5. REDUCTION OF PAPR OF SC-FDMA SIGNALS USING DFT SPREADING TECHNIQUE WITH GAUSSIAN PULSE SHAPING

5.1. INTRODUCTION

A low complexity modified alternative signal (MAS-S) technique with sequential optimization for reducing PAPR in OFDM-OQAM systems is developed in the preceding chapter. Here both the aspects of PAPR reduction and complexity are considered. Orthogonal frequency division multiple access (OFDMA) is an efficient technique for providing better quality of service (QOS) to the users by eliminating fading of signals and also for providing high data rates in multimedia services. Single carrier FDMA (SC-FDMA) technique that makes use of single carrier modulation has similar performance and the equivalent complexity as that of OFDMA. The inherent advantage of SC-FDMA is the lower PAPR achieved by it due to its single carrier structure [43]. SC-FDMA transmits single carrier signal which has lower PAPR than that of OFDMA system which transmits multicarrier signal having higher PAPR.

In this chapter, we focus on reduction of PAPR of SC-FDMA signals using both raised cosine (RC) pulse shaping and Gaussian pulse shaping and compare their performance. It is shown that DFT spreading technique with Gaussian pulse shaping has better performance than that with raised cosine pulse shaping. So, three algorithms based on this technique namely Localized Frequency Division Multiple Access (LFDMA), Distributed Frequency Division Multiple Access (DFDMA) and Interleaved Frequency Division Multiple Access (IFDMA) are developed. PAPR analysis of SC-FDMA signals by employing DFT spreading technique with raised cosine pulse shaping is performed in [44]. Here the authors implemented the technique with and without raised cosine pulse shaping and have shown that PAPR reduction performance with raised cosine pulse shaping yields better results. However in our thesis work, it is shown that PAPR reduction using Gaussian pulse shaping is better compared to that with raised cosine pulse shaping.

First, SC-FDMA system is characterized and the block diagram of SC-FDMA transceiver is discussed. Thereafter, the DFT spreading technique algorithms namely LFDMA, DFDMA and IFDMA are developed and the algorithm steps are discussed. IFFT outputs for these three algorithms are derived in the next section. The pulse
shaping using raised cosine and Gaussian pulse shapings is discussed in sec. 5.5. Also, the significance of Gaussian pulse shaping in reducing PAPR is discussed in this chapter. Simulation results are plotted and discussed in detail for LFDMA, DFDMA and IFDMA algorithms in chapter 6.

5.2. CHARACTERIZATION OF SC-FDMA SYSTEM

In this section, we describe the block diagram of an SC-FDMA system. The building blocks of SC-FDMA system are essentially same as that of OFDMA system.

Figure 5.1 : Block diagram of SC-FDMA system

Figure 5.1 shows the block diagram of SC-FDMA system. At the transmitter side, the time domain symbols are given as input to the modulator which modulates the serially inputted data. The serial data is converted into parallel form by means of a serial to parallel converter. The subcarrier mapping is made after performing the FFT.
The subcarrier mapped symbols in frequency domain are converted into time domain by IFFT operation and inputted to digital to analog converter (DAC) which converts the digital data to analog form. The output of DAC is passed through the channel whose output is inputted to the receiver.

At the receiver side, the analog data is converted into digital form and converted into frequency domain by FFT operation followed by subcarrier de-mapping operation. The de-mapped symbols are transformed to time domain by performing IFFT. The data which is in parallel form is converted into serial form and then demodulated to obtain the demodulated signal.

5.2.1. Necessity of PAPR reduction in SC-FDMA system

Peak to average power ratio occurs as a result of the summing of carriers together. The peak power increases corresponding to the number of carriers. After linear region, the scalar relationship does not exist and the amplifier moves into the saturation region. When the devices are made to work in the saturation region, it results in distortion which is a major drawback.

SC-FDMA systems incorporate power amplifiers at the transmitter to obtain the required transmission power. For achieving the maximum output power efficiency, the high power amplifiers (HPA) are usually made to work at or near the saturation region. The nonlinear characteristic of these high power amplifiers is very sensitive to the variation in signal amplitudes. Hence it is essential to develop effective algorithms to reduce PAPR in SC-FDMA systems.

5.3. DFT SPREADING TECHNIQUE

In this technique, the allocation of subcarriers to subscribers is done using the three algorithms namely localized FDMA (LFDMA), distributed FDMA (DFDMA) and interleaved FDMA (IFDMA).

The allocation of subcarriers using the three algorithms is shown in Figures 5.2, 5.3 and 5.4. As an example, consider the total subcarriers as \( N = 12 \). Here it is assumed that \( M = 4 \) DFT outputs are allocated. The spreading factor is \( L = 3 \). \( X(0), X(1), X(2), X(3) \) denote the outputs that are allocated using LFDMA, DFDMA
and IFDMA algorithms. Now these three algorithms with the help of diagrams are described by considering these values for each algorithm.

**Figure 5.2: LFDMA algorithm**

In LFDMA algorithm, $\mathcal{M}$ successive outputs are allocated in a total of $N$ subcarriers and the remaining unused subcarriers are filled with zeroes. It is observed that that Fig. 5.2 corresponds to LFDMA algorithm. Here $\mathcal{M}=4$ outputs namely $X(0), X(1), X(2), X(3)$ are consecutively allocated and the remaining $N-\mathcal{M}=8$ subcarriers are padded with zeroes.

**Figure 5.3: DFDMA algorithm**

In DFDMA algorithm, $\mathcal{M}$ outputs are allocated over the entire range of $N$ subcarriers alternatively and the remaining subcarriers are filled with zeroes. The DFDMA algorithm where $\mathcal{M}=4$ DFT outputs $X(0), X(1), X(2), X(3)$ are allocated alternatively over the entire range of $N=12$ subcarriers is represented in Fig. 5.3. Here the remaining outputs are filled with zeroes.

**Figure 5.4: IFDMA algorithm**

In IFDMA algorithm, $\mathcal{M}$ DFT outputs are allocated over $N$ subcarriers in equal intervals. Here spreading factor $L=N/\mathcal{M}$ is defined where $\mathcal{M}$ is the number of outputs that are allocated. Fig. 5.4 shows IFDMA algorithm where $\mathcal{M}=4$ DFT outputs $X(0), X(1), X(2), X(3)$ are allocated over the entire range of $N=12$ subcarriers with equidistance. Here the spreading factor $L=N/\mathcal{M}$ is 3. The rest of the unused subcarriers are padded with zeroes.
5.4. EXPRESSIONS FOR IFFT OUTPUTS FOR LFDMA, DFDMA AND IFDMA ALGORITHMS

Here, in this section, the expressions for IFFT outputs for LFDMA, IFDMA and DFDMA algorithms are derived. The IFFT output in each algorithm is obtained after the FFT and subcarrier mapping operations. The importance of IFFT outputs is that it gives the details about the time domain signal which depends on factors like scaling, phase rotation, weighting factor, etc.

5.4.1. IFFT output for LFDMA algorithm

Let us again assume a total number of $N$ subcarriers and let $\tilde{M}$ be the successive outputs that are allocated and the remaining subcarriers are filled with zeroes.

Now $n = Lm + l$ where $L$ is the spreading factor Here $0 \leq l \leq L - 1$ and $0 \leq m \leq \tilde{M} - 1$.

The output of this algorithm after the subcarrier mapping is

$$X_{\text{LFD}}(k) = X(k), \quad k = 0, 1, \ldots, \tilde{M} - 1$$

$$= 0 \quad \text{otherwise}$$

so that the IFFT output sequence is given by

$$x_{\text{LFD}}(n) = x_{\text{LFD}}(Lm + l), \quad 0 \leq m \leq \tilde{M} - 1$$

Now

$$x_{\text{LFD}}(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_{\text{LFD}}(k) \exp \left( \frac{j2\pi kn}{N} \right)$$

Now $x_{\text{LFD}}(n)$ can be expressed as

$$x_{\text{LFD}}(n) = \frac{1}{LM} \sum_{k=0}^{\tilde{M}-1} X(k) \exp \left( \frac{j2\pi kn}{N} \right)$$

If $l=0$, then from (5.4),

$$x_{\text{LFD}}(n) = \frac{1}{LM} \sum_{k=0}^{\tilde{M}-1} X(k) \exp \left( \frac{j2\pi mk}{\tilde{M}} \right)$$

$$= \frac{1}{L} x(m)$$

If $l \neq 0$, then from (5.4),

$$x_{\text{LFD}}(n) = x_{\text{LFD}}(Lm + l)$$

$$= \frac{1}{LM} \sum_{k=0}^{\tilde{M}-1} X(k) \exp \left( \frac{j2\pi kn}{\tilde{M}} \right)$$

We know that the DFT of $x(n)$ is formulated as

$$X(k) = \sum_{r=0}^{\tilde{M}-1} x(r) \exp \left( -\frac{j2\pi kr}{\tilde{M}} \right)$$

$$= \frac{1}{L\tilde{M}} \sum_{k=0}^{\tilde{M}-1} \sum_{r=0}^{\tilde{M}-1} x(r) \exp \left( -\frac{j2\pi kr}{\tilde{M}} \right) \exp \left\{ j2\pi \left( \frac{m}{\tilde{M}} + \frac{l}{LM} \right) k \right\}$$
Now, \( X(k) = \frac{1}{L M} \sum_{k=0}^{M-1} \sum_{r=0}^{M-1} x(r) \exp \left\{ j2\pi \left( \frac{m-r}{M} + \frac{l}{LM} \right) \right\} \) \( (5.8) \)

On simplifying (5.8), we obtain

\[
x_{LFD}(n) = \frac{1}{LM} \sum_{r=0}^{N-1} \frac{x(r)}{1 - \exp \left\{ j2\pi \left( \frac{m-r}{M} + \frac{l}{LM} \right) \right\}} \quad (5.9)
\]

From (5.9) it is observed that the IFFT output for LFDMA depends on the scaled version of the input signal by \( 1/L \) along with a weighting factor.

### 5.4.2. IFFT output for IFDMA algorithm

Consider \( k = pL \) where \( p \) is an integer that denotes the allocated DFT output. Here \( L \) represents the spreading factor.

The DFT output following subcarrier mapping is

\[
X_{IFD}(k) = X \left( \frac{k}{L} \right), \quad k = Lp
\]

Here \( p = 0, 1, \ldots, \bar{M} - 1 \)

The IFFT output is \( x_{IFD}(n) \) where \( n = \bar{M} l + m \)

Here \( l = 0, 1, \ldots (L - 1), m = 0, 1, \ldots (\bar{M} - 1) \)

\[
x_{IFD}(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_{IFD}(k) \exp \left( \frac{j2\pi kn}{N} \right)
\]

Substituting \( k = Lp, N = L\bar{M} \) in (5.11), we obtain

\[
x_{IFD}(n) = \frac{1}{LM} \sum_{p=0}^{M-1} X(Lp) \exp \left( \frac{j2\pi pn}{M} \right)
\]

\[
= \frac{1}{LM} \sum_{p=0}^{\bar{M}-1} X(p) \exp \left( \frac{j2\pi pm}{\bar{M}} \right)
\]

Substituting \( n = \bar{M} l + m \) in (5.13),

\[
x_{IFD}(n) = \frac{1}{LM} \sum_{p=0}^{\bar{M}-1} X(p) \exp \left( \frac{j2\pi (l+m)p}{\bar{M}} \right)
\]

On simplifying (5.14), we obtain

\[
x_{IFD}(n) = \frac{1}{LM} \sum_{p=0}^{\bar{M}-1} X(p) \exp \left( \frac{j2\pi pm}{\bar{M}} \right)
\]

\[
= \frac{1}{L} IDFT \{ X(p) \}
\]

\[
= \frac{1}{L} x(m)
\]

(5.15)

From (5.15) it is evident that the IFFT expression for IFDMA is the original signal \( x(m) \) multiplied by a factor of \( 1/L \).
5.4.3. IFFT output for DFDMA algorithm

Next, the output for this algorithm is

\[ X_{DFD}(k) = X(p), \ p = 0,1,\ldots,\tilde{M} - 1 \]  

(5.16)

Here \( k = Lp + r, \ r = 0,1,\ldots, (\tilde{M} - 1) \)

The IFFT output is given by

\[ x_{DFD}(n) = x_{DFD}(L\tilde{M} + l) = \frac{1}{N} \sum_{k=0}^{N-1} X_{DFD}(k) \exp\left(\frac{2\pi kn}{N}\right) \]  

(5.17)

Substituting \( k = Lp + r \) in (5.18), we get

\[ x_{DFD}(n) = \frac{1}{\tilde{M}} \sum_{p=0}^{\tilde{M}-1} X(p) \exp\left(\frac{2\pi (Lp + r)n}{N}\right) \]  

(5.19)

Substituting \( n = L\tilde{M} + l \) and \( N = L\tilde{M} \) in (5.20),

\[ x_{DFD}(n) = \frac{1}{\tilde{M}} \sum_{p=0}^{\tilde{M}-1} X(p) \exp\left(\frac{2\pi Lp(L\tilde{M} + l)}{N}\right) \exp\left(\frac{j2\pi mn}{N}\right) \]  

(5.20)

\[ x_{DFD}(n) = \frac{1}{\tilde{M}} \exp\left(\frac{j2\pi mn}{N}\right) \sum_{p=0}^{\tilde{M}-1} X(p) \exp\left(\frac{j2\pi lp}{\tilde{M}}\right) \]  

(5.21)

The IFFT output finally becomes

\[ x_{DFD}(n) = \frac{1}{L} \exp\left(\frac{j2\pi mn}{N}\right) x(m) \]  

(5.22)

From (5.22), it is obvious that the IFFT output of DFDMA is obtained by phase rotating the input signal \( x(m) \) with a scaling factor.

5.5. PULSE SHAPING IN SC-FDMA SYSTEM

In SC-FDMA systems, PAPR is the disadvantage and the focus is to reduce PAPR to a minimum. PAPR reduction in SC-FDMA systems is carried out by employing the DFT spreading techniques which make use of pulse shaping. Using pulse shaping, PAPR can be decreased by reducing the peak power of the transmitted signal and this does not influence the bandwidth utilization. Pulse shaping is incorporated in the transmitter section of SC-FDMA system to band limit the signal and hence to reduce its peak power. This in turn reduces the PAPR.

Consider an SC-FDMA system that uses pulse shaping before transmission as illustrated in Fig. 5.5. Here the input signal is \( x(n) \) defined for \( 0 \leq n \leq N-1 \) and let \( f_c \) be the carrier frequency.
Let $X(k)$ denote the DFT of $x(n)$ in the interval $0 \leq k \leq N-1$. After subcarrier mapping, let $X(p)$ specify the output for the range $0 \leq p \leq M-1$ and let $x(p)$ represent the output after the IFFT operation of $X(p)$ for $0 \leq p \leq M-1$. The band pass signal of SC-FDMA signal is given as

$$x(t) = \sum_{p=0}^{M-1} x(p) \ast s(t - pT_s)$$  \hfill (5.23)

In (5.23), $s(t)$ is the baseband pulse and ‘$\ast$’ denotes convolution. In our work, discussion is made on two prominent pulse shaping techniques which are the raised cosine pulse shaping and Gaussian pulse shaping and we make a performance comparison in reducing PAPR. Now we discuss about the characteristics of these two pulses.

### 5.5.1. Pulse shaping using raised cosine pulse

The raised cosine pulse is prominently used in pulse shaping techniques. This pulse in time domain is

$$s_{RC}(t) = \sin\left(\frac{t}{T_s}\right) \cos\left(\frac{\pi \beta}{2} \frac{t}{T_s}\right) \frac{1 - 4\beta^2 t^2}{T_s^2}$$ \hfill (5.24)

where $\beta$ is the roll-off factor which varies from 0 to 1 and $T_s$ is the symbol duration.

By making use of raised cosine pulse shaping, (5.23) can be represented as

$$x(t) = \sum_{p=0}^{M-1} x(p) \ast s_{RC}(t - pT_s)$$ \hfill (5.25)

The two-sided frequency response of raised cosine pulse is shown in Fig. 5.6. The response shows the plot between $\frac{f}{w}$ and $P(f)$ as $\frac{f}{w}$ ratio is varied in steps of 0.5. The
ratio $\frac{f}{w}$ denotes the ratio of frequency to bandwidth. The plot is indicated for various values of roll-off factors $\beta = 0, 0.5, 1$.

Figure 5.6: Frequency response of raised cosine pulse

5.5.2. Pulse shaping using Gaussian pulse

Another important pulse used as pulse shaping filter to decrease the PAPR of SC-FDMA signals is the Gaussian pulse and in time domain, it is given by

$$s_G(t) = \frac{\sqrt{\pi}}{\alpha} \exp \left( -\frac{\pi^2 t^2}{\alpha^2} \right)$$  (5.26)

Here $\alpha$ is the parameter related to the bandwidth $B_T$ of the Gaussian pulse defined as

$$\alpha = \frac{0.5887}{B_T}$$  (5.27)

Substituting Gaussian pulse shaping, the pass band signal in (5.23) can be modified as

$$x(t) = \sum_{p=0}^{R-1} x(p) * s_G(t - pT_s)$$  (5.28)

Figure 5.7: Time domain representation of Gaussian pulse
The Gaussian pulse has a narrow bandwidth, sharp cut-off and pulse area preservation properties and hence efficiently used in pulse shaping. Because of these features, Gaussian pulse is prominently made use of in wireless communications for pulse shaping. The Gaussian pulse waveform for different values of roll off factors is represented in Fig. 5.7.

5.5.3. Significance of Gaussian pulse shaping in reducing PAPR

Consider the transmitted signal with raised cosine pulse as specified in (5.25). Let the Fourier transform of the output signal be \( X(f) \) and the peak power of this signal be equal to \( P_{RC} \). We know that the convolution in time domain corresponds to multiplication in frequency domain. Also, the Fourier transform of Gaussian pulse is another Gaussian signal.

As (5.25) involves convolution of \( x(p) \) with \( s_{RC}(t - pT_s) \), the Fourier transform of \( x(t) \) is given by

\[
X(f) = X(p)S_{RC}(f)\exp(-j2\pi fpT_s)
\]

The peak power is determined as

\[
P_{RC} = \int_{f=-\infty}^{\infty} |X(f)|^2
\]

Now (5.30) can be written as

\[
P_{RC} = \int_{f=-\infty}^{\infty} |X(p)S_{RC}(f)|^2
\]

As we know that \(|\exp(-j2\pi fpT_s)| = 1\), (5.31) can be expressed as

\[
P_{RC} = \int_{f=-\infty}^{\infty} |X(p)S_{RC}(f)|^2
\]

Now consider the transmitted signal with Gaussian pulse specified in (5.28). As stated above, let the peak power of this transmitted signal be equal to \( P_G \).

Proceeding as above, the peak power \( P_G \) of transmitted signal is given by

\[
P_G = \int_{f=-\infty}^{\infty} |X(p)S_{G}(f)|^2
\]

Now, comparing (5.32) and (5.33), it is evident that \( P_G < P_{RC} \) because of the narrow bandwidth of the Gaussian pulse compared to that of raised cosine pulse. This in turn reduces PAPR when Gaussian pulse shaping is used. So, SC-FDMA system with Gaussian pulse shaping achieves less PAPR compared to that with raised cosine pulse shaping.
5.6. SUMMARY

In this chapter, characterization of SC-FDMA system is made and next DFT spreading techniques namely LFDMA, DFDMA and IFDMA algorithms are developed for reducing PAPR in SC-FDMA systems. Then the concept of pulse shaping is introduced and the discussions about the raised cosine and Gaussian pulse shapings and their characteristics are made. Then the significance of Gaussian pulse shaping in reducing PAPR is described. The simulation results for LFDMA, DFDMA and IFDMA algorithms are plotted in chapter 6 and their performance in reducing PAPR is compared. We will show that these techniques employing Gaussian pulse shaping attain good performance in reducing PAPR compared to that with raised cosine pulse shaping.

Simulation results and related discussions are made for the algorithms developed in chapters 3, 4 and 5 in the next chapter. Also, analytical results are plotted for O-PTS algorithm and compared with the simulation results.