CHAPTER - 2

GENERAL ANALYSIS OF \((s, S)\) INVENTORY SYSTEM
WITH DEFECTIVE SUPPLIES

2.1. Introduction.

\((s, S)\) inventory systems have been studied by several researchers. Single commodity inventory problem of \((s, S)\) type was first analyzed by Arrow, Karlin and Scraf [1]. Its applications were also discussed by various researchers. Daniel and Ramanarayanan [7] discussed \((s, S)\) inventory with random lead time and unit demand. Ramanarayanan and Jacob [40] studied this system with bulk demand. Murthy and Ramanarayanan [30, 29, 28] considered two ordering level inventories with different lead times and rest time to the server. They also discussed inventory system exposed to calamity with \(SCBZ\) arrival property and two \((s, S)\) inventory systems with perishable units. Kun-Shan Wu and Liang-Yoh Ouyang [20] have analyzed \((Q, r, L)\) Inventory model with defective items.

In this chapter, we discuss the \((s, S)\) inventory system with defective supplies. The supply is accepted only when at least \(s + 1\) units are good. Otherwise the whole lot is rejected and fresh order is made for the supply of \(S - s\) units. This model can be fitted into real life situation in almost all types of inventories. In mass manufacturing, testing of units are done only randomly. It is often noticed that a supply may contain defective items. In this chapter we consider such a situation and obtain inventory

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level probabilities and expected inventory levels for different values of arrival rates and lead times.

2.2. Model - 1: General Case.

Let $S$ be the maximum capacity of a warehouse. At the beginning let the inventory be full and the stock level from $S$ goes on decreasing due to incoming demands. The demands are assumed to occur for one unit at a time and the time interval between the arrivals of two consecutive demands forms a family of independent identically distributed (i.i.d) random variables with cumulative density function $F(.)$. Assume that an order is placed for $S-s$ units when the stock level drops to $s$. Let the lead time distribution for the supply be $G(x)$. We assume that an arrival order lot may contain some defective items and the number of defective items in the lot is a random variable. If the number of perfect (good) units supplied is less than $s+1$, the whole lot is rejected and fresh order is made for the supply of $S-s$ units. The supply is accepted only when at least $s+1$ units are good. Let $p_i$ be the probability that $i$ units are defective out of $S-s$ units with $p_i > 0$ and

$$\sum_{i=0}^{S-s} p_i = 1.$$

We note that the rejection probability of a lot $q = \sum_{j=S-2s}^{S-s} p_j$. Let the probability density function of lead time with $k$ perfect items are supplied be $g(x,k)$. We find

$$g(x,k) = p_{S-s-k} \int_0^x \sum_{n=0}^{\infty} q^n g^n(u) g(x-u) du$$

(2.1)
for $s + 1 \leq k \leq S - s$ where $\ast$ denotes convolution. Let its cumulative distribution function be denoted by $G(x, k)$ for $s + 1 \leq k \leq S - s$.

### 2.2.1. Inventory Level Probabilities.

We note that the probability density function of time between two consecutive orders for $S - s$ units when the inventory level falls to $s$ is given by

\[
s_h(t) = \sum_{m=s+1}^{S-s} \left( \sum_{n=0}^{s-1} \int_0^t f^m(u)[G(x, m) - G(u, m)]f(x-u)du \int_{x-u}^{\infty} f_{n-t-1}(t-x)dx \right)
\]

The above equation may be explained as follows. The first term is the part of the probability density function when $m$, for $s + 1 \leq m \leq S - s$ good units are supplied to the inventory considering the inventory is not in the dry period. The realization time points are as follows.

\begin{center}
\begin{tikzpicture}
\draw[->] (0,0) -- (5,0) node[right] {$t$};
\draw[very thick] (0,0) -- (0.5,0);\draw[very thick] (0.5,0) -- (1,0);\draw[very thick] (1,0) -- (2,0);\draw[very thick] (2,0) -- (2.5,0);\draw[very thick] (2.5,0) -- (3,0);\draw[very thick] (3,0) -- (3.5,0);\draw[very thick] (3.5,0) -- (4,0);\draw[very thick] (4,0) -- (4.5,0);
\draw (0.5,0) node[below] {$0$};\draw (1,0) node[below] {$u$};\draw (2,0) node[below] {$y$};\draw (3,0) node[below] {$x$};\draw (3.5,0) node[below] {$t$};
\end{tikzpicture}
\end{center}

\text{figure - i}

At time 0 an order is made for $S - s$ units. At time $y$ the first successful order is realized with $m$ ($s + 1 \leq m \leq S - s$) good units following immediately after the $n$-th demand which has occurred at time $u$. The next demand after the realization of the order occurs at $x$. At time $t$ the $(m - n - 1)$ th demand occurs and an order is made for $S - s$ units. The second term of equation (2.2) is the part of the pdf considering the inventory is in the dry period.
After an order is made at time 0 for \( S - s \) units the \((s + k)\)th demand for \( 0 \leq k \leq \infty \) occurs at time \( u \) during the dry period and the first successful order is realized at time \( y \) before the next demand which occurs at time \( x \). At time \( t \), the \((m - s - 1)\)th demand occurs and an order is made for \( S - s \) units. Let

\[
h_1(x) = \sum_{k=0}^{\infty} h^*_s(x)
\]

be the renewal density function of such order due to inventory level falls to \( s \) at time 0 and at time \( x \).

Now we calculate the probabilities of various inventory levels at time \( t \).

Let \( \Pi_i(t) = P(\text{inventory level is } i \text{ at time } t \mid \text{at time 0 the inventory level is } S \text{ and demand process starts}) \) for \( 0 \leq i \leq S \). We note that

\[
\Pi_S(t) = \overline{F}(t) + \int_0^t \int_0^x f^{S-s}(u)h_1(x - u)du\overline{F}(t - x)G(t - x, S - s)dx
\]

(2.5)

where \( \overline{F}(t) = 1 - F(t) \).

The first term is the probability that no demand has occurred during \((0, t)\). The second term is the probability that the first order is made at time \( u \), the last order is made at time \( x \), after which during \((x, t)\) no demand occurs and the order is realized with \( S - s \) good units.
The inventory level probabilities for levels in \([S - s + 1, S - 1]\) are as follows:

\[
\Pi_{S-i}(t) = \int_0^t f^{*i}(u)\bar{F}(t-u)du
\]

\[+
\int_0^t \left[ \int_0^x f^{*S-s}(u)h_1(x-u)du \sum_{k=0}^{i-1} \int_0^{t-x} f_{s,S-i+k}(v)[F_k(t-x-v) - F_{k+1}(t-x-v)]dv \right] dx
\]

\[+
\int_0^t \int_0^x f^{*S-s}(u)h_1(x-u)du
\]

\[\left[ \sum_{j=0}^i \int_0^{t-x} f^{*j}(v)\bar{F}(t-x-v)[G(t-x,S-s-i+j) - G(v,S-s-i+j)]dv \right] dx \quad (2.6)
\]

for \(1 \leq i \leq s - 1\).

The first term of equation (2.6) is the probability that the level falls to \(S - i\) and remains there till \(t\). The second term is the probability that several orders are made and realized, the last order is made at \(x\), during the interval \((0, t-x)\) at the time point \(v\) the inventory level falls to \(S-i+k\) due to the occurrence of the first demand after the acceptance of order and exactly \(k\) demands occur during \((0, t-x-v)\). The third term is the probability that several orders are made and realized, the last order is made at \(x\) during the interval \((0, t-x)\) at \(v\) the \(j\)th demand occurs, no demand occurs after \(v\) and the order is realized with \(S-s-i+j\) good units.

We use the transition time probability density function of jump from level \(s\) at time 0 at the time of an order is made to level \(m\) immediately due to the first demand which occurs after the first accepted order subsequent to rejecting defective orders for \(S-s+1 \leq m \leq S-1\). It
may be seen as follows.

\[ f_{s,m}(t) = \sum_{j=0}^{s-1} \int_0^t f^s(u)[G(t, m + 1 + j - s) - G(u, m + 1 + j - s)]f(t - u)du \]

(2.7)

for \( S - s + 1 \leq m \leq S - 1 \).

The inventory level probabilities for levels in \([s + 1, S - s]\) are as follows:

\[ \Pi_{s-i}(t) = \int_0^t f^s(u)F(t - u)du + \int_0^t \int_0^x f^{S-s}(u)h_1(x - u)du \]

\[ + \sum_{k=0}^{i-1} \int_0^{t-x} \int_0^{t-x} f_{s,S-i+k}(v)[F_k(t - x - v) - F_{k+1}(t - x - v)]dvdx \]

\[ + \int_0^t \int_0^x f^{S-s}(u)h_1(x - u)du \]

\[ + \int_0^{t-x} \sum_{n=0}^{\infty} f^{s+n}(v)F(t - x - v)[G(t - x, S - i) - G(v, S - i)]dvdx \]

(2.8)

for \( s \leq i \leq S - s - 1 \).

The first term of equation (2.8) is the probability that the inventory level falls to \( S - i \) and remains there. The second term is the probability that several orders are realized and an order is made at \( x \), after the supply is accepted the next demand makes the inventory level to fall to \( S - i + k \) and exactly \( k \) demands occur in the remaining period. The transition time probability density function is given below in equation (2.9). The last term is the probability that the several orders are realized, an order is made subsequently the inventory becomes dry, the level jumps to \( S - i \)
due to supply and remains there. Considering inventory dry period the
transition time probability density function is given by

$$f_{s,m}(t) = \int_{0}^{t} \sum_{n=0}^{\infty} f_{s,s+n}(v) [G(t, m + 1) - G(v, m + 1)] f(t - v) dv$$  \hspace{1cm} (2.9)

for $s + 1 \leq m \leq S - s$.

Similarly we can obtain

$$\Pi_i(t) = \int_{0}^{t} \int_{0}^{x} f_{s}^{S-s}(u) h_1(x - u) du \frac{d}{dx} \Pi(t) \left( \sum_{k=s+1}^{S-s} \int_{t-x}^{\infty} g(v, k) dv \right) dx$$

$$\times [F_{s-i}(t - x) - F_{s-i+1}(t - x)] dx$$  \hspace{1cm} (2.10)

for $1 \leq i \leq s - 1$ and

$$\Pi_0(t) = \int_{0}^{t} \int_{0}^{x} f_{s}^{S-s}(u) h_1(x - u) du \left( \sum_{k=s+1}^{S-s} \int_{t-x}^{\infty} g(v, k) dv \right) F_s(t - x) dx.$$  \hspace{1cm} (2.11)

All the inventory level probabilities are listed above in equations
(2.5) to (2.12). In the next section we take up a special case.

2.3. Model - 2: Markovian Model.

In this section we consider a Markovian model in which the distri-
bution functions are all exponentials. Assuming the maximum capacity
of the inventory as $S$ and ordering level on $s$, let us study the general
model when demands occur for one unit at a time with exponential in-
ter occurrence time distribution with parameter \( \lambda \) and exponential lead
time distribution with parameter \( \mu \). With \( p_i, i = 0, 1, 2 \ldots S - s \) and \( q \) as
defined in the previous model, we list the various states of the continu-
ous time Markov chain underlying the model. Let us consider the state
space of the system as

\[
\Omega = \{ i : i \text{ units are in inventory and } i = 0, 1, 2, \ldots, S \}.
\]

Then the infinitesimal generator of the finite state space continuous time
Markov chain is given as follows.
The infinitesimal generator $Q$ is a square matrix of order $S + 1$ in which all the unmarked entries are 0 and the steady state probability vector $\Pi$ of the system satisfies the following

$$\Pi Q = 0 \text{ and } \Pi e = 1$$

(2.13)

where $\Pi = (\Pi, \Pi_{S-1}, \ldots, \Pi_3, \Pi_2, \Pi_1, \Pi_0)$, $e = (1, 1, \ldots, 1)^t$ and $\Pi_i$ is the steady state probability that inventory level is $i$ for $0 \leq i \leq S$. We note that

$$\Pi_i = \left(\frac{\lambda}{\lambda + \mu q}\right)^{s-i} \Pi_s \text{ for } 1 \leq i \leq s - 1$$

(2.14)

$$\Pi_0 = \left(\frac{\lambda}{\mu q}\right) \left(\frac{\lambda}{\lambda + \mu q}\right)^{s-1} \Pi_s$$

(2.15)

$$\Pi_s = \left(\frac{\mu \pi_0}{\lambda}\right) \Pi_s$$

(2.16)

$$\Pi_{S-i} = \frac{\mu}{\lambda} \left[ \sum_{k=1}^{i} \sum_{j=0}^{k} \pi_{k-j} \left(\frac{\lambda}{\lambda + \mu q}\right)^j \Pi_s \right] + \left(\frac{\mu \pi_0}{\lambda}\right) \Pi_s, \text{ for } 1 \leq i \leq s,$$

(2.17)

$$\Pi_{S-i} = \frac{\mu}{\lambda} \left[ \sum_{k=s+1}^{i} \sum_{j=0}^{s} \pi_{k-j} \left(\frac{\lambda}{\lambda + \mu q}\right)^j \Pi_s \right] + \frac{\mu}{\lambda} \left[ \sum_{k=1}^{s} \sum_{j=0}^{k} \pi_{s-j} \left(\frac{\lambda}{\lambda + \mu q}\right)^j \Pi_s \right] + \left(\frac{\mu \pi_0}{\lambda}\right) \Pi_s$$

(2.18)
for $s + 1 \leq i \leq S - 2s - 1$.

$$\Pi_{2s-i} = \frac{\mu}{\lambda} \left[ \sum_{k=0}^{i} \sum_{j=0}^{s-1-k} p_{S-2s-1-j} \left( \frac{\lambda}{\lambda + \mu q} \right)^{k+j+1} \Pi_s \right]$$

$$+ \frac{\mu}{\lambda} \left[ \sum_{k=s+1}^{S-2s-1} \sum_{j=0}^{s} p_{k-j} \left( \frac{\lambda}{\lambda + \mu q} \right)^{j} \Pi_s \right]$$

$$+ \frac{\mu}{\lambda} \left[ \sum_{k=1}^{s} \sum_{j=0}^{k} p_{k-j} \left( \frac{\lambda}{\lambda + \mu q} \right)^{j} \Pi_s \right] + \left( \frac{\mu p_0}{\lambda} \right) \Pi_s \quad (2.19)$$

for $0 \leq i \leq s - 1$.

Now we find $\Pi_s$. Let

$$a = \frac{\lambda}{\lambda + \mu q}, \quad b = \frac{\lambda}{\mu q} \quad \text{and} \quad c = \frac{\mu p_0}{\lambda}. \quad (2.20)$$

Use the relation $\Pi e = 1$ and the equations (2.13) to (2.19) to find

$$\Pi_s = \left( \sum_{i=1}^{s-1} a^{s-i} \right) + ba^{s-1} + c + 1 + \frac{\mu}{\lambda} \left\{ \sum_{i=1}^{s} \left( \sum_{k=1}^{k} \sum_{j=0}^{k} p_{k-j} a^{j} + p_0 \right) \right\}$$

$$+ \sum_{i=s+1}^{S-2s-1} \left( \sum_{k=i}^{i} \sum_{j=0}^{s} p_{k-j} a^{j} + \sum_{k=1}^{k} \sum_{j=0}^{k} p_{k-j} a^{j} + p_0 \right)$$

$$+ \sum_{i=0}^{s-1} \left( \sum_{k=0}^{i} \sum_{j=0}^{s-1-k} p_{S-2s-1-j} a^{k+j+1} + \sum_{k=s+1}^{S-2s-1} \sum_{j=0}^{s} p_{k-j} a^{j} + \sum_{k=1}^{k} \sum_{j=0}^{k} p_{k-j} a^{j} + p_0 \right) \right\}^{-1} \quad (2.21)$$

2.4. Numerical Example.

Let the maximum capacity of the warehouse be 8 ($S = 8$) and the reorder level is 2 ($s = 2$). The infinitesimal generator of the finite state space continuous time Markov Chain is as follows.
\[
Q' = \begin{bmatrix}
-\lambda & \lambda & & & \\
-\lambda & -\lambda & \lambda & & \\
-\lambda & -\lambda & -\lambda & \lambda & \\
& -\lambda & -\lambda & -\lambda & \lambda \\
\mu p_0 & \mu p_1 & \mu p_2 & \mu p_3 & -\lambda - \mu \bar{q} & \lambda \\
\mu p_0 & \mu p_1 & \mu p_2 & \mu p_3 & -\lambda - \mu \bar{q} & \lambda \\
& \mu p_0 & \mu p_1 & \mu p_2 & \mu p_3 & -\mu \bar{q}
\end{bmatrix}
\]

\[
\Pi_1 = \left( \frac{\lambda}{\lambda + \mu \bar{q}} \right) \Pi_2 \\
\Pi_0 = \left( \frac{\lambda}{\mu \bar{q}} \right) \left( \frac{\lambda}{\lambda + \mu \bar{q}} \right) \Pi_2 \\
\Pi_8 = \left( \frac{\mu p_0}{\lambda} \right) \Pi_2 \\
\Pi_7 = \frac{\mu}{\lambda} \left[ p_1 + p_0 \left( \frac{\lambda}{\lambda + \mu \bar{q}} \right) + p_0 \right] \Pi_2 \\
\Pi_6 = \frac{\mu}{\lambda} \left[ p_2 + p_1 \left( 1 + \frac{\lambda}{\lambda + \mu \bar{q}} \right) + p_0 \left( \frac{\lambda}{\lambda + \mu \bar{q}} \right) \left( 1 + \frac{\lambda}{\mu \bar{q}} \right) + p_0 \right] \Pi_2 \\
\Pi_5 = \frac{\mu}{\lambda} \left[ p_3 + p_2 \left( 1 + \frac{\lambda}{\lambda + \mu \bar{q}} \right) + p_1 \left( 1 + \frac{\lambda}{\lambda + \mu \bar{q}} + \frac{\lambda}{\mu \bar{q}} \right) \left( \frac{\lambda}{\lambda + \mu \bar{q}} \right) \right] \Pi_2 \\
\quad + \frac{\mu}{\lambda} \left[ p_0 \left( 1 + \frac{\lambda}{\lambda + \mu \bar{q}} + \frac{\lambda}{\mu \bar{q}} \right) \frac{\lambda}{\lambda + \mu \bar{q}} \right] \Pi_2 \\
\Pi_4 = \frac{\mu}{\lambda} \left[ p_3 \left( 1 + \frac{\lambda}{\lambda + \mu \bar{q}} \right) + \left( 1 + \frac{\lambda}{\lambda + \mu \bar{q}} + \frac{\lambda}{\mu \bar{q}} \right) \left( \frac{\lambda}{\lambda + \mu \bar{q}} \right) \left( p_2 + p_1 + p_0 \right) \right] \Pi_2
\]
\[ \Pi_3 = \frac{\mu}{\lambda} \left[ \left( 1 + \frac{\lambda}{\lambda + \mu q} + \frac{\lambda}{\mu q} + \frac{\lambda}{\lambda + \mu q} \right) (p_3 + p_2 + p_1 + p_0) \right] \Pi_2. \]  

(2.29)

Use the relation \( \Pi_\epsilon = 1 \) and the equations (2.22) to (2.29) to find

\[ \Pi_2 = [a(1 + b) + c + 1 + \frac{\mu}{\lambda} \{q + (1 + a)(2q + 2(p_0 + p_1) + p_2 + p_0(1 + b)) \}
+ ab(2(p_0 + p_1) + p_2 + q)]^{-1}. \]  

(2.30)

For fixed values of \( \lambda = 1, \mu = 2, p_0 = 0.1, p_1 = 0.2, p_2 = 0.2 \) and \( p_3 = 0.1 \), we compute the components of \( \Pi \). Here

\[ q = p_0 + p_1 + p_2 + p_3 = 0.6. \]

The steady state probability vector

\[ \Pi = (\Pi_8, \Pi_7, \Pi_6, \Pi_5, \Pi_4, \Pi_3, \Pi_2, \Pi_1, \Pi_0) \]

is given by \( \Pi_8 = 0.0195, \Pi_7 = 0.0672, \Pi_6 = 0.1312, \Pi_5 = 0.1831, \Pi_4 = 0.2067, \Pi_3 = 0.2140, \Pi_2 = 0.0973, \Pi_1 = 0.0442, \Pi_0 = 0.0369 \). The sum of steady state probabilities is found to be 1.000 (corrected to 3 decimals).

**Case 2.4.1.** For the fixed value of lead time rate \( \mu = 2 \), the steady probability vectors corresponding to the values of demand rate \( \lambda = 1, 2, 3, 4 \) and 5 are given in Table-2.4.1 and Table-2.4.2.
Table - 2.4.1 (Supplies with defective units \((A)\) \((p_0 = 0.1, p_1 = 0.2, p_2 = 0.2, p_3 = 0.1)\)

<table>
<thead>
<tr>
<th>(\lambda)</th>
<th>(\Pi_6)</th>
<th>(\Pi_7)</th>
<th>(\Pi_8)</th>
<th>(\Pi_4)</th>
<th>(\Pi_5)</th>
<th>(\Pi_3)</th>
<th>(\Pi_2)</th>
<th>(\Pi_1)</th>
<th>(\Pi_0)</th>
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<td>1</td>
<td>0.0195</td>
<td>0.0672</td>
<td>0.1312</td>
<td>0.2067</td>
<td>0.2140</td>
<td>0.0973</td>
<td>0.0442</td>
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<tr>
<td>2</td>
<td>0.0121</td>
<td>0.0440</td>
<td>0.0961</td>
<td>0.1815</td>
<td>0.1941</td>
<td>0.1213</td>
<td>0.0758</td>
<td>0.1264</td>
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</tr>
<tr>
<td>3</td>
<td>0.0082</td>
<td>0.0306</td>
<td>0.0736</td>
<td>0.1584</td>
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<tr>
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<tr>
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Table - 2.4.2 (Supplies without defective units\((B)\) \((p_0 = 1)\)

<table>
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<tr>
<th>(\lambda)</th>
<th>(\Pi_6)</th>
<th>(\Pi_7)</th>
<th>(\Pi_8)</th>
<th>(\Pi_4)</th>
<th>(\Pi_5)</th>
<th>(\Pi_3)</th>
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<td>0.0550</td>
<td>0.0183</td>
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<td>2</td>
<td>0.0800</td>
<td>0.1200</td>
<td>0.1600</td>
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<td>0.0800</td>
<td>0.0400</td>
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</tr>
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<td>0.1753</td>
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For a fixed value of lead time rate \(\mu = 2\), we compute the expected inventory levels for the demand rate \(\lambda = 1, 2, 3, 4\) and 5 are given in Table-2.4.3 and the corresponding graph is given in figure-2.4.2

### Expected Inventory Levels

<table>
<thead>
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<th>(A)</th>
<th>(B)</th>
</tr>
</thead>
<tbody>
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</tr>
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<td>2</td>
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</tr>
<tr>
<td>5</td>
<td>2.1190</td>
<td>3.5259</td>
</tr>
</tbody>
</table>

Table - 2.4.3 and Figure -2.4.2
From Table 2.4.1 and 2.4.2, we note that above reorder level $s = 2$, the inventory level probabilities (i.l.p.) decrease while the inter occurrence time increases, in both the cases of supplies with defective items and without defective items. But below the reorder level $s = 2$, the i.l.p. increase.

It is clear from Table-2.4.3 and figure-2.4.2 that the expected inventory levels(e.i.l.) decreases while the inter occurrence time increases. They also show that the e.i.l. in the case of supplies without defective units are more than the case of supplies with defective units.

**Case 2.4.2.** For the fixed value of demand rate $\lambda = 1$, the steady state probability vectors corresponding to the values of lead time rate $\mu = 1, 2, 3, 4$ and 5 are given in Table-2.4.4 and Table-2.4.5.

**Table - 2.4.4 (Supplies with defective units (A) ($p_0 = 0.1, p_1 = 0.2, p_2 = 0.2, p_3 = 0.1$))**

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$\Pi_5$</th>
<th>$\Pi_7$</th>
<th>$\Pi_6$</th>
<th>$\Pi_5$</th>
<th>$\Pi_4$</th>
<th>$\Pi_3$</th>
<th>$\Pi_2$</th>
<th>$\Pi_1$</th>
<th>$\Pi_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0121</td>
<td>0.0440</td>
<td>0.0961</td>
<td>0.1486</td>
<td>0.1815</td>
<td>0.1941</td>
<td>0.1213</td>
<td>0.0758</td>
<td>0.1264</td>
</tr>
<tr>
<td>2</td>
<td>0.0195</td>
<td>0.0672</td>
<td>0.1312</td>
<td>0.1831</td>
<td>0.2067</td>
<td>0.2140</td>
<td>0.0973</td>
<td>0.0442</td>
<td>0.0369</td>
</tr>
<tr>
<td>3</td>
<td>0.0234</td>
<td>0.0787</td>
<td>0.1470</td>
<td>0.1965</td>
<td>0.2141</td>
<td>0.2188</td>
<td>0.0781</td>
<td>0.0279</td>
<td>0.0155</td>
</tr>
<tr>
<td>4</td>
<td>0.0259</td>
<td>0.0854</td>
<td>0.1557</td>
<td>0.2033</td>
<td>0.2173</td>
<td>0.2205</td>
<td>0.0648</td>
<td>0.0191</td>
<td>0.0079</td>
</tr>
<tr>
<td>5</td>
<td>0.0276</td>
<td>0.0899</td>
<td>0.1613</td>
<td>0.2074</td>
<td>0.2189</td>
<td>0.2212</td>
<td>0.0553</td>
<td>0.0138</td>
<td>0.0046</td>
</tr>
</tbody>
</table>

**Table - 2.4.5 (Supplies without defective units (B) ($p_0 = 1$))**

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$\Pi_5$</th>
<th>$\Pi_7$</th>
<th>$\Pi_6$</th>
<th>$\Pi_5$</th>
<th>$\Pi_4$</th>
<th>$\Pi_3$</th>
<th>$\Pi_2$</th>
<th>$\Pi_1$</th>
<th>$\Pi_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0800</td>
<td>0.1200</td>
<td>0.1600</td>
<td>0.1600</td>
<td>0.1600</td>
<td>0.1600</td>
<td>0.0800</td>
<td>0.0400</td>
<td>0.0400</td>
</tr>
<tr>
<td>2</td>
<td>0.1101</td>
<td>0.1468</td>
<td>0.1651</td>
<td>0.1651</td>
<td>0.1651</td>
<td>0.1651</td>
<td>0.0550</td>
<td>0.0183</td>
<td>0.0092</td>
</tr>
<tr>
<td>3</td>
<td>0.1246</td>
<td>0.1557</td>
<td>0.1661</td>
<td>0.1661</td>
<td>0.1661</td>
<td>0.1661</td>
<td>0.0415</td>
<td>0.0104</td>
<td>0.0035</td>
</tr>
<tr>
<td>4</td>
<td>0.1331</td>
<td>0.1597</td>
<td>0.1664</td>
<td>0.1664</td>
<td>0.1664</td>
<td>0.1664</td>
<td>0.0333</td>
<td>0.0067</td>
<td>0.0017</td>
</tr>
<tr>
<td>5</td>
<td>0.1388</td>
<td>0.1619</td>
<td>0.1665</td>
<td>0.1665</td>
<td>0.1665</td>
<td>0.1665</td>
<td>0.0278</td>
<td>0.0046</td>
<td>0.0009</td>
</tr>
</tbody>
</table>
For a fixed value of demand rate $\lambda = 1$, we compute the expected inventory levels for the lead time rates $\mu = 1, 2, 3, 4$ and $5$ are given in Table-2.4.6 and the corresponding graph is given in figure-2.4.3.

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.3514</td>
<td>4.5600</td>
</tr>
<tr>
<td>2</td>
<td>4.0363</td>
<td>5.0092</td>
</tr>
<tr>
<td>3</td>
<td>4.2995</td>
<td>5.1696</td>
</tr>
<tr>
<td>4</td>
<td>4.4357</td>
<td>5.2512</td>
</tr>
<tr>
<td>5</td>
<td>4.5184</td>
<td>5.3006</td>
</tr>
</tbody>
</table>

Table - 2.4.6 and Figure -2.4.3

From Table-2.4.4 and Table-2.4.5, we note that the above reorder level $s = 2$, the i.l.p. increase while the lead time rate increases, in both the cases of supplies with defective items and without defective items. But other levels the i.l.p. decreases.

It is clear from Table-2.4.6 and Figure-2.4.3 that the e.i.l. increases while the lead time rate increases. They also show that the e.i.l. in the case of supplies without defective units are more than the case of supplies with defective units.