CHAPTER - 4

METHODS OF ANALYSIS

4.1 INTRODUCTION

This chapter deals with the different methods of analysis and their governing equations. Namely, there are three analytical methods

- Kern method
- Bell-Delaware method
- Pressure drop model based on FEM

To design a shell and tube heat exchanger, it involves determination of certain variables like heat transfer coefficient, pressure drop on both shell and tube side for which empirical correlations are required [6, 98]. Many researchers were made analysis to explain the methods in determination of shell side pressure drop model in STHX and also published [94, 99, 100 and 101]. The review all those method was given by Palen and Taborek [6].

The empirical correlations given in the Bell-Delaware is quite tedious as it requires many of input parameters. This is only method which accounts the leakages. Normally, authors choose to prefer this method for designing any shell and tube heat exchangers. This method was used by many authors [13, 14, 16, 30, 34 and 102] for designing and manufacturing the heat exchangers. Kern method is simplest method analysed by [9, 37, 66 and 67] for shell and tube heat exchanger.

There are seven case studies are discussed in this chapter, each case study is termed as model. The model-1, model-2 and model-3 are deals with Kern method. The model-4, model-5 and model-6 are deals with Bell-Delaware method. The model-7 deals with pressure drop model.

4.2 KERN METHOD

This method is very simple and allows rapid calculation of shell-side coefficients and pressure drop. This method cannot be adequately used because it does not account leakages between baffle-to-shell and tube-to-baffle and is limited to fixed baffle cut (25%). The model-1, model-2 and model-3 are deals with Kern method. The governing equations and the input parameters are remain same for
model-4, model-5 and model-6. This method used by Hadidi at el. [9], Mohanty et al. [37] and Asadi at el. [103] as single objective function. In the present work the model-1, model-2 and model-3 are deals with single objective optimization.

Model - 1: Optimization of shell and tube heat exchangers using Flower pollination algorithm. (Single objective function i.e. Total cost).

Model - 2: Optimization using bat algorithm on shell and tube heat exchangers by Kern method. (Single objective function i.e. Total cost).

Model – 3: Optimization of shell and tube heat exchangers by minimizing the entropy generation using bat algorithm. (Single objective function i.e. entropy generation).

4.2 THERMAL AND ANALYTICAL MODELLING

The heat exchanger surface area is given by [19]

\[ A = \frac{q}{U \Delta T_{LM} F} \]  

(4.2.1)

Where \( q \) is the heat quantity, \( U \) is the overall heat transfer coefficient, \( \Delta T_{LM} \) is the logarithmic mean temperature difference, \( F \) is the correction factor.

The heat transfer rate is given by,

\[ q = \dot{m}_s c_{ps} (T_{hi} - T_{ho}) = \dot{m}_t c_{pt} (T_{co} - T_{cl}) \]  

(4.2.2)

\[ U = \frac{1}{\frac{1}{h_o} + R_{o,f} + \frac{d_0}{d_i} (R_{i,f} + \frac{1}{h_i})} \]  

(4.2.3)

Where \( R_{o,f} \) and \( R_{i,f} \) are the fouling resistance taken from the literature [104].

\[ d_i = 0.8d_0 \]  

(4.2.4)

4.2.2 TUBE SIDE

\[ h_t = \frac{k_t}{d_i} \left[ 3.657 + \frac{0.0677(Re_t Pr_t)^{0.5}}{1 + 0.1 Pr_t (Re_t^{0.5})^{0.3}} \right] \]  

(4.2.5)

\((Re_t < 2300)\)
\[
\begin{align*}
    h_t &= \frac{k_t}{d_i} \left( \frac{f_i(R_e - 1000) P_{t1}}{1 + 12.7 \sqrt{\frac{P_{t1}}{P_{t1}^2 - 1}}} \left[ 1 + \left( \frac{d_i}{L} \right)^{0.67} \right] \right) \quad (4.2.6) \\
    \text{where} \quad (2300 < R_e < 10000) \\
    h_t &= 0.027 \frac{k_t}{d_i} R_e^{0.8} P_{t1}^{\frac{1}{3}} \left( \frac{\mu_t}{\mu_w} \right)^{0.14} \quad (4.2.7) \\
    \text{(} R_e > 10000) \\
    f_t &= (1.82 \log_{10} R_e - 1.64)^{-2} \quad (4.2.8)
\end{align*}
\]

where \( f_t \) is the Darcy friction factor given as [105]
\[
\begin{align*}
    P_{t1} &= \frac{\mu_t c_{pt}}{k_t} \quad (4.2.9) \\
    R_{t1} &= \frac{\rho_t v_t d_i}{\mu_t} \quad (4.2.10)
\end{align*}
\]

Where \( P_{t1} \) and \( R_{t1} \) are tube side Prandtl and Reynolds number.
\[
\nu_t = \frac{\dot{m}_t}{\rho_t \pi d_i^2} \frac{N_p}{N_t} \quad (4.2.11)
\]

Where \( N_p \) number of passes and \( N_t \) is the number of tubes [6, 105]
\[
N_t = k_1 \left( \frac{d_s}{d_o} \right)^{n_1} \quad (4.2.12)
\]

Where \( k_1 \) and \( n_1 \) are coefficients that are taken values according to flow arrangement and number of passes.
\[
L = \frac{A}{\pi d_o N_t} \quad (4.2.13)
\]

### 4.2.3 SHELL SIDE

\[
\begin{align*}
    h_s &= 0.36 \frac{k_s}{D_e} R_e^{0.55} P_{t1}^{1/3} \left( \frac{\mu_t}{\mu_w} \right)^{0.14} \quad (4.2.14) \\
    D_e &= \frac{4(p_t^2 - (\pi d_o^2/4))}{\pi d_o} \quad (4.2.15) \\
    \text{(for square pitch)} \\
    D_e &= \frac{4(0.43 p_t^2 - (0.5 \pi d_o^2/4))}{0.5 \pi d_o} \quad (4.2.16) \\
    \text{(for triangular pitch)}
\end{align*}
\]
where $D_e$ is the hydraulic diameter on shell and computed as given by [19,113]

$$p_r_s = \frac{\rho_s C_{ps}}{k_s} \quad (4.2.17)$$

$$Re_s = \frac{\rho_s v_s D_e}{\mu_s} \quad (4.2.18)$$

$$v_s = \frac{m_s}{\alpha_s \rho_s} \quad (4.2.19)$$

Where $v_s$ is the shell side flow velocity and can be obtained $[6, 105],

$$a_s = d_s B \left(1 - \frac{d_e}{p_t}\right) \quad (4.2.20)$$

where $a_s$ is the cross section area normal to flow.

$$C_i = P_t - d_0 \quad (4.2.21)$$

where $C_i$ shell side clearance.

### 4.2.4 LOGARITHMIC MEAN TEMPERATURE DIFFERENCE

$$\Delta T_{lm} = \frac{(T_{hi} - T_{co}) - (T_{ho} - T_{ci})}{\ln((T_{hi} - T_{co})/(T_{ho} - T_{ci}))} \quad (4.2.22)$$

$$F = \frac{\sqrt{R^2 + 1}}{R - 1} \cdot \ln \left(\frac{1 - P}{1 - P_R}\right) \quad (4.2.23)$$

$$R = \frac{T_{hi} - T_{ho}}{T_{co} - T_{ci}} \quad (4.2.24)$$

$$P = \frac{T_{co} - T_{ci}}{T_{hi} - T_{ci}} \quad (4.2.25)$$

### 4.2.5 PRESSURE DROP AND OBJECTIVE FUNCTION

For a fixed heat capacity heat exchanger, increasing the flow velocity will cause a rise of heat transfer coefficient and will cause more pressure drop which results in extra running cost.

$$\Delta P_t = \Delta P_{tubelength} + \Delta P_{tubeelbow} = \rho_t v_t^2 \cdot \frac{L}{d_t} \cdot f_t + p \cdot N_p \quad (4.2.26)$$

Assumed $p = 4$ from Kern et al. [19] and assumed $p = 2.5$ Sinnott et al. [105].

$$\Delta P_s = f_s \left(\frac{\rho_s v_s^2}{2}\right) \cdot \frac{L}{B} \cdot \left(\frac{d_s}{D_e}\right) \quad (4.2.27)$$

Where $f_s$ is the friction factor.
\[ f_s = 2b_0Re_s^{0.15} \] (4.2.28)

\[ b_0 = 0.72 \] [28]valid for \( Re_t < 40000. \)

\[ P = \frac{1}{n} \left( \frac{m_t}{\rho_t} \Delta P_t + \frac{m_s}{\rho_s} \Delta P_s \right) \] (4.2.29)

\[ C_{inc} = 8000 + 259.2A_{ct}^{0.91} \] (4.2.30)

Where \( C_{inc} \) is the capital investment for exchangers both shell and tubes made out of stainless steel [106].

\[ C_{oc} = Pk_{elt} \tau \] (4.2.31)

\[ C_{opc} = \sum_{k=1}^{n_y} \frac{C_{oc}}{(1+i)^k} \] (4.2.32)

\[ C_{totc} = C_{inc} + C_{opc} \] (4.2.33)

Where \( C_{totc} \) is total cost taken as the objective function, which includes energy cost \( (k_{elt}) \), capital investment \( (C_{inc}) \), total discounted operating cost \( (C_{opc}) \) and annual operating cost \( (C_{oc}) \) [8].

### 4.2.6 Entropy Generation

Entropy minimization using genetic algorithm was carried out by Jiangfeng et al. [107] as single objective. Especially, in these studies the performance analyses of heat exchangers are generally based on first and second law of thermodynamics as given reference [108, 109, 110, 111, and 112]. In the present work, the entropy minimization is carried out using bat algorithm (BA).

\[ \dot{S}_{gen,\Delta T} = J_1^0 \left( \frac{m_c \rho d \tau}{T} \right)_{h,c} = (mC_p)_{h} \ln \frac{T_{ho}}{T_{hi}} + (\dot{m}C_p)_{c} \frac{T_{co}}{T_{ci}} \] (4.2.34)

\[ \dot{S}_{gen,\Delta P} = \left( \frac{-\Delta \rho}{\rho} \ln \frac{T_{o}}{T_i} \right)_{h,c} \]

\[ = m_1 \frac{\Delta \rho}{\rho} \ln \frac{T_{ho}/T_{hi}}{T_{ho}/T_{hi}} + m_2 \frac{\Delta \rho_2}{\rho_2} \ln \frac{T_{co}/T_{ci}}{T_{co}/T_{ci}} \] (4.2.35)

\[ \dot{S}_{gen} = \dot{S}_{gen,\Delta T} + \dot{S}_{gen,\Delta P} \] (4.2.36)

\[ T_{ho} = T_{hi} - \epsilon(T_{hi} - T_{ci})C^{**} \] (4.2.37)

\[ T_{co} = T_{ci} - \epsilon(T_{hi} - T_{ci}) \] (4.2.38)
\[ S_{\text{mod,ent}} = \frac{S_{\text{gen}}T_{ci}}{q} \]  

(4.2.39)

where \( S_{\text{mod,ent}} \) is the objective function represents modified entropy generation number for the case study [113].

**4.3 BELL-DELAWARE METHOD**

This method is the most reliable, accurate and complete method because it accounts the leakages between baffles and shell, and between baffles and tubes. The results obtained by using empirical correlations of this method are quite matching with the experimental results with small deviation. Hence large numbers of people choose to prefer this method for calculating thermal and hydraulic performance of a heat exchanger. From the open literature, Sanaye at el. [13], Fettaka at el. [14], Fesanghary at el. [56], Sun et al. [89] and Jiangfeng et al. [107], used this method. The model-4, model-5 and model-6 are deals with Bell-Delaware method and the governing equations and input parameter remains same.

- **Model - 4:** Multi-objective optimization using genetic algorithm on shell and tube heat exchanger by Bell-Delaware method.

- **Model - 5:** A Comparison between multi-objective and single objective optimization using genetic and cuckoo search algorithms by Bell-Delaware method.

- **Model - 6:** Multi-objective optimization using bat algorithm on shell and tube exchanger by Bell-Delaware method. (Multi-objective functions)

The optimized results of the above models are discussed in the chapter 5 Results and discussion.

**4.3.1 THERMAL AND ANALYTICAL MODELLING**

The heat exchanger selected is of type 1-2 TEMA E single phase flow shell and tube heat exchanger with a fixed tube sheet and segmental baffles
4.3.2 HEAT EXCHANGER DESIGN FORMULATIONS

To design the size of a heat exchanger the three important parameters are to be determined. They are overall heat transfer coefficient, tube side pressure drop and shell side pressure drop.

\[
U = \frac{1}{\frac{1}{h_0} + R_{o,f} + \frac{d_0 \ln(d_0/d_1)}{2K_w} + R_{i,f} \frac{d_a}{d_i} + \frac{1}{h_i d_i}} \quad (4.3.1)
\]

where \( U \) is the overall heat transfer coefficient, \( R_{i,f}, R_{o,f} \) are fouling resistance on tube side and shell side respectively, \( K_w \) is the thermal conductivity of the wall.

\[
A_{t,t} = \pi L d_0 N_t \quad (4.3.2)
\]

where \( A_{t,t} \) is the total heat transfer surface area on tube side.

\[
NTU_{max} = \frac{UA_{t,t}}{C_{min}} \quad (4.3.3)
\]

Where \( NTU \) is the number of transfer of units

\[
\varepsilon = \frac{2}{(1+C^{**}) + (1+C^{**2})^{0.5} \cot h \left( \frac{NTU}{2(1+C^{**2})^{0.5}} \right)} \quad (4.3.4)
\]

Where \( C^{**} \) is heat capacity ratio.

4.3.3 SHELL SIDE

The shell side computations are significantly more intricate than those for the tube side. This is because on shell side four bypass streams and one principal cross flow stream were involved.

\[
h_0 = h_x = h_{id} J_c J_{id} J_r \quad (4.3.5)
\]

where \( h_{id} \) is the heat transfer coefficient in ideal tube bank for pure cross flow.

\[
h_{id} = J_s c_{p,s} \left( \frac{n_s}{\chi_s} \right) \left( \frac{k_s}{c_p,s \mu_s} \right)^{2/3} \left( \frac{\mu_s}{\mu_{s,w}} \right)^{0.14} \quad (4.3.6)
\]

Where \( J_s \) is the Colburn factor, \( \mu_s \) is the viscosity on shell side, \( \mu_{s,w} \) is the viscosity of fluid wall layer on shell side.

Segmental baffle window correction factor (\( J_c \)):

\[
J_c = 0.55 + 0.72 F_c \quad (4.3.7)
\]

\[
F_c = 1 - 2(F_{w}) \quad (4.3.8)
\]
where $F_c$ is the fraction of number of tubes in pure cross between baffle cuts.

\[ F_w = \frac{\theta_{ctl}}{360} - \sin \frac{\theta_{ctl}}{2\pi} \]  \hspace{1cm} (4.3.9)

where $F_w$ is the fraction of number of tubes in one baffle window.

\[ \theta_{ctl} = 2 \cos^{-1} \left[ \frac{D_s}{D_{ctl}} \left[ 1 - 2 \left( \frac{B_c}{100} \right) \right] \right] \]  \hspace{1cm} (4.3.10)

\[ D_{ctl} = D_{otl} - d_o \]  \hspace{1cm} (4.3.11)

Correction factor for baffle leakage ($J_L$)

\[ J_L = 0.44(1 - r_s) + [1 - 0.44(1 - r_s)] e^{-2.2r_{lm}} \]  \hspace{1cm} (4.3.12)

\[ r_s = \frac{S_{sb}}{S_{sb} + S_{tb}} \]  \hspace{1cm} (4.3.13)

\[ r_{lm} = \frac{S_{sb} + S_{tb}}{S_m} \]  \hspace{1cm} (4.3.14)

\[ S_m = B \left[ L_{bb} + \frac{D_{ctl}}{P_{T,eff}} (P_T - d_o) \right] \]  \hspace{1cm} (4.3.15)

where $B$ central baffle spacing.

\[ S_{sb} = \pi D_s \left( \frac{L_{sb}}{2} \right) \left( \frac{360 - \theta_{ds}}{360} \right) \]  \hspace{1cm} (4.3.16)

\[ \theta_{ds} = 2 \cos^{-1} \left[ 1 - 2 \left( \frac{B_c}{100} \right) \right] \]  \hspace{1cm} (4.3.17)

\[ S_{tb} = S_{tb1} N_t (1 - F_w) \]  \hspace{1cm} (4.3.18)

\[ S_{tb1} = \left\{ \frac{\pi}{4} \left[ (d_o - L_{tb})^2 - d_o^2 \right] \right\} \]  \hspace{1cm} (4.3.19)

\[ D_{otl} = D_s - L_{bb} \]  \hspace{1cm} (4.3.20)

where $L_{bb}$ is the inside shell diameter to tube bundle bypass clearance and $L_{tb}$ is the diametral clearance between tube and outside diameter and baffle hole.

where $L_{sb}$ is the diametral clearance between shell diameter and baffle diameter.

Correction factor for by pass between tube bundle and shell ($J_b$)

\[ J_b = \exp \left\{ -C_{bb} F_{sbp} \left( 1 - 3\sqrt{2r_{ss}} \right) \right\} \]  \hspace{1cm} (4.3.21)

\[ F_{sbp} = \frac{s_b}{S_m} \]  \hspace{1cm} (4.3.22)
\[ S_b = B[(D_s - D_{otl}) + L_{pl}] \]  \hspace{1cm} (4.3.23)

\[ D_{ctl} = D_s - (L_{bb} + d_o) = D_{otl} - d_o \]  \hspace{1cm} (4.3.24)

\[ r_{ss} = \frac{N_{ss}}{N_{tcc}} \]  \hspace{1cm} (4.3.25)

where \( N_{ss} \) is the number sealing strips used in one baffle.

\[ N_{tcc} = \frac{D_s}{P_p} \left[ 1 - 2 \left( \frac{B_c}{100} \right) \right] \]  \hspace{1cm} (4.3.26)

**Correction factor for factor unequal baffle spacing \( (J_s) \)**

\[ J_s = \frac{(N_b-1) + (L_t)^{(1-n)} + (L_d)^{(1-n)}}{(N_b-1) + (L_t) + (L_d)} \]  \hspace{1cm} (4.3.27)

\[ N_b = \frac{L_{ti}}{B} - 1 \]  \hspace{1cm} (4.3.28)

\[ L_{ti} = L_{to} - 2L_{ts} \]  \hspace{1cm} (4.3.29)

\[ L_{ts} = 0.1D_s \]  \hspace{1cm} (4.3.30)

where \( L_{ti} \) is summation of all baffle spacing’s. \( L_{ts} \) is the width of the tubesheet.

**Correction factor for adverse temperature gradient in laminar flow \( (J_r) \)**

Case (i) If \( R_e \leq 20 \)

\[ J_r = (J_r)_r \left[ \frac{1}{N_c} \right]^{0.18} = \frac{1.51}{(N_c)^{0.18}} \]  \hspace{1cm} (4.3.31)

\[ N_c = (N_{tcc} + N_{tcw})(N_b + 1) \]  \hspace{1cm} (4.3.32)

where \( N_c \) is the total number of tube rows crossed in entire heat exchanger.

\[ N_{tcc} = \frac{D_s}{P_p} \left[ 1 - 2 \left( \frac{B_c}{100} \right) \right] \]  \hspace{1cm} (4.3.33)

\[ N_{tcw} = \frac{0.8}{P_p} \left[ D_s \left( \frac{B_c}{100} \right) - \left( \frac{D_s - D_{ctl}}{2} \right) \right] \]  \hspace{1cm} (4.3.34)

\[ N_b = \frac{L_{ti}}{B} - 1 \]  \hspace{1cm} (4.3.35)
Case (ii) If \( 20 \leq R_e \leq 100 \)

\[ J_r = (J_r)_r + \left[ \frac{20-R_e}{80} \right] (J_r)_r - 1 \]  

(4.3.36)

Case (ii) If \( R_e \geq 100 \)

\[ J_r = 1 \]  

(4.3.37)

The total pressure drop on shell side is given below.

\[ \Delta p_s = \Delta p_c + \Delta p_e + \Delta p_w \]  

(4.3.38)

where \( \Delta p_s \) is the shell side total pressure drop, \( \Delta p_c \) is the pressure drop in cross flow, \( \Delta p_e \) is the pressure drop at shell side inlet and outlet region, \( \Delta p_w \) is the pressure drop at window region.

A single point calculation of heat transfer and pressure drop will give unrealistic results when there is a variation of flow is involved on shell side. This is valid in the case of combination of laminar and turbulent flow exists because the thermal performance is different in those two regimes. Therefore it is necessary to perform the calculations in zone-wise. The number of zones will determine the variations in pressure drop and thus the Reynolds number [8].

4.3.4 TUBE SIDE

The heat transfer coefficient on tube side (\( h_t \)) from Shah et al. [6]

\[ h_t = \left( \frac{K_t}{d_t} \right) 0.024 R_e^{0.8} P_{rt}^{0.4} \quad \text{for} \quad 2500 < R_e < 1.24 	imes 10^5 \]  

(4.3.39)

where \( K_t \) is tube side thermal conductivity, \( P_{rt} \) is the Prandtl number on tube side, \( R_e \) is Reynolds number on tube side.

Mass velocity unequivocally impacts the heat transfer coefficient. Increasing the mass velocity, pressure drop builds more quickly than heat exchange coefficient.

At high velocities, it prompts erosion.

\[ R_e = \frac{\dot{m}_t d_i}{\mu_t A_{o.t.p}} \]  

(4.3.40)

where \( A_{o.t.p} \) is tube side cross flow section area / pass is follows:

\[ A_{o.t.p} = 0.25 \pi d_i^2 N_t/n_p \]  

(4.3.41)

where \( n_p \) is the number of tube passes, \( m_t \) is mass flow rate in the tube.

\[ \Delta p_t = \frac{\rho_t}{2\rho_i} \left[ \left( 1 - \sigma^2 + K_{cc} \right) + 2 \left( \rho_i/\rho_0 - 1 \right) + \frac{4f_iL}{d_i} \rho_i (1/\rho) m_t - \left( 1 - \sigma^2 + \right] 

\]
\[ K_{ee} \rho_i / \rho_0 \]  

where \( K_{ee} \) and \( K_{ec} \) are tube entrance and exit pressure loss coefficients.

\[ f_t = 0.00128 + 0.1143(Re_t)^{-0.311} \text{ for } 4000 < Re_t < 10^7 \]  

(4.3.42)

(4.3.43)

4.3.5 OBJECTIVE FUNCTIONS AND DESIGN PARAMETERS

In the present paper, the two objective functions are effectiveness and total cost. Each objective function has several decision variables. They are baffle cut, baffle spacing, pitch, length of the tube, number of tubes and tube layout pattern. The two objectives have their maxima and minima. There is no single solution to this multi-objective problem. The multi-objective genetic algorithm gives a set of optimal solutions. The solutions are also known as Pareto front.

The total cost includes both investment cost and operating cost and heat transfer area

\[ C_{inc} = 8500 + 409A_t^{0.85} \]  

(4.3.44)

where \( A_t \) is the total heat transfer area at tube outside. \( C_{inc} \) is investment cost for both shell and tube [106]. The tube material is Admiralty (70% Cu, 30% Ni).

\[ C_{opc} = \sum_{K=1}^{n_y} \frac{C_{oc}}{(1+i)^k} \]  

(4.3.45)

\[ C_{oc} = P k_{ell} \tau \]  

(4.3.46)

\[ P = \frac{1}{\eta} \left( \frac{\dot{m}_t}{\rho_t} \Delta p_t + \frac{\dot{m}_s}{\rho_s} \Delta p_s \right) \]  

(4.3.47)

\[ C_{totc} = C_{inc} + C_{opc} \]  

(4.3.48)

where \( P \) is pumping power, \( n_y \) is the equipment life period, \( i \) is annual discount rate, \( k_{ell}, \tau \) and \( \eta \) are price of electrical energy, hours of operation per year and pump efficiency.

4.3.6 NUMBER OF TUBES

This is also one of the important parameter which causes trade-off between objective functions. As the number of tubes increases, the shell diameter increases. Hence the number of tubes is not considered for maximizing effectiveness or minimizing total cost because the shell diameter is kept constant from the designer’s perspective.
The equations for exit temperatures are as follows.

\[ T_{ho} = T_{hi} - \varepsilon \frac{c_{min}}{c_s} (T_{hi} - T_{ci}) \]

\[ T_{co} = T_{ci} + \varepsilon \frac{c_{min}}{c_t} (T_{hi} - T_{ci}) \]

where \( T_{hi} \) and \( T_{ho} \) are shell side inlet and outlet temperatures, \( T_{ci} \) and \( T_{co} \) are tube side inlet and outlet temperatures.

### 4.4 PRESSURE DROP MODEL BASED ON FEM.

Model - 7: Optimization of shell and tube heat exchanger using FEM based pressure drop model.

The FEM based pressure drop model is the latest model developed by Parikshit et al. [38]. No other authors are used this FEM based model for determination of pressure drop. This method is exclusively different from Bell-Delaware and Kern method. This method is chosen for optimization and the results are compared with the open literature [13]. The experimental results of the pressure drop are given by Bell K. J. et al. [102] by using pressure tapping instrumented tubes.

In this model, the entire shell and tube heat exchanger is discretised into number of elements in the direction of flow based on number of baffles. It is discretised into four elements, they are two elements in mid-region and another two elements in window region.

#### 4.4.1 GEOMETRICAL MODEL

In this model, the elements are discretised in such a way that it resembles with rectangular tube bundles as in Zukauskas. For each element, Zukauskas [99] friction factor is used. The Fig. 4.4.1 shows the distribution all the four elements. The following are the empirical correlations are given below.
a. One baffle space divided into elements (1, 2, 3, 4).

b. Cross section of the baffle space.

c. Circular cross section split into Rectangular elements.

d. Flow direction and associated pitches.

Fig: 4.4.1.a, b, c, d. Discretized elements in the shell and tube heat exchanger.

For inline tube arrangement, the minimum cross flow area used is given by

\[
A_{min} = \frac{(X_t - d_0)Sx(i)}{x_t} + Sx_e \quad (4.4.1)
\]

\[
X_1 = 2 \sqrt{- \left(\frac{D_2 - 2l_t}{2}\right)^2 + \left(\frac{Dot}{2}\right)^2} \quad (4.4.2)
\]

For staggered arrangement, the minimum cross flow area is given by

\[
A_{min} = \frac{(X_t - d_0)Sx(i)}{x_t} + Sx_e \quad \text{for} \ (X_t - d_0) \leq (X_d - d_0) \quad (4.4.3)
\]
or

\[ A_{\min} = \frac{2(X_d-d_0)Sx_e(i)}{X_t} + Sx_e \text{ for } (X_t - d_0) > (X_d - d_0) \quad (4.4.4) \]

For the minimum cross flow area at the baffle is given by

\[ A_{b\min} = \frac{1}{2} \left( \frac{D_e}{2} \right)^2 \left( \theta_b - \sin \theta_b \right) - N_{tw} \pi \frac{d_o^2}{4} \quad (4.4.5) \]

### 4.4.2 YAW CORRECTION FACTOR

Many investigations reported in the literature, the direction of flow of fluid have not been considered for the determination of pressure drop. Many authors have assumed that fluid flow is perpendicular to the axis of the shell. However, Kapale et al. [115] have proposed that fluid flow in the window region is parallel to the axis of the tube and in the mid-section it is flowing at an angle. When the friction factors given by Gunter et al. [115] are applied the result showed the deviation up to 85% from the experimental values of Halle at el. [116]. But in the Parikshit et al. [38] model using FEM, it is proposed that flow of fluid is at an angle both in the window region and mid-section region is shown in Fig.4.4.2. Further, it is shown in Fig.4.4.3 that the flow of fluid angle in inter-baffle regions is different from that of inlet and outlet sections.

The Yaw angle (\( \psi \)) decides the flow patterns of the fluid. The Zukuskas [99] correction factor is in the form of graph and is correlated by Schlunder [3]. The following is the correlations to get the Yaw correction factors. Yaw correction factor for inline tube arrangement is given below.
Fig: 4.4.2(a, b). Flow in Inter-baffle region when $\frac{D_3}{S} \geq 1$.

Fig. 4.4.3 Flow in inlet and out section.

\[ K_\psi = 1.107 \exp(-0.301 \psi^{-2.412}) \]  \hspace{1cm} (4.4.6)

\[ K_\psi = 1.245 \exp(-0.478 \psi^{-1.733}) \]  \hspace{1cm} (4.4.7)

The Euler number ($E_u$) obtained from [117] is multiplied with correction factor ($K_\psi$). So that the corrected Euler number ($E_{uc}$) can be determined.

\[ E_{uc} = K_\psi E_u \]  \hspace{1cm} (4.4.8)
4.4.3 FEM MODEL

In STHX, the flow of fluid is complex on shell side. The entire heat exchanger is discretised into number of elements in the direction of flow based on the number of baffles. The number of elements obtained is four between the baffles on discretization. In order to evaluate the pressure drop, initially the fluid angle is determined it is the characteristics of the any element. Then the friction factor is determined for each and every element. Using this, the stiffness matrix is calculated for every element and the global stiffness matrix is obtained by the assembling of these elemental stiffness matrices. Using the known boundary conditions, the pressure at each node is determined. Pressure drop example in a pipe network is given by Lewis et al. [18] and the related formula are given in the reference [119].

For nozzles the coefficient \( k \) of stiffness matrix within the shell for each element is given by

\[
k = \frac{2A_{\text{min}}^2}{Eu_c\rho Q_n}
\]

The above equation or elements are summed up at end of the exit section and at the beginning of inlet section.

\[
k = \frac{2A_{\text{nozzle}}^2}{K_n\rho Q}
\]

For spring element the coefficient \( k \) of the stiffness matrix is given below

\[
k = \frac{A_{\text{min}}^2}{K_1\rho Q}
\]

Fig: 4.4.4. P1, P2, P3, P4, P5 are the pressures at nodes and \( \Delta P_1, \Delta P_2, \Delta P_3, \Delta P_4 \) are the pressure drop in the inlet region.
The pressure drop are calculated in each zone is shown in Fig. 4.4.4 and assembled in the matrix given below to find overall pressure drop. The pressure element is shown below,

\[
\begin{bmatrix}
  k & -k \\
  -k & k
\end{bmatrix}
\begin{bmatrix}
P_i \\
P_j
\end{bmatrix} = \begin{bmatrix}
Q \\
-Q
\end{bmatrix}
\tag{4.4.12}
\]

The pressure element in the no tube in window (NTIW) region is shown below

\[
\begin{bmatrix}
  1 & -1 \\
  -1 & 1
\end{bmatrix}
\begin{bmatrix}
P_i \\
P_j
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\tag{4.4.13}
\]