CHAPTER 1

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1.1 Motivation

Notion of fuzzy relation equations was first given by Sanchez [133] in 1976. In many methods of describing a system it is assumed that there exists a functional relation between the input and the output variables of the system. Statistical methods are then widely used for determining an acceptable mathematical model of the process. In many ill-defined processes, decision algorithms may be set up based on numerical and non-numerical (linguistic) kinds of information. One way of modeling such fuzzy systems is by means of fuzzy relation equations. In fact, the idea of fuzzy relation equations used as a model of a system could be useful for a wide class of problems where we are faced with fuzzy information, i.e. economical, social or industrial.

Moreover, one can find wide range of applications of fuzzy logic, particularly, in fuzzy control and knowledge engineering, fuzzy modeling, fuzzy diagnosis and also applications in fields such as decision analysis, psychology, medicine, economics, and sociology [1,24,64,126,151]. The majority of fuzzy inference systems can be implemented by using the fuzzy relation equations [147]. Fuzzy relation equations can also be used for processes of compression/decompression of images and videos [49,50,79,96,97].

Resolution problem of fuzzy relation equations is one of the most important and widely studied problems in the field of fuzzy sets and systems. The first step for the resolution of fuzzy relation equations is to establish the existence of the solution. The solution of fuzzy relation equations lies on a lattice structure. The solution set of a consistent system can be characterized by unique maximum solution and finitely many minimal solutions, or dually, by a unique minimum solution and finitely many maximal solutions. If the
solution set is empty, approximate solutions can be found. It has been observed in the literature that the solvability and the solution set of fuzzy relation equations changes according to the selection of compositions, t-norms and the structure of lattice.

Fuzzy relation equations are powerful tools to analyze linear and non-linear systems. Since solutions to a system of fuzzy relation equations may not be unique in general, solutions with some particular features are usually desired, for instance, Sanchez [134] suggested that the study of fuzzy relation equations can be extended to the study of the degree of fuzziness of the solutions. This immediately leads to the problem of optimizing an objective function (or multiple objective functions) subject to a system of fuzzy relation equations. The solution set of fuzzy relation equations is, in general, a non-convex set, therefore traditional/classical methods for solving linear and nonlinear programming problems cannot be applied to fuzzy linear and nonlinear optimization problems equipped with fuzzy relation equations.

It has been observed that finding a simple solving algorithm for the resolution of fuzzy relation equations is still a challenge today. The problem of solving a fuzzy optimization problem is a NP-hard problem in terms of computational complexity. Although there is no polynomial time algorithm to completely solve the fuzzy relation equations unless \( P = NP \), a universal algorithm, nevertheless, is still desirable.

In this situation, a universal algorithm with effectively less computational complexity is very useful for solving linear or nonlinear fuzzy optimization problems. Moreover, use of advanced soft computing techniques are needed to keep pace with the development in this field. This motivation forms the basis of research.

1.2 Objectives and methodology

The major objective behind the work lies in the exposition of fuzzy relation equations and study of fuzzy linear, nonlinear and multiobjective optimization problems with fuzzy relation equations as constraints. The two major types of fuzzy relation equations are with
sup-$\otimes$ composition and inf-$\rightarrow$ composition, where $\otimes$ and $\rightarrow$ denotes a t-norm and a residuation operation (implication), respectively. The optimization models considered are with fuzzy relation equations subject to sup-$\otimes$ composition. Characterization of the feasible domain, complete solution set and establishing necessary conditions for solvability of fuzzy relation equations is discussed.

Linear, nonlinear and multiobjective optimization models are designed subject to fuzzy relation equations with different compositions as constraints and models are characterized for obtaining optimal solutions/decidable solutions/satisficing solutions. In case of linear/nonlinear programming problems with fuzzy relation equations having no unique solution, notion of approximate solutions is given. Applications of fuzzy relation equations is discussed in image compression and decompression/reconstruction. Tools, techniques and algorithms are developed based upon uncoded techniques as well as coded techniques.

Methodologies used to carry out the work are as follows:

(i) Uncoded techniques:

- Fuzzy relational calculus
- Modified classical techniques
- Modern heuristic techniques

(ii) Coded techniques (soft computing techniques):

- Genetic algorithms
- Evolutionary theory
- Graphical algorithms
- Hybrid techniques

Figure 1.1 shows the flowchart of the objectives of the research work.
1.3 Survey of related literatures

A new paradigm of mathematics based upon the concept of a fuzzy set was introduced by Zadeh in 1965 [172]. He introduced the notion of a fuzzy subset of a set as a method for representing uncertainty. The mathematical foundation of fuzzy logic has been discussed by Gottwald [39,42], Hájek [45], Novák et al. [99] and Belohlavek [9]. General discussion on fuzzy logic can be found in Wang et al. [156], Zadeh [177-179].

The basic ideas of fuzzy relations and the concepts of similarity and fuzzy orderings were introduced by Zadeh [172,173]. Binary fuzzy relations were further investigated by Rosenfeld [128], Yager [165] and Ovchinnikov [102]. The notion of fuzzy relation equations based upon the max-min composition was first proposed and investigated by Sanchez in 1976 [133]. The fundamental results for fuzzy relation equations with max-

Infinite fuzzy relation equations in complete Brouwerian lattices can be found in Wang [155] and Qu and Wang [124]. Han et al. [47] studied resolution of matrix equations over arbitrary Brouwerian lattices. Sessa [139] discussed unique solution of finite fuzzy relation equations in complete Brouwerian lattices. More work on fuzzy relation equations over Brouwerian lattices can be found in [154,164]. Wang and Chang [152] discussed interval valued fuzzy relation equations. Yeh [169] studied minimal solutions of max-min fuzzy relation equations. Fuzzy relations equations over continuous t-norms have been studied by Shieh [140,141].

Gottwald and Pedrycz [37,38] and Gottwald [40,41] studied the solvability indices of fuzzy relation equations. Approximate solutions of fuzzy relation equations can be found in Yuan and Klir [65,170]. Numerical methods for approximately solving fuzzy relation equations have been studied by Pedrycz [109]. More literature in this regard can be found in Pedrycz et al. [110,113,116]. Pedrycz [115] discussed statistical methods in case of approximate solutions of fuzzy relation equations. The use of genetic algorithms for
solving fuzzy relation equations was suggested by Sanchez [135]. Results in this direction have been obtained by Negoită et al. [95].


The linear optimization problem with different kinds of fuzzy relation equations as constraints is an important area of research. The problem of minimizing a linear objective function subject to a system of max-min equations was first investigated by Fang and Li [27] and later by Wu et al. [158] and Wu and Guu [159]. It was shown that such an optimization problem can be decomposed into two subproblems, one of which can be solved analytically while the other can be polynomially reduced to a 0-1 integer programming problem. Optimization problem with max-product composition was considered by Loetamonphong and Fang [77]. Pandey [105] studied the optimization of fuzzy relation equations with continuous t-norms and with linear objective function. Pandey and Srivastava [103] gave efficient procedure for optimization of linear objective function subject to fuzzy relation equations as constraints. More work in this regard can be found in Pandey [104,106] and Pandey and Srivastava [107]. Wu [160] and Khorram and Ghodousian [60] studied a linear optimization problem with max-average fuzzy relation equations.
The extension to nonlinear optimization problem with fuzzy relation equations as constraints was first proposed by Lu and Fang [80]. Nonlinear optimization with max-average fuzzy relation equations has been discussed by Khorram and Hassanzadeh [62]. Wang [153] proposed multiobjective mathematical programming problem with fuzzy relation equations as constraints. On the other hand Loetamonphong et al. [78] studied the problem of multiobjective optimization with max-min fuzzy relation equations. Khorram and Zarei [63] considered multiobjective optimization problem with max average fuzzy relation equations.

Wu et al. [162] and Li and Fang [73] studied linear fractional programming problem with max-Archimedean fuzzy relation equations. Chakraborty and Gupta [16] discussed multiobjective fuzzy linear fractional programming problem. Geometric programming problem with fuzzy relation equations has been studied by Wu [161] and Yang and Cao [168]. More work on study of optimization problems with fuzzy relation equations as constraints can be found in [31-33,43,44,71,86,120,125,150].

1.4 Organization

The presentation of thesis is organized as follows. Chapter 2 discusses a brief explanation of the origination of fuzzy sets, their meaning, properties and operations on fuzzy sets, fuzzy arithmetic, fuzzy relations, fuzzy relation equations and fuzzy logic. In Chapter 3, complete exposition of fuzzy relation equations, conditions of solvability of fuzzy relation equations, fuzzy optimization models and introduction to genetic algorithms is given. In Chapter 4, a linear optimization problem with fuzzy relation equations as constraints is considered and the optimal solution of the problem is achieved by two methods, one by establishing dynamic programming procedure for solving fuzzy linear programming problem and other by value matrix method and its reduction rules. In Chapter 5, concept of covering is established for a linear optimization problem with fuzzy relation equations as constraints. The covering problem is solved by developing a binary coded genetic algorithm. In Chapter 6, real coded genetic algorithm is developed for solving a nonlinear optimization problem with fuzzy relation equations as constraints.
Chapter 7 discusses a multiobjective optimization problem with fuzzy relation equations as constraints. Concept of Pareto optimality is used to develop a genetic algorithm to solve the problem. In Chapter 8, an optimization model is discussed with fuzzy relation equations as constraints having no unique solution. Problem is solved by introducing approximate solutions of fuzzy relation equations. In Chapter 9, a multiobjective linear fractional programming problem with max-Archimedean fuzzy relation equations as constraints is defined and a binary coded genetic algorithm is developed to solve the problem. Application of fuzzy relation equations in image processing is discussed in Appendix A.

The reference list at the end of the dissertation comprises mainly the works cited in the text and notes, and covers relevant books and significant papers on the study of fuzzy relation equations and optimization problems.