CHAPTER - 5

TWO \((s, S)\) INVENTORY SYSTEMS WITH BINARY CHOICE OF DEMANDS AND OPTIONAL ACCESSORIES WITH \(SCBZ\) ARRIVAL PROPERTY

5.1. Introduction.

Two \((s, S)\) inventory systems in series is discussed by Chenniappan and Ramanarayanan [5]. Thangaraj and Ramanarayanan [51] have treated \((s, S)\) inventory system with two ordering levels. Daniel and Ramanarayanan [7] discussed \((s, S)\) inventory with random lead time rate and unit demand. Ramanarayanan and Jacob [40] studied this system with bulk demand. Authors [28, 29] have discussed the inventory system exposed to calamity with \(SCBZ\) arrival property and also have discussed two \((s, S)\) inventory system with perishable units.

In this chapter two inventory models are treated under Markovian environment. In model 1, the inventory has two brands of units. The arriving customer may demand any one of the two demands with a fixed probability. When units are similar, the manufactories of brand name plays the major role in business. Model 1 considers such a situation. In Model 2, two different units are available in the inventory. Units of one type are purchased by customer and units of second type are optional. This is always the case of companies offering financial package or credit with certain percentage of interest for loan along with supply of units for sales. In these above two models we introduce a concept in this area which has been introduced as Setting the Clock Back to Zero (SCBZ) by Raja Rao [39] for arrival process. The demand (arrival) rate changes

\(^{1}\)It is accepted and in due course of publication in International Journal of Contemporary Mathematical Sciences.
after an exponential time from one rate to another and becomes the initial one upon arrival of demand.

The above two models are studied for steady state inventory probabilities using the block partitioned methods of *NEUTS*. Numerical results are also presented.

5.2. **Model - 1: Binary Choice of Demands.**

The following are the assumptions of the model.

(i) There are two types of units *A* and *B* stored with maximum capacity of the inventory for each unit is $S$.

(ii) The arrival rate of unit demand is $\lambda_1$. If it does not occur within an exponential time with parameter $c$ the arrival rate changes to $\lambda_2$. Immediately upon arrival, the rate becomes $\lambda_1$.

(iii) Whenever a demand occurs type *A* unit is sold with probability $p$ if available and type *B* unit is sold with probability $q$ if available where $p + q = 1$. When a demand occurs for a non-available type, the demand is lost.

(iv) When the level falls to $s$ for type *A* or *B* units an order is placed for $S - s$ such units. The lead time rate distribution for type *A* and *B* are independent and exponential with parameters $\mu_1$ and $\mu_2$ respectively.

The inter arrival time distribution of demands may be derived as follows. Let $Y$ be the time between two consecutive demands. Let its *probability density function* be $h(y)$. Considering the truncation point $\tau_0$.
in which the parameter changes we note

\[ h(y) = \begin{cases} 
\lambda_1 e^{-\lambda_1 y} & \text{if } y \leq \tau_0 \\
\lambda_2 e^{-\lambda_2 y} e^{\tau_0(\lambda_2 - \lambda_1)} & \text{if } y > \tau_0.
\end{cases} \] (5.1)

Noting the truncation point \( \tau_0 \) itself is a random variable with pdf \( ce^{-c\tau_0} \), we find

\[ h(y) = \lambda_1 e^{-\lambda_1 y} e^{-cy} + \lambda_2 e^{-\lambda_2 y} \int_0^y e^{\tau_0(\lambda_2 - \lambda_1)} ce^{-c\tau_0} d\tau_0 \]

which reduces to

\[ h(y) = \frac{(\lambda_1 - \lambda_2)(c + \lambda_1)}{(c + \lambda_1 + \lambda_2)} e^{-y(\lambda_1 + c)} + \frac{c\lambda_2 e^{-\lambda_2 y}}{(c + \lambda_1 + \lambda_2)} \] (5.2)

and the distribution function of time between unit demands with varying parameter after exponential time is given by

\[ H(y) = 1 - \dot{p} e^{-y(\lambda_1 + c)} - q e^{-\lambda_2 y} \] (5.3)

where \( p = \frac{\lambda_1 - \lambda_2}{c + \lambda_1 - \lambda_2} \) and \( q = \frac{c}{c + \lambda_1 - \lambda_2} \). Also we note that \( p + q = 1 \).

For Markovian models it is advantageous to set up the infinitesimal generator for finding probabilities. The state of the system may be written as follows.

\[ S = \{(i, j, k) : 0 \leq i \leq S, 0 \leq j \leq S \text{ and } k = 1, 2\}. \] (5.4)

The system is in state \((i, j, k)\) when \( i \) units of type \( A \) and \( j \) units of type \( B \) are available when the rate of arrival of demand \( \lambda_k \) for \( k = 1, 2 \).

Let \( \mathbf{i} \) be the vector of states of the system arranged as follows,

\[ \mathbf{i} = \{(i, S, 1), (i, S, 2), (i, S - 1, 1), (i, S - 1, 2), \ldots, (i, 0, 1), (i, 0, 2)\} \]
for $0 \leq i \leq S$. The infinitesimal generator $Q$ ($Q$ is a matrix of order $2(S + 1)(S + 1)$ and submatrices $T, T_o$ and $A$ of order $2(S + 1)$ are given below.
\[ Q = \begin{bmatrix}
T & A \\
A & \mu_1 I \\
& \cdots
\end{bmatrix}
\]
\[
\begin{array}{cccccccc}
\text{(5.5.1)} & \text{(5.5.2)} & \text{(5.5-1.1)} & \text{(5.5-1.2)} & \cdots & \text{(5.5+1.1)} & \text{(5.5+1.2)} & \text{(5.5+1.3)} & \cdots & \text{(5.5+1.1)} & \text{(5.5+2.1)} & \text{(5.5+2.2)} & \cdots \\
-\lambda_1 & \epsilon & \lambda_1 & & & \epsilon & \lambda_1 & & \cdots & & & & \\
-\lambda_2 & \epsilon & \lambda_2 & & & \epsilon & \lambda_2 & & \cdots & & & & \\
-\lambda_1 - \mu_2 & \epsilon & \lambda_1 & & & \epsilon & \lambda_1 & & \cdots & & & & \\
-\lambda_2 - \mu_2 & \epsilon & \lambda_2 & & & \epsilon & \lambda_2 & & \cdots & & & & \\
-\lambda_1 - \epsilon - \mu_2 & \epsilon & \lambda_1 & & & \epsilon & \lambda_1 & & \cdots & & & & \\
-\lambda_2 - \epsilon - \mu_2 & \epsilon & \lambda_2 & & & \epsilon & \lambda_2 & & \cdots & & & & \\
& \mu_2 & \cdot & \cdot & \cdots & \cdot & \cdot & \cdots & \cdot & \cdot & \cdots & \cdot & \cdot & \cdots \\
\end{array}
\]
Let $\Pi$ be the vector of steady state probabilities associated with $Q$ satisfying

$$\Pi Q = 0 \quad \text{and} \quad \Pi e = 1 \quad (5.9)$$

where

$$\Pi = (\Pi_s, \Pi_{s-1}, \ldots, \Pi_1, \Pi_0)$$

and

$$e = (1, 1, 1, \ldots, 1)^t.$$  

We note from equation (5.9)

$$\Pi_s (T - \mu_1 I) + \Pi_{s+1} A = 0$$

which implies

$$\Pi_s = \Pi_{s+1} A ((\mu_1 I - T))^{-1}. \quad (5.10)$$

Similarly we may note

$$\Pi_{s-i} = \Pi_{s+1} [A ((\mu_1 I - T))^{-1}]^{i+1} \quad \text{for} \quad 0 \leq i \leq s - 1. \quad (5.11)$$
Similarly we can find

\[ \Pi_0 = \Pi_{s+1} \left[ A((\mu_1 I - T))^{-1} \right]^s \left[ A((\mu_1 I - T_0))^{-1} \right]. \tag{5.12} \]

Considering the first column

\[ \Pi_s T + \Pi_s \mu_1 I = 0 \]
gives

\[ \Pi_s = \Pi_{s+1} \left[ A((\mu_1 I - T))^{-1} \right] (\mu I)(-T)^{-1}. \tag{5.13} \]

Step by step we can write down other partitioned parts of \( \Pi \) vectors in terms of block \( \Pi_{s+1} \). Considering the second column we obtain

\[ \Pi_s A + \Pi_{s-1} T + \Pi_{s-1} \mu_1 I = 0 \]

and

\[ \Pi_{s-1} = \Pi_s A(-T)^{-1} + \Pi_{s-1} (\mu_1 I)(-T)^{-1} \]

We note

\[ \Pi_{s-1} = \Pi_{s+1} \left[ \left[ A((\mu_1 I - T))^{-1} \right]^2 \right] [(\mu_1 I)(-T)^{-1}] \]
\[ + \left[ A((\mu_1 I - T))^{-1} \right] (\mu_1 I)(-T)^{-1} A(-T)^{-1}. \]

It is straightforward to obtain

\[ \Pi_{s-k} = \Pi_{s+1} \sum_{i=0}^{k} \left[ A((\mu_1 I - T))^{-1} \right]^{k+1-i} [(\mu_1 I)(-T)^{-1}] [A(-T)^{-1}]^i \tag{5.14} \]

for \( 1 \leq k \leq s - 1 \). We further note that

\[ \Pi_{s-s+1} A + \Pi_{s-s} T + \Pi_0 \mu_1 I = 0 \]
presents

\[ \Pi \ s_{-s} = \Pi_{s+1} \left[ A((\mu_1 I - T))^{-1} \right]^s [A(\mu_1 I - T_0)^{-1}] [A(-T)^{-1}] \]
\[ + \Pi_{s+1} \left[ \sum_{i=0}^{s-1} \left[ A((\mu_1 I - T))^{-1} \right]^s \left[ (\mu_1 I)^{-1} \right] [A(-T)^{-1}]^{i+1} \right] \]

(5.15)

Proceeding in this manner we can obtain

\[ \Pi \ s_{-s-k} = \Pi_{s+1} \left\{ \left[ A((\mu_1 I - T))^{-1} \right]^s [A(\mu_1 I - T_0)^{-1}] [A(-T)^{-1}] \right\} \]
\[ + \left[ \sum_{i=0}^{s-1} \left[ A((\mu_1 I - T))^{-1} \right]^s \left[ (\mu_1 I)^{-1} \right] [A(-T)^{-1}]^{i+1} \right] \] \[ \times \left[ \sum_{k=0}^{S-2s-2} \left[ A(-T)^{-1} \right]^k \right] + 1 + \sum_{i=0}^{s-1} \left[ A((\mu_1 I - T))^{-1} \right]^{i+1} \]
\[ + \left[ A((\mu_1 I - T))^{-1} \right]^s [A((\mu_1 I - T_0)^{-1})] \] \[ \varepsilon = 1 \] (5.16)

for \(0 \leq k \leq S - 2s - 2\).

All blocks of the \(\Pi\) vector are written in terms of \(\Pi_{s+1}\). The vector \(\Pi_{s+1}\) may be calculated using the law of the total probability

\[ \Pi_{s+1} \left\{ \sum_{k=0}^{s-1} \left[ \sum_{i=0}^{k} \left[ A((\mu_1 I - T))^{-1} \right]^{k+1-i} \left[ (\mu_1 I)^{-1} \right] \right] \right\} \]
\[ + \left[ A((\mu_1 I - T))^{-1} \right]^s [A(\mu_1 I - T_0)^{-1}] [A(-T)^{-1}] \]
\[ + \sum_{i=0}^{s-1} \left[ A((\mu_1 I - T))^{-1} \right]^s \left[ (\mu_1 I)^{-1} \right] [A(-T)^{-1}]^{i+1} \]
\[ \times \left( \sum_{k=0}^{S-2s-2} \left[ A(-T)^{-1} \right]^k \right) + 1 + \sum_{i=0}^{s-1} \left[ A((\mu_1 I - T))^{-1} \right]^{i+1} \]
\[ + \left[ A((\mu_1 I - T))^{-1} \right]^s \left[ A((\mu_1 I - T_0)^{-1}) \right] \] \[ \varepsilon = 1 \] (5.17)

This gives

\[ \Pi_{s+1} = \frac{a^t}{|a|^2} \] (5.18)
where \( a \) is given by

\[
\begin{align*}
\frac{\text{se}}{\text{oil/}} & = \left\{ \sum_{k=0}^{s-1} \left[ \sum_{i=0}^{k} \left[ A((\mu_1 I - T))^{-1} \right]^{k+1-i} \left( (\mu_1 I)(-T)^{-1} \right)^i \right] \right. \\
& + \left\{ \left[ A((\mu_1 I - T))^{-1} \right]^s \left[ A((\mu_1 I - T_0))^{-1} \right] \right. \\
& \left. + \sum_{i=0}^{s-1} \left[ A((\mu_1 I - T))^{-1} \right]^s \left[ (\mu_1 I)(-T)^{-1} \right] \left[ A(-T)^{-1} \right]^{i+1} \right\} \\
& \left. \times \left( \sum_{k=0}^{S-2s-2} \left[ A(-T)^{-1} \right]^k \right) + 1 + \sum_{i=0}^{s-1} \left[ A((\mu_1 I - T))^{-1} \right]^{i+1} \right. \\
& \left. + \left[ A((\mu_1 I - T))^{-1} \right]^s \left[ A((\mu_1 I - T_0))^{-1} \right] \right\} e. \\
\end{align*}
\]

(5.19)

Equations (5.12) to (5.19) present all the probabilities of the system.

5.3. **Model - 2: Optional Accessories.**

The following are the assumptions of the model 2.

(i) There are two types of units \( A \) and \( B \) stored with maximum capacity of the inventory for each unit is \( S \).

(ii) The arrival rate of unit demand is \( \lambda_1 \). If it does not occur within an exponential time with parameter \( c \) the arrival rate changes to \( \lambda_2 \). Immediately upon arrival, the rate becomes \( \lambda_1 \).

(iii) Whenever a demand occurs type \( A \) and type \( B \) units are together demanded with probability \( p \) and type \( A \) unit is alone demanded with probability \( q \) where \( p + q = 1 \). When unit \( B \) is not available unit \( A \) is alone supplied and when unit \( A \) is not available the demand is lost.

(iv) When the level falls to \( s \) for type \( A \) or \( B \) units an order is placed for \( S - s \) such units. The lead time rate distribution for type \( A \)
and $B$ are independent and exponential with parameters $\mu_1$ and $\mu_2$ respectively.

We may note that the cumulative distribution function and probability density function of arrival time of demand is as given in equations (5.3) and (5.2). The state of the system is given by

$$S = \{(i, j, k) : 0 \leq i \leq S, 0 \leq j \leq S \text{ and } k = 1, 2\}. \quad (5.20)$$

The system is in state $(i, j, k)$ when $i$ units of type $A$ and $j$ units of type $B$ are available when the rate of arrival of demand $\lambda_k$ for $k = 1, 2$. Let $\dot{i}$ be the vector of states of the system arranged as follows,

$$\dot{i} = \{(i, S, 1), (i, S, 2), (i, S - 1, 1), (i, S - 1, 2), \ldots, (i, 0, 1), (i, 0, 2)\}$$

for $0 \leq i \leq S$. The infinitesimal generator of the system is a matrix of order $2(S + 1)(S + 1)$. It has same block partitioned structure as given in model 1 in (5.5). However the sub-matrices $T, T_o$ and $A$ are different as given below.
\[(5.21)\]
\begin{align*}
\begin{array}{cccc}
0.0, 0.01, 0.5 & 0.0, 0.01, 0.5 & 0.0, 0.01, 0.5 & 0.0, 0.01, 0.5 \\
0.0, 0.01, 0.5 & 0.0, 0.01, 0.5 & 0.0, 0.01, 0.5 & 0.0, 0.01, 0.5 \\
\vdots & \vdots & \vdots & \vdots \\
0.0, 0.01, 0.5 & 0.0, 0.01, 0.5 & 0.0, 0.01, 0.5 & 0.0, 0.01, 0.5 \\
0.0, 0.01, 0.5 & 0.0, 0.01, 0.5 & 0.0, 0.01, 0.5 & 0.0, 0.01, 0.5 \\
\end{array}
\end{align*}

(5.22)
For $1 \leq j \leq S$, we have the following submatrix $A$.

$$
A =
\begin{bmatrix}
\lambda_1q & 0 & \lambda_1p & 0 & 0 \\
\lambda_2q & 0 & \lambda_2p & 0 & 0 \\
0 & 0 & \lambda_1q & 0 & \lambda_1p \\
0 & 0 & \lambda_2q & 0 & \lambda_2p \\
& & \lambda_1q & 0 & \lambda_1p \\
& & \lambda_2q & 0 & \lambda_2p \\
& & & \ddots & \ddots \\
& & & & \lambda_1q & 0 & \lambda_1p \\
& & & & \lambda_2q & 0 & \lambda_2p \\
& & & & 0 & 0 & \lambda_1 \\
& & & & 0 & 0 & \lambda_2 \\
\end{bmatrix}
$$

(5.23)

Since the sub matrices are alone different and the infinitesimal generator has the structure given in (5.5) the equations (5.10) to (5.20) derived for model 1 present the steady state probability of model 2 where equations (5.21), 5.22 and (5.23) are used for $T$, $T_0$ and $A$.

5.4. Numerical Example.

5.4.1. Numerical Example for Model - 1.

Let the maximum capacity of the inventory be $3(S = 3)$ and the reorder level is $1(s = 1)$. The infinitesimal generator of the finite state space continuous time Markov chain is as follows.

$$
\begin{array}{ccc}
& 3 & 2 & 1 & 0 \\
3 & T & A & & \\
2 & & T & A & \\
1 & \mu_1 I & T - \mu_1 I & A & \\
0 & \mu_1 I & T_0 - \mu_1 I & & \\
\end{array}
$$

where $T$, $A$ and $T_0$ are as given below.
For fixed values of $\lambda_1 = 1$, $\lambda_2 = 2$, $\mu_1 = 1$, $\mu_2 = 2$, $c = 1$, $p = 0.7$ and $q = 0.3$, the i.l.p (inventory level probabilities) are given as follows.

The sum of steady state probabilities is found to be 1.0000.

We find the expected inventory levels (e.i.l.), for different cases.
Case 5.4.1. (Varying first demand rate $\lambda_1$)

For the fixed values of $\lambda_2 = 1$, $\mu_1 = 1$, $\mu_2 = 2$, $c = 1$, $p = 0.7$ and $q = 0.3$, we compute the expected inventory levels e.i.l $\lambda_1 = 1, 2, 3, 4$ and $5$.

<table>
<thead>
<tr>
<th>$\lambda_1$</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.6098</td>
<td>2.2498</td>
</tr>
<tr>
<td>2</td>
<td>1.3609</td>
<td>2.1933</td>
</tr>
<tr>
<td>3</td>
<td>1.1961</td>
<td>2.1457</td>
</tr>
<tr>
<td>4</td>
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<td>2.1016</td>
</tr>
<tr>
<td>5</td>
<td>0.9939</td>
<td>2.0587</td>
</tr>
</tbody>
</table>

Table - 5.4.1 and Figure - 5.4.1

When first demand rate $\lambda_1$ increases, the expected inventory levels of $A$ and $B$ decreases.

Case 5.4.2. (Varying second demand rate $\lambda_2$)

For the fixed values of $\lambda_1 = 1$, $\mu_1 = 1$, $\mu_2 = 2$, $c = 1$, $p = 0.7$ and $q = 0.3$, we compute the expected inventory levels e.i.l $\lambda_2 = 1, 2, 3, 4$, and $5$.

<table>
<thead>
<tr>
<th>$\lambda_2$</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.8252</td>
<td>2.3466</td>
</tr>
<tr>
<td>2</td>
<td>1.6098</td>
<td>2.2498</td>
</tr>
<tr>
<td>3</td>
<td>1.4995</td>
<td>2.1874</td>
</tr>
<tr>
<td>4</td>
<td>1.4313</td>
<td>2.1423</td>
</tr>
<tr>
<td>5</td>
<td>1.3846</td>
<td>2.1877</td>
</tr>
</tbody>
</table>

Table - 5.4.2 and Figure - 5.4.2

When second demand rate $\lambda_2$ increases, the expected inventory levels of $A$ and $B$ decreases except of $B$ for $\lambda_2 = 5$. 
Case 5.4.3. (Varying lead time rate of inventory $A$)

For the fixed values of $\lambda_1 = 1$, $\lambda_2 = 2$, $\mu_2 = 2$, $c = 1$, $p = 0.7$ and $q = 0.3$, we compute the expected inventory levels $e.i.l \mu_1 = 1, 2, 3, 4$ and 5.

<table>
<thead>
<tr>
<th>$\mu_1$</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.6098</td>
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<tr>
<td>2</td>
<td>2.0126</td>
<td>2.2468</td>
</tr>
<tr>
<td>3</td>
<td>2.1706</td>
<td>2.2456</td>
</tr>
<tr>
<td>4</td>
<td>2.2527</td>
<td>2.2450</td>
</tr>
<tr>
<td>5</td>
<td>2.3027</td>
<td>2.2447</td>
</tr>
</tbody>
</table>

Table - 5.4.3 and Figure - 5.4.3

When the lead time rate of inventory $A$ increases, the expected inventory levels of $A$ increase but $e.i.l$ of $B$ decrease slightly.

Case 5.4.4. (Varying lead time rate of inventory $B$)

For the fixed values of $\lambda_1 = 1$, $\lambda_2 = 2$, $\mu_1 = 1$, $c = 1$, $p = 0.7$ and $q = 0.3$, we compute the expected inventory levels $e.i.l \mu_2 = 1, 2, 3, 4$ and 5.

<table>
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<th>$\mu_2$</th>
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<td>2.0115</td>
</tr>
<tr>
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<td>1.6098</td>
<td>2.2498</td>
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<tr>
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<td>1.6164</td>
<td>2.3330</td>
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<td>1.6204</td>
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<tr>
<td>5</td>
<td>1.6231</td>
<td>2.3999</td>
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</table>

Table - 5.4.4 and Figure - 5.4.4

When the lead time rates of inventory $B$ increase, the expected inventory levels of $A$ and $B$ increase.
Case 5.4.5. (Varying $c$)

For the fixed values of $\lambda_1 = 1$, $\lambda_2 = 2$, $\mu_1 = 1$, $\mu_2 = 2$, $p = 0.7$ and $q = 0.3$, we compute the expected inventory levels $e.i.l$ $c = 1, 2, 3, 4$ and 5.

<table>
<thead>
<tr>
<th>$c$</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.6098</td>
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</tr>
<tr>
<td>2</td>
<td>1.5309</td>
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<td>1.4627</td>
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<td>5</td>
<td>1.4453</td>
<td>2.2097</td>
</tr>
</tbody>
</table>

Table - 5.4.5 and Figure - 5.4.5

When the rate $c$ increases, the expected inventory levels of $A$ and $B$ decrease slightly.

Case 5.4.6. (Equal probability of demand of inventories $A$ and $B$)

For the fixed values of $\lambda_1 = 1$, $\lambda_2 = 2$, $\mu_1 = 1$, $\mu_2 = 2$, $p = 0.5$ and $q = 0.5$, we compute the expected inventory levels $e.i.l$ $c = 1, 2, 3, 4$ and 5.

<table>
<thead>
<tr>
<th>$c$</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
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<td>2.1106</td>
</tr>
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<td>2</td>
<td>1.7488</td>
<td>2.0760</td>
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<td>3</td>
<td>1.7127</td>
<td>2.0571</td>
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<tr>
<td>4</td>
<td>1.6902</td>
<td>2.0454</td>
</tr>
<tr>
<td>5</td>
<td>1.6750</td>
<td>2.0376</td>
</tr>
</tbody>
</table>

Table - 5.4.6 and Figure - 5.4.6

When the rate $c$ increases and the probabilities of demands of $A$ and $B$ are same, the expected inventory levels of $A$ and $B$ decrease.

In all the cases the $e.i.l$ of $A$ are less than that of $B$ except the Case 5.4.3.
5.4.2. Numerical Example for Model - 2.

Let the maximum capacity of the inventory be \(3(S = 3)\) and the reorder level is \(1(s = 1)\). The infinitesimal generator of the finite state space continuous time Markov chain is as follows.

\[
\begin{bmatrix}
3 & 2 & 1 & 0 \\
3 & T & A & \\
2 & T & A & \\
1 & \mu_1 I & T - \mu_1 I & A \\
0 & \mu_1 I & T_0 - \mu_1 I & \\
\end{bmatrix}
\]

where \(T, A\) and \(T_0\) are as given below.

\[
T = \begin{bmatrix}
-\lambda_1 - c & c & \\
-\lambda_2 & 0 & c \\
-\lambda_2 & -\lambda_1 - c - \mu_2 & 0 \\
\mu_2 & -\lambda_1 - c - \mu_2 & c \\
\mu_2 & \lambda_2 & -\lambda_1 p - c - \mu_2 & c \\
\mu_2 & \mu_2 & & -\lambda_2 - \mu_2 \\
\end{bmatrix}
\]

\[
T_0 = \begin{bmatrix}
-c & c & \\
0 & 0 & \\
-c & c & \\
0 & 0 & \\
\mu_2 & -c - \mu_2 & c \\
\mu_2 & -\mu_2 & 0 \\
\mu_2 & -\mu_2 - c & c \\
\mu_2 & & -\mu_2 \\
\end{bmatrix}
\]
For fixed values of $\lambda_1 = 1$, $\lambda_2 = 2$, $\mu_1 = 1$, $\mu_2 = 2$, $c = 1$ $p = 0.7$ and $q = 0.3$, the i.l.p (inventory level probabilities) are given as follows.

<table>
<thead>
<tr>
<th></th>
<th>$\pi_0$</th>
<th>$\pi_1$</th>
<th>$\pi_2$</th>
<th>$\pi_3$</th>
<th>$\pi_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$ 0.0439</td>
<td>0.0898</td>
<td>0.0582</td>
<td>0.0319</td>
<td>0.0677</td>
<td>0.1182</td>
</tr>
</tbody>
</table>

The sum of above probabilities is found to be 1.0000.

We find the expected inventory levels (e.i.l), for different cases.

**Case 5.4.7. (Varying first demand rate $\lambda_1$)**

For the fixed values of $\lambda_2 = 2$, $\mu_1 = 1$, $\mu_2 = 2$, $c = 1$ $p = 0.7$ and $q = 0.3$, we compute the expected inventory levels e.i.l $\lambda_1 = 1, 2, 3, 4$ and $5$.

<table>
<thead>
<tr>
<th>$\lambda_1$</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.3706</td>
<td>1.8103</td>
</tr>
<tr>
<td>2</td>
<td>1.1000</td>
<td>1.7001</td>
</tr>
<tr>
<td>3</td>
<td>0.9373</td>
<td>1.6447</td>
</tr>
<tr>
<td>4</td>
<td>0.8321</td>
<td>1.6134</td>
</tr>
<tr>
<td>5</td>
<td>0.7595</td>
<td>1.5939</td>
</tr>
</tbody>
</table>

Table - 5.4.7 and Figure - 5.4.7
When first demand rate $\lambda_1$ increases, the expected inventory levels of $A$ and $B$ decreases.

**Case 5.4.8. (Varying second demand rate $\lambda_2$)**

For the fixed values of $\lambda_1 = 1$, $\mu_1 = 1$, $\mu_2 = 2$, $c = 1$, $p = 0.7$ and $q = 0.3$, we compute the expected inventory levels $e.i.l \lambda_2 = 1, 2, 3, 4$ and $5$.

<table>
<thead>
<tr>
<th>$\lambda_2$</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.6000</td>
<td>1.9393</td>
</tr>
<tr>
<td>2</td>
<td>1.3706</td>
<td>1.8103</td>
</tr>
<tr>
<td>3</td>
<td>1.2672</td>
<td>1.7562</td>
</tr>
<tr>
<td>4</td>
<td>1.2086</td>
<td>1.7268</td>
</tr>
<tr>
<td>5</td>
<td>1.1711</td>
<td>1.7086</td>
</tr>
</tbody>
</table>

Table - 5.4.8 and Figure - 5.4.8

When second demand rate $\lambda_2$ increases, the expected inventory levels of $A$ and $B$ decrease.

**Case 5.4.9. (Varying lead time rates of inventory $A$)**

For the fixed values of $\lambda_1 = 1$, $\lambda_2 = 2$, $\mu_2 = 2$, $c = 1$, $p = 0.7$ and $q = 0.3$, we compute the expected inventory levels $e.i.l \mu_1 = 1, 2, 3, 4$ and $5$.

<table>
<thead>
<tr>
<th>$\mu_1$</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.3706</td>
<td>1.8103</td>
</tr>
<tr>
<td>2</td>
<td>1.8491</td>
<td>1.6986</td>
</tr>
<tr>
<td>3</td>
<td>2.0529</td>
<td>1.6679</td>
</tr>
<tr>
<td>4</td>
<td>2.1619</td>
<td>1.6560</td>
</tr>
<tr>
<td>5</td>
<td>2.2289</td>
<td>1.6503</td>
</tr>
</tbody>
</table>

Table - 5.4.9 and Figure - 5.4.9

When the lead time rate of inventory $A$ increases, the expected inventory levels of $A$ and $B$ decrease slightly.
Case 5.4.10. (Varying lead time rate of inventory $B$)

For the fixed values of $\lambda_1 = 1$, $\lambda_2 = 2$, $\mu_1 = 1$, $c = 1$, $p = 0.7$ and $q = 0.3$, we compute the expected inventory levels $e.i.l \mu_2 = 1, 2, 3, 4$ and 5.

<table>
<thead>
<tr>
<th>$\mu_2$</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.3706</td>
<td>1.8103</td>
</tr>
<tr>
<td>2</td>
<td>1.3706</td>
<td>2.1402</td>
</tr>
<tr>
<td>3</td>
<td>1.3706</td>
<td>2.2600</td>
</tr>
<tr>
<td>4</td>
<td>1.3706</td>
<td>2.3205</td>
</tr>
<tr>
<td>5</td>
<td>1.3706</td>
<td>2.3568</td>
</tr>
</tbody>
</table>

Table - 5.4.10 and Figure - 5.4.10

When the lead time rates of inventory $B$ increase, the expected inventory levels of $A$ remain same and the $e.i.l$ of $B$ increase.

Case 5.4.11. (Varying $c$)

For the fixed values of $\lambda_1 = 1$, $\lambda_2 = 2$, $\mu_1 = 1$, $\mu_2 = 2$, $p = 0.7$ and $q = 0.3$, we compute the expected inventory levels $e.i.l c = 1, 2, 3, 4$ and 5.

<table>
<thead>
<tr>
<th>$c$</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.3706</td>
<td>2.1402</td>
</tr>
<tr>
<td>2</td>
<td>1.2800</td>
<td>2.1143</td>
</tr>
<tr>
<td>3</td>
<td>1.2334</td>
<td>2.1018</td>
</tr>
<tr>
<td>4</td>
<td>1.2055</td>
<td>2.0945</td>
</tr>
<tr>
<td>5</td>
<td>1.1817</td>
<td>2.0899</td>
</tr>
</tbody>
</table>

Table - 5.4.11 and Figure - 5.4.11

When the rate $c$ increases, the expected inventory levels of $A$ and $B$ decrease slightly.
Case 5.4.12. (Equal probability of demand of inventories $A$ and $B$)

For the fixed values of $\lambda_1 = 1$, $\lambda_2 = 2$, $\mu_1 = 1$, $\mu_2 = 2$, $p = 0.5$ and $q = 0.5$, we compute the expected inventory levels $e.i.l \; c = 1, 2, 3, 4$ and 5.

<table>
<thead>
<tr>
<th>$e.i.l$</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>$A$</td>
<td>$B$</td>
</tr>
<tr>
<td>1</td>
<td>1.3706</td>
<td>2.2432</td>
</tr>
<tr>
<td>2</td>
<td>1.2800</td>
<td>2.2245</td>
</tr>
<tr>
<td>3</td>
<td>1.2334</td>
<td>2.2154</td>
</tr>
<tr>
<td>4</td>
<td>1.2055</td>
<td>2.2101</td>
</tr>
<tr>
<td>5</td>
<td>1.1871</td>
<td>2.2067</td>
</tr>
</tbody>
</table>

Table - 5.4.12 and Figure - 5.4.12

When the rate $c$ increases and the probabilities of demands of $A$ and $B$ are same, the expected inventory levels of $A$ and $B$ decrease.

In all cases the $e.i.l$ of $A$ are less than that of $B$ except the Case 5.4.9.