

Preface

In modern analysis, the concept of convergence theory includes the study of analysis, operators and transformation theory, topology and functional analysis, which provides a general framework for finding solutions of different problems in pure and applied mathematics. Above all, a study of convergence theory in itself provides a new insight and understanding into the processes and techniques of elementary analysis, which we are accustomed to use in our daily calculations. Many generalizations of usual convergence have been appeared in the literature so far, out of which statistical summability have become an active area of research in recent years.

Over the years statistical convergence has been studied under different names in the fields of fourier analysis, ergodic theory and number theory. The idea of statistical convergence was introduced by Fast (1) and Schoenberg (2) independently nearly six decades ago. This notion has very useful applications in different areas of mathematics such as Turnpike Theory, Summability Theory, Strong Integral Summability, Different Sequence Spaces and their geometry, Statistical Approximations, Fuzzy Sequence Spaces and Compact Matrix Operators.

To the obvious question regarding the significance of study of generalized convergence in different spaces, the answer is that there is no loss of generality in such study, rather there is a gain in terms of easy verification of generalized cases.

The contribution of this thesis is to present a study of general cases of convergence theory in different spaces (Normed and Banach spaces, double sequence spaces, Probabilistic Normed spaces and Random 2-normed spaces). In the present thesis, we have given a generalization of some classical methods of summability and introduced some summability theories in different spaces followed by characterization of certain properties of these generalized convergence methods on above mentioned spaces. Also, some particular cases for these convergence methods are also given.

The present thesis entitled *Characterization of Generalized Convergence in Some Spaces* is broadly divided into six chapters. The results of each chapter have been published in at least one research paper as mentioned in the beginning of the chapter. The materials of this description are organized as follows:

CHAPTER 1 is an introductory chapter. In this chapter, we have given a short survey on some basic definitions, notations and preliminary results, which are already known in the literature, that forms the background of our subject concerning the introduction of these spaces and theory of summability of sequences in considered spaces with some of its applications.

CHAPTER 2 introduces the notion of generalized weak convergence with the help of generalized de la Vallée-Poussin mean and ideals in normed spaces, which is a new method of summability, including the most classical methods as their special cases. Further, their algebraic and topological properties are also studied and some concrete examples are given to characterize the importance of this summability method.

CHAPTER 3 continues with the generalization of weak convergence using the notions of lacunary sequences and ideals in Banach spaces. In this chapter, we have defined weak lacunary statistical limit and cluster points in Banach spaces, discussed weak lacunary statistical convergence via ideals and defined generalized limit and

cluster points in these spaces. Further, we derived some interesting inclusion relations between these convergence methods and the collections of generalized limit and cluster points in these spaces.

CHAPTER 4 introduces (λ, μ) - statistical convergence of order $\tilde{\alpha}$ of double sequences. Further, we studied the general case of this summability method defined on double sequence space. Moreover, we deduced some known or new results as particular cases.

CHAPTER 5 investigates the theory of S_θ -limit and S_θ -cluster points in probabilistic normed spaces. Further, we studied S_θ -limit superior and S_θ -limit inferior in these spaces. The partial materials of this CHAPTER have been presented in the *International Conference on History and Development of Mathematical Sciences* (ICHDMS 2012), MDU, Rohtak, Haryana .

CHAPTER 6 is devoted to study of generalized statistical convergence in random 2-normed spaces. In this chapter, we have defined the terminology Λ -statistical convergence of order α and Λ -statistical Cauchy sequences of order α in random 2-normed spaces. We have extended some results of Esi and Braha (3) using the notion of λ -statistical convergence of order α given by Çolak and Bektaş (4).

Future Scope briefly gives some future directions of the field where some scope of the related work exists.

The results, examples and remarks, in this thesis, are specified with double decimal numbering. The first number indicates the chapter, the second represents the section, and the third refers to the number of result, example or remark as the case may be in a particular chapter. For example, the form 2.1.3 refers to the third paragraph (lemma, theorem, example or remark) appearing in Section 1 of Chapter 2.

Separate Conclusion Subsection is also given at the end of each chapter. The thesis ends with a fairly exhaustive bibliography of books and research articles consulted for the work.

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