \text{erfc} \left( \alpha_{6} - \alpha_{7} \right) + e^{\alpha_{1}} \text{erfc} \left( \alpha_{6} + \alpha_{7} \right) \right\}$$
\[
T_4(y,t) = \left(\frac{t}{2} - \alpha_y\right) e^{-\alpha_y \text{erfc}(\alpha_y - \alpha_T)} + \left(\frac{t}{2} + \alpha_y\right) e^{\alpha_y \text{erfc}(\alpha_y + \alpha_T)}
\]

\[
T_5(y,t) = \left(\frac{e^{-A_T t}}{2}\right) \left\{ e^{-\frac{\sqrt{\Delta y}}{\sqrt{\Delta T}} t} \text{erfc}(\alpha_y - \sqrt{(\Gamma - A_T)t}) + e^{\frac{\sqrt{\Delta y}}{\sqrt{\Delta T}} t} \text{erfc}(\alpha_y + \sqrt{(\Gamma - A_T)t}) \right\}
\]

\[
T_6(y,t) = \left(\frac{1}{2}\right) \left\{ e^{-\alpha_y \text{erfc}(\alpha_y - \alpha_T)} + e^{\alpha_y \text{erfc}(\alpha_y + \alpha_T)} \right\}
\]

\[
T_7(y,t) = \left(\frac{e^{-A_T t}}{2}\right) \left\{ e^{-\frac{\sqrt{\Delta y}}{\sqrt{\Delta T}} \alpha_y \text{erfc}(\alpha_y - \sqrt{(\zeta - A_T)t}) + e^{\frac{\sqrt{\Delta y}}{\sqrt{\Delta T}} \alpha_y \text{erfc}(\alpha_y + \sqrt{(\zeta - A_T)t}) \right\}
\]

\[
F_1(y,t) = \left(\frac{e^{A_T t}}{2}\right) \left\{ e^{-\frac{\sqrt{\Delta y}}{\sqrt{\Delta T}} \alpha_y \text{erfc}(\alpha_y - \sqrt{(a - A_T)t}) + e^{\frac{\sqrt{\Delta y}}{\sqrt{\Delta T}} \alpha_y \text{erfc}(\alpha_y + \sqrt{(a - A_T)t}) \right\}
\]

\[
F_2(y,t) = \left(\frac{e^{A_T t}}{2}\right) \left\{ e^{-\frac{\sqrt{\Delta y}}{\sqrt{\Delta T}} \alpha_y \text{erfc}(\alpha_y - \sqrt{(a - A_T)t}) + e^{\frac{\sqrt{\Delta y}}{\sqrt{\Delta T}} \alpha_y \text{erfc}(\alpha_y + \sqrt{(a - A_T)t}) \right\}
\]

\[
F_3(y,t) = \left(\frac{t}{2} - \frac{y}{4\sqrt{\Delta T}}\right) e^{-\frac{\sqrt{\Delta y}}{\sqrt{\Delta T}} \alpha_y \text{erfc}(\alpha_y - \sqrt{(\zeta - A_T)t}) + \left(\frac{t}{2} + \frac{y}{4\sqrt{\Delta T}}\right) e^{\frac{\sqrt{\Delta y}}{\sqrt{\Delta T}} \alpha_y \text{erfc}(\alpha_y + \sqrt{(\zeta - A_T)t}) \right\}
\]

\[
F_4(y,t) = \left(\frac{e^{A_T t}}{2}\right) \left\{ e^{-\frac{\sqrt{\Delta y}}{\sqrt{\Delta T}} \alpha_y \text{erfc}(\alpha_y - \sqrt{(a - A_T - A_T)t}) + e^{\frac{\sqrt{\Delta y}}{\sqrt{\Delta T}} \alpha_y \text{erfc}(\alpha_y + \sqrt{(a - A_T - A_T)t}) \right\}
\]

\[
F_5(y,t) = \left(\frac{e^{A_T t}}{2}\right) \left\{ e^{-\frac{\sqrt{\Delta y}}{\sqrt{\Delta T}} \alpha_y \text{erfc}(\alpha_y - \sqrt{(A - A_T - A_T)t}) + e^{\frac{\sqrt{\Delta y}}{\sqrt{\Delta T}} \alpha_y \text{erfc}(\alpha_y + \sqrt{(A - A_T - A_T)t}) \right\}
\]

\[
F_6(y,t) = \left(\frac{e^{A_T t}}{2}\right) \left\{ e^{-\frac{\sqrt{\Delta y}}{\sqrt{\Delta T}} \alpha_y \text{erfc}(\alpha_y - \sqrt{(A - 2A_T - A_T)t}) + e^{\frac{\sqrt{\Delta y}}{\sqrt{\Delta T}} \alpha_y \text{erfc}(\alpha_y + \sqrt{(A - 2A_T - A_T)t}) \right\}
\]

\[
F_7(y,t) = T_6(y,t)
\]

\[
F_8(y,t) = T_7(y,t)
\]

\[
F_9(y,t) = \left(\frac{e^{-A_T t}}{2}\right) \left\{ e^{-\frac{\sqrt{\Delta y}}{\sqrt{\Delta T}} \alpha_y \text{erfc}(\alpha_y - \sqrt{(\zeta - A_T)t}) + e^{\frac{\sqrt{\Delta y}}{\sqrt{\Delta T}} \alpha_y \text{erfc}(\alpha_y + \sqrt{(\zeta - A_T)t}) \right\}
\]
\[ F_{i0}(y,t) = T_i(y,t) \]
\[ F_{i1}(y,t) = T_3(y,t) \]
\[ F_{i2}(y,t) = T_4(y,t) \]
\[ F_{i3}(y,t) = \left( \frac{e^{-A_2 t}}{2} \right) \left( e^{-\sqrt{S_2 \Gamma} t} \text{erfc} \left( \alpha_6 - \sqrt{\Gamma - A_2} t \right) + e^{\sqrt{S_2 \Gamma} (\Gamma - A_2)} \text{erfc} \left( \alpha_6 + \sqrt{\Gamma - A_2} t \right) \right) \]
\[ F_{i4}(y,t) = T_4(y,t) \]

Mass transfer coefficient

Another important physical quantity is the mass transfer coefficient, i.e. the Sherwood number which is in non-dimensional form is given by

\[ Sh = -\left( \frac{\partial C}{\partial y} \right)_{y=0} = \left( \frac{\sqrt{S_2 \pi}}{2 \sqrt{\Gamma}} + t \sqrt{S_2 \Gamma} \right) \text{erfc} \sqrt{\Gamma t} - \frac{\sqrt{S_2 \pi}}{2 \sqrt{\Gamma}} - t \sqrt{S_2 \Gamma} - \frac{\sqrt{tS_2 \pi}}{\sqrt{\pi}} e^{-t \gamma} \quad (3.21) \]

Heat transfer coefficient

Expression for rate of heat transfer at the plate i.e. the rate of heat transfer in terms of Nusselt number is given by

\[ Nu = -\left( \frac{\partial T}{\partial y} \right)_{y=0} = \left( \frac{1}{t_1} \right) \left[ N_1 - H(t-t_1)N_2 \right] + A_4 (N_3 - N_6) + A_6 (N_5 - N_7) + A_4 (N_4 - N_1) \quad (3.22) \]
Skin friction coefficient

The skin friction at the plate in non-dimensional form is given by

\[
\tau = -\left( \frac{\partial F}{\partial y} \right)_{y=0}
\]

\[
= f_1 - A_{38} f_2 - A_{39} f_3 - A_{40} f_4 - A_{41} f_5 - A_{42} f_6 + A_{43} H(t-t_1) f_{21} + A_{44} H(t-t_1) f_{31} + A_{45} f_7 + A_{46} f_8 + A_{47} f_9 + A_{48} f_{10} - A_{49} H(t-t_1) f_{71} - A_{50} H(t-t_1) f_{81} - A_{51} H(t-t_1) f_{91} + A_{52} f_{10} + A_{53} f_{12} + A_{54} f_{13} + A_{55} f_{14}
\]

(3.23) Where

\[
N_1 = \left( t \sqrt{\frac{\zeta}{A_t}} + \frac{1}{2\sqrt{\zeta A_t}} \right) \text{erfc} \left( \sqrt{\frac{\zeta}{A_t}} t \right) - \frac{1}{2\sqrt{\zeta A_t}} \sqrt{\frac{t}{\pi A_t}} - e^{-t} \sqrt{\frac{t}{\pi A_t}}
\]

\[
N_2 = \left( t-t_1 \right) \sqrt{\frac{\zeta}{A_t}} + \frac{1}{2\sqrt{\zeta A_t}} \right) \text{erfc} \left( \sqrt{\frac{\zeta}{A_t}} \left( t-t_1 \right) \right) - \frac{1}{2\sqrt{\zeta A_t}} \left( t-t_1 \right) \sqrt{\frac{\zeta}{A_t}} - e^{-\left( t-t_1 \right)} \left( t-t_1 \right) \sqrt{\frac{1}{\pi A_t}}
\]

\[
N_3 = \left( \sqrt{S_t} \Gamma \right) \text{erfc} \left( \Gamma - t \right) - e^{-\Gamma} \sqrt{\frac{S_t}{\pi t}} - \sqrt{S_t} \Gamma
\]

\[
N_4 = \left( 2 \sqrt{\frac{S_t}{\Gamma} + t} \sqrt{S_t} \Gamma \right) \text{erfc} \left( \Gamma - t \right) - \frac{S_t}{2\Gamma} \sqrt{\frac{S_t}{\Gamma} - \sqrt{S_t} \Gamma} - e^{-\Gamma} \sqrt{\frac{t S_t}{\pi}}
\]

\[
N_5 = e^{-A_t} \left\{ \text{erfc} \left( \sqrt{(\Gamma - A_t)} t \right) \sqrt{S_t} \left( \Gamma - A_t \right) - e^{-\left( \Gamma - A_t \right)} \sqrt{\frac{S_t}{\pi t}} - \sqrt{S_t} \left( \Gamma - A_t \right) \right\}
\]

\[
N_6 = \sqrt{\frac{\zeta}{A_t}} \text{erfc} \sqrt{\frac{\zeta}{A_t}} t - \frac{\zeta}{A_t} \sqrt{\frac{1}{\pi A_t}} e^{-t}
\]

\[
N_7 = e^{-A_t} \left( \sqrt{\frac{\zeta - A_t}{A_t}} \text{erfc} \left( \sqrt{\zeta - A_t} t \right) - \sqrt{\frac{\zeta}{A_t}} \sqrt{\frac{t}{\pi A_t}} - e^{-\left( \zeta - A_t \right) t} \right)
\]
\[ f_1 = e^{\alpha t} \left( \sqrt{a - A_t} \text{erfc}\left( \sqrt{(a - A_t)t} \right) - \sqrt{a - A_t} - \frac{e^{-(a-A_t)t}}{\sqrt{\pi t}} \right) \]

\[ f_2 = \left( \sqrt{-A_t} \text{erfc}\left( \sqrt{-A_t}t \right) - \sqrt{-A_t} - \frac{e^{(A_t)t}}{\sqrt{\pi t}} \right) \]

\[ f_3 = \left( t \sqrt{-A_t} + \frac{1}{2 \sqrt{-A_t}} \right) \text{erfc}\left( \sqrt{-A_t}t \right) - \frac{t}{\sqrt{\pi}} e^{A_t t} - t \sqrt{-A_t} - \frac{1}{2 \sqrt{-A_t}} \]

\[ f_4 = e^{-A_t} \left( \sqrt{-A_t} \text{erfc}\left( \sqrt{-A_t - A_t}t \right) - \sqrt{-A_t - A_t} - \frac{e^{(-A_t - A_t)t}}{\sqrt{\pi t}} \right) \]

\[ f_5 = e^{-A_t} \left( \sqrt{-A_t} \text{erfc}\left( \sqrt{-A_t - A_t}t \right) - \sqrt{-A_t} - \frac{e^{(-A_t - A_t)t}}{\sqrt{\pi t}} \right) \]

\[ f_6 = e^{-A_3 t} \left( \sqrt{-A_2 - A_t} \text{erfc}\left( \sqrt{-A_2 - A_t}t \right) - \sqrt{-A_2 - A_t} - \frac{e^{(-A_2 - A_t)t}}{\sqrt{\pi t}} \right) \]

\[ f_{23} = \left( \sqrt{-A_t} \text{erfc}\left( \sqrt{-A_t}(t-t) \right) - \sqrt{-A_t} - \frac{e^{A_t(t-t)}}{\sqrt{\pi(t-t)}} \right) \]

\[ f_{34} = \left( (t-t) \sqrt{-A_t} + \frac{1}{2 \sqrt{-A_t}} \right) \text{erfc}\left( \sqrt{-A_t}(t-t) \right) - \frac{(t-t)}{\sqrt{\pi}} e^{A_t(t-t)} - (t-t) \sqrt{-A_t} - \frac{1}{2 \sqrt{-A_t}} \]

\[ f_{41} = e^{-A_3 t} \left( \sqrt{-A_2 - A_t} \text{erfc}\left( \sqrt{-A_2 - A_t}t \right) - \sqrt{-A_2 - A_t} - \frac{e^{(-A_2 - A_t)t}}{\sqrt{\pi t}} \right) \]

\[ f_7 = \sqrt{\frac{\zeta}{A_t}} \text{erfc}\left( \frac{\zeta}{A_t} \right) - \sqrt{\frac{\zeta}{A_t}} - \frac{1}{\sqrt{\pi A_t}} e^{-\frac{\zeta^2}{A_t}} \]

\[ f_8 = \left( t \sqrt{\frac{\zeta}{A_t}} + \frac{1}{2 \sqrt{\zeta A_t}} \right) \text{erfc}\left( \sqrt{\zeta t} \right) - \frac{1}{2 \sqrt{\zeta A_t}} t \sqrt{\frac{\zeta}{A_t}} - e^{-\frac{\zeta^2}{A_t}} \frac{t}{\sqrt{\pi A_t}} \]

\[ f_9 = e^{-A_t} \left( \sqrt{\frac{\zeta - A_t}{A_t}} \text{erfc}\left( \sqrt{(\zeta - A_t)t} \right) - \sqrt{\frac{\zeta - A_t}{A_t}} - \frac{1}{\sqrt{\pi A_t}} e^{-\frac{(\zeta - A_t)^2}{A_t}} \right) \]
\[ f_{10} = e^{-A_{1}t} \left( \sqrt{\frac{\zeta - A_{1}}{A_{1}}} \text{erfc}\left(\frac{\zeta - A_{1}}{A_{1}}t\right) - \frac{1}{\sqrt{\pi A_{1}}} e^{-\left(\zeta - A_{1}\right)^{2}t} \right) \]

\[ f_{71} = \sqrt{\frac{\zeta}{A_{1}}} \text{erfc}\left(\frac{\zeta}{A_{1}}(t-t_{1})\right) - \frac{1}{\sqrt{\pi A_{1}(t-t_{1})}} e^{-\zeta(t-t_{1})} \]

\[ f_{81} = \left( t-t_{1} \right) \left( \sqrt{\frac{\zeta}{A_{1}}} + \frac{1}{2\sqrt{\zeta A_{1}}} \right) \text{erfc}\left(\sqrt{\zeta(t-t_{1})}\right) - \frac{1}{2\sqrt{\zeta A_{1}}} \left( t-t_{1} \right) \sqrt{\frac{\zeta}{A_{1}}} e^{-\zeta(t-t_{1})} \frac{(t-t_{1})}{\pi A_{1}} \]

\[ f_{91} = e^{-A_{1}(t-t_{1})} \left( \sqrt{\frac{\zeta - A_{1}}{A_{1}}} \text{erfc}\left(\frac{\zeta - A_{1}}{A_{1}}(t-t_{1})\right) - \frac{\zeta - A_{1}}{A_{1}} - \frac{1}{\sqrt{\pi A_{1}(t-t_{1})}} e^{-\left(\zeta - A_{1}\right)^{2}(t-t_{1})} \right) \]

\[ f_{11} = \left( \sqrt{S_{c}\Gamma} \right) \text{erfc}\sqrt{\Gamma t} - e^{-T_{1}} \frac{S_{c}}{\sqrt{\pi}} - \sqrt{S_{c}\Gamma} \]

\[ f_{12} = \left( 2 \sqrt{S_{c}} \frac{\Gamma}{t} + t \sqrt{S_{c}\Gamma} \right) \text{erfc}\sqrt{\Gamma t} - \sqrt{S_{c}} \frac{\Gamma}{t} - t \sqrt{S_{c}\Gamma} - e^{-T_{1}} \frac{S_{c}}{\sqrt{\pi}} \]

\[ f_{13} = e^{-A_{1}t} \left( \text{erfc}\left(\sqrt{\left(\Gamma - A_{24}\right)t}\right) \sqrt{S_{c}(\Gamma - A_{24})} - e^{-\left(\Gamma - A_{24}\right)t} \sqrt{S_{c}(\Gamma - A_{24})} \right) \]

\[ f_{14} = e^{-A_{1}t} \left( \text{erfc}\left(\sqrt{\left(\Gamma - A_{3}\right)t}\right) \sqrt{S_{c}(\Gamma - A_{3})} - e^{-\left(\Gamma - A_{3}\right)t} \sqrt{S_{c}(\Gamma - A_{3})} \right) \]

### 3.3 Results and Discussion:

Figures 3.2(a) and 3.2(b) depict the consequences of thermal Grashof number on the primary and secondary fluid velocities. From these figures it is evident that, the flow velocity enhances with increasing values of thermal Grashof number. The similar nature of the primary and secondary velocities of the fluid is noticed in the case of solutal Grashof number, which is evident in the figures 3.3(a) and 3.3(b). This is due to relative strength of buoyancy force to viscous force. Since, fluid flow in this problem is induced due to free convection arising as a result of buoyancy force; therefore, thermal buoyancy force will obviously tend to accelerate fluid flow in both the primary and secondary flow directions throughout the boundary layer region. Figures 3.4(a) and 3.4(b) illustrate the effect of radiation parameter on primary and
secondary fluid velocities. From these figures it is noticed that the velocity of the fluid decreases with increasing values of radiation parameter. Figures 3.5(a) and 3.5(b) exhibit the effect of radiation absorption on the primary and secondary velocity of the fluid. It is observed that primary velocity as well as secondary velocity increase for increasing values of heat generation parameter. Figures 3.6(a) and 3.6(b) depict the influence of heat absorption parameter on primary and secondary velocities of the fluid. Both velocities of the fluid decrease for increasing values of heat absorption parameter ($\zeta$). It is perceived from figures 3.7(a) and 3.7(b) that primary and secondary velocities decrease for increasing values of Schmidt number. Figures 3.8(a) and 3.8(b) demonstrate the effect of chemical reaction parameter on primary and secondary fluid velocities. It is evident from these figures that primary and secondary velocities decrease for increasing values of chemical reaction parameter. Figures 3.9(a) and 3.9(b) illustrate the effect of Hall current on the primary and secondary velocities of the fluid. It is evident from these figures that primary and secondary velocities increase for increasing values of Hall parameter. This implies that Hall current tends to accelerate fluid flow in both the primary and secondary flow directions throughout the boundary layer region. This is due to the reason that Hall current induces secondary flow in the flow-field. Figures 3.10(a) and 3.10(b) present the influence of porosity parameter on the primary and secondary velocities of the fluid. It is observed from these figures that primary and secondary velocities increases for increasing values of permeability parameter.

In order to examine the effect of critical time for rampedness and isothermal, numerical values for fluid temperature $T$ are computed from the analytical solution (3.19) and are shown graphically against boundary layer coordinate $y$ in figures 3.11-3.19. Figure 3.11 illustrates the effect of radiation parameter on ramped temperature. It is evident from this figure that the temperature of the fluid increases for raising values of $Nr$. Figure 3.12 depicts the effect of heat absorption parameter on ramped temperature. It is noticed from this figure that temperature of the fluid decreases for increasing values of heat absorption parameter. Figure 3.13 exhibits the effect of Prandtl number on temperature. It is evident from this figure that velocity of the fluid decreases for increasing values of Prandtl number. It is perceived from the Figure 3.14 that the temperature of the fluid increases for increasing values of
radiation absorption parameter. From figure 3.15 it is noticed that the temperature of the fluid increases when time $t$ increases. When $t > t_1$ the temperature becomes uniform and it is equal to 1 which agrees condition (3.16). As we know that for isothermal plate, temperature of the plate is uniform, i.e. in non-dimensional form fluid temperature $T = 1$ at the plate for every value of time $t$. This means that nature of fluid temperature is same for both ramped temperature and isothermal plate when $t > t_1$. However, fluid temperature is getting enhanced in the flow-field whether $t \leq t_1$ or $t > t_1$, it is evident from figures 3.15-3.19. From the figures 3.20 and 3.21 it is noticed that the concentration of the fluid decreases for increasing values of chemical reaction parameter and Schmidt number. Figure 3.22 exhibits the variations in concentration of the fluid for different values of time $t$. It is evident from the figure that the concentration of the fluid increases for increasing values of time $t$.

The numerical values for rate of mass transfer at the plate i.e. $\left( \frac{\partial C}{\partial y} \right)_{y=0}$, are computed from the analytical expression (3.21), and presented in table 3.1 for various values of Schmidt number and chemical reaction parameter. Table 3.1 exhibits the rate of mass transfer at the plate. It is noticed that Sherwood number increases for increasing values of Schmidt number and chemical reaction parameter. The numerical values of rate of heat transfer at the plate i.e. $\left( \frac{\partial T}{\partial y} \right)_{y=0}$, computed from the analytical expression (3.22), are presented in the Table 3.2. From this table it is noticed that Nusselt number increases for increasing values of Prandtl number, heat absorption parameter, radiation parameter whereas it decreases for increasing values of radiation absorption parameter. The numerical values of skin friction coefficient at the plate i.e. $\left( \frac{\partial F}{\partial y} \right)_{y=0}$, computed from the analytical expression (3.23), are presented in the Table 3.3. From this table it is noticed that Skin friction increases for increasing values of thermal Grashof number, solutal Grashof number and radiation parameter whereas it decreases for increasing values of magnetic parameter, Hall parameter, radiation absorption parameter, permeability parameter, heat absorption parameter.

In order to confirm the accuracy of methodology a comparison has made with the results of Seth et al. [96] by taking into account the effect of radiation parameter
on temperature which is depicted in figure 3.23. A good agreement is noticed in this comparison.

Figure 3.2(a): Effect of thermal Grashof number on primary velocity

Figure 3.2(b): Effect of thermal Grashof number on secondary velocity
Figure 3.3(a): Impact of solutal Grashof on primary velocity

Figure 3.3(b): Impact of solutal Grashof on secondary velocity
Figure 3.4(a): Influence of radiation parameter on primary velocity

Figure 3.4(b): Effect of radiation parameter on secondary velocity
Figure 3.5(a): Effect of radiation absorption parameter on primary velocity

Figure 3.5(b): Effect of radiation absorption parameter on secondary velocity
Figure 3.6(a): Effect of heat absorption parameter on primary velocity

Figure 3.6(b): Effect of heat absorption parameter on secondary velocity
Figure 3.7(a): Effect of Schmidt number on primary velocity

Figure 3.7(b): Effect of Schmidt number on secondary velocity
Figure 3.8(a): Effect of chemical reaction on primary velocity

Figure 3.8(b): Effect of chemical reaction on secondary velocity
Figure 3.9(a): Effect of Hall parameter on primary velocity

Figure 3.9(b): Effect of Hall parameter on secondary velocity
Figure 3.10(a): Effect of porosity parameter on primary velocity

Figure 3.10(b): Effect of porosity parameter on secondary velocity
Figure 3.11: Effect of radiation parameter on temperature

Figure 3.12: Effect of heat absorption parameter on temperature
Figure 3.13 Impact of Prandtl number on temperature

Figure 3.14 Impact of radiation absorption parameter on temperature
Figure 3.15: Effect of time parameter on temperature.

Figure 3.16: Effect of radiation absorption parameter on temperature ($t > t_1$)
Figure 3.17: Effect of Prandtl number on temperature ($I > l_1$)

Figure 3.18: Effect of radiation parameter on temperature ($I > l_1$)
Figure 3.19: Effect of heat absorption parameter on temperature ($t > t_1$)

Figure 3.20: Effect of chemical reaction on concentration
Figure 3.21: Effect of Schmidt number on concentration

Figure 3.22: Effect of time parameter on concentration
Figure 3.23: Comparison of the present results with that of Seth et al. [96]

Table 3.1: Variations in rate of mass transfer, when t=0.9, \( t_1 = 1 \).

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Table 3.2: Variations in rate of heat transfer under the effect of several parameters, when $t=0.9$

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### Table 3.3: Variations in skin friction when $t=0.9$, $Sc=0.22$, $a=1$, $\Gamma =0.5$

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