Conclusion: Present Aspects and Future Prospects

In this thesis, we have focused on various aspects of current research in lepton masses, mixings and flavor violation in supersymmetric theories. First we have studied neutrino mass generation and mixings in the context of a grand unified theory (GUT). We have considered an SO(10) model with the Higgs fields (10+\overline{126}), which has been generously considered as the most eminent candidate for the minimal SO (10) GUT. Next, we predicted the possibility of detection of new supersymmetric particles at second run of LHC where we explore charged
lepton flavor violation decays (cLFV) like $\mu \to e\gamma$ in various universal and non universal mass models like mSUGRA, NUHM, NUSM and NUGM, in a supersymmetric scenario embedded in the SO (10) GUT. The charged lepton flavour violating processes have not been so far observed, despite many experimental searches. The experimental searches at MEG has set stringent upper bounds on the cLFV processes. There are several ongoing and future experiments for improving the sensitivity of different cLFV processes. Improvements for $\mu \to e\gamma$ decay of MEG experiment has presently reached $\text{BR}(\mu \to e\gamma) \leq 4.2 \times 10^{-13}$. The LHC will search for superpartners of SM particles, and if detected, it will provide evidence in support of BSM theories. In chapter 4, we have discussed the potential of the Long baseline neutrino experiment LBNE (Fermilab, USA) to explain two of the most important unknowns of neutrino physics namely: the leptonic CPV phase $\delta_{CP}$ and octant of atmospheric angle.

Apart from studying the physics potential of the long-baseline neutrino experiments in determining the unknown oscillation parameters, in this thesis we have also studied how the presence of small non unitarity effects in the leptonic mixing matrix can significantly constrain the absolute value of the lightest neutrino mass. Neutrino oscillation can only probe the mass squared differences of the neutrinos but not their absolute masses. The cosmological constraints of the sum of the $v$ masses bound is $\sum_i m(v_i) < 0.23$ eV from CMB, Planck 2015 data (CMB15+ LRG+ lensing + $H_0$). Also Tritium beta decay experiments measures the absolute mass of neutrinos. Bounds on sum of the absolute $v$ mass is $\sum_i m(v_i) < 2$eV from tritium beta decay. The cLFV decays such as $\mu \to e\gamma$, $\mu \to \tau\gamma$ and $\tau \to e\gamma$ are also affected due to the non unitarity effects of the PMNS matrix. Predictions presented in this thesis may shed light on the underlying new physics if future experiments confirm the presence of cLFV searches at LHC and the presence of small non unitarity effects in the neutrino mixing matrix.

In chapter 2, we have determined neutrino oscillation parameters in a realistic minimal SO(10) model, using updated values of running quark and lepton masses. We find that these
values conforms with the latest global fit values of ν oscillation parameters. We would like
to do R.G.E (Renormalization Group Equations) study of the neutrino oscillation parameter
in our future work. Renormalisable SO (10) embedded with and without supersymmetry
(SUSY) — is compelling and predictive in constraining fermion mass matrices. Its underlying
\( SU(4)_C \) symmetry connects the quark and lepton Yukawa couplings in a renormalisable
gauge theory. In SO(10), we have

\[ 16 \otimes 16 = 10 \oplus 120 \oplus 126 \quad (6.1) \]

and thus Higgs fields giving mass to the 16 dimensional fermions belongs to the 10, 120
and 126 dimensional representations of Higgs. We have used the model with 10 and 126
dimensional Higgs for generating neutrino oscillation parameters. The 126 dimensional
Higgs representation contains color singlet as submultiplets which transform as a triplet
under \( SU(2)_L \) and a singlet under \( SU(2)_R \). Type II Seesaw mechanism mediates through
induced scalar triplet vev and can generate small neutrino masses in SO(10) model. The
126_H Higgs vev relates the mass of the Majorana neutrinos to both the Dirac mass as well as
charged fermion masses and this unique feature makes the model a predictive one.

In chapter 3, we have studied the rare cLFV process \( \mu \rightarrow e\gamma \) in \( \mu - \tau \) symmetric SUSY
SO(10) theories, using the type I seesaw mechanism, in mSUGRA, NUHM, NUGM and
NUSM models where we have predicted the possibility of detection of new sparticles at
next run of LHC. We have used the value of the Higgs mass central value of 125.9 GeV as
measured at LHC, the latest global data on the reactor mixing angle \( \theta_{13} \) for neutrinos, and the
latest constraints on BR(\( \mu \rightarrow e\gamma \)) as projected by MEG. We found that in mSUGRA a very
heavy region for gauginos is allowed by the future MEG bound of BR(\( \mu \rightarrow e\gamma \)), though in
the NUHM case a low region of \( M_{1/2} \) is also allowed. We also observed that NUGM allows,
in general, a wider parameter space, as compared to both mSUGRA and NUHM. Here
BR(\( \mu \rightarrow e\gamma \)) is found to increase with increase in \( m_0 \), which could be due to the particular
ratios of gaugino masses. In NUGM, we find that the allowed values of $|A_0|$ are shifted towards the heavier side (compared to mSUGRA and NUHM). In NUSM, the allowed $M_{1/2}$ parameter space at low energies becomes constrained as compared to the other three models. For a Higgs boson mass around 125 GeV, $M_{1/2}$ lies between $4 \text{ TeV} \leq M_{1/2} \leq 6 \text{ TeV}$ and $m_0 \geq 2 \text{ TeV}$ is favored. The branching ratio of $\mu \rightarrow e\gamma$ does not change significantly with variation of first and second generation sfermion masses at the GUT scale, in the completely non-universal NUSM model. Any observation of heavy particles at the next run of LHC could help us understand and discriminate among these various universal and non universal models, in reference to stringent limit put by LHC on cLFV decays. This in turn could contribute towards a better understanding of theories beyond the standard model.

In chapter 4 we have shown how the entanglement of the quadrant of leptonic CPV phase and octant of atmospheric mixing angle $\theta_{23}$ at LBNE/DUNE, can be broken via leptogenesis and baryogenesis. Here, we considered the effect of near detector only in LBNE, on sensitivity of CPV phase measurement, but similar conclusions would hold for the effect of reactor experiments as well. This study was done for both normal and inverted hierarchy, higher and lower octants. We find that nonsupersymmetric SO(10) model is favored for baryogenesis with respect to the supersymmetric model. In the nonsupersymmetric model [? ], the baryon asymmetry is around the required order of magnitude $Y_B \sim 10^{-10}$. We did a parameter scan in 3 dimensional space $-\delta_{CP}, \theta_{13}$ and $\Delta m_{31}^2$ that encode the breaking of the entanglement of the quadrant of CPV phase and Octant of $\theta_{23}$ in light of the latest constraints on $|Y_B|$, $5.7 \times 10^{-10} \leq Y_B < 6.7 \times 10^{-10}$, by taking neutrino oscillation mixings and mass scales as indicated by the latest global fits. The current data shows a preference of $\delta_{CP}$ towards $1.5\pi$. From our analysis, one of the leptonic CPV phase determined in IH, LO of $\theta_{23}$ case is $\delta_{CP} = 276.5^0$ or $1.536\pi$ corresponding to $\theta_{13} \sim 9.0667^0, 9.0974^0, 9.3167^0 - 9.4417^0, 9.5167^0$ and $\Delta m_{31}^2 \in [-2.45 \times 10^{-3}, -2.21 \times 10^{-3}] eV^2$ which is near to the preferred data $\delta_{CP} = \frac{3}{2} \pi$. Our analysis of $\delta_{CP} = 1.43\pi$ of IH, HO of $\theta_{23}$ case is close to the best fit value of $\delta_{CP} = 1.48\pi$. 
for inverted ordering from global fit results and also $\delta_{CP} = 1.436\pi$ of NH, HO of $\theta_{23}$ case is favored with the recent hint of $\delta_{CP} = 1.41\pi$ for normal hierarchy.

In chapter 5, we have considered the possibility that the neutrino mixing matrix (considering charged lepton mass matrix to be diagonal), $U_{PMNS}$ could be non unitary, and then calculated the limits on non unitary parameters $\eta_{\mu e}$, $\eta_{\tau e}$ and $\eta_{\tau \mu}$ from latest constraints on branching ratios of cLFV decays. We have analysed how the non unitarity of $U_{PMNS}$ can affect flavored and unflavored leptogenesis and the generation of baryon asymmetry of the universe in this regard. In this context we also calculated the values of lightest $\nu$ mass, dirac CPV phase $\delta_{CP}$ and majorana phases $\alpha$ and $\beta$, such that $Y_B$ lies in the present day constraints $(5.8 \times 10^{-10} < Y_B < 6.6 \times 10^{-10})$ using type I see saw mechanisms for producing light $\nu$ masses. It is worth mentioning that all these four quantities are unknown yet, and future experiments will be measuring them, and thus our analysis could shed new light in this area.

6.1 New Results Presented In The Thesis (Highlights)

- In chapter 2, we found that our predictions of $\nu$ oscillation parameters using a model with 10 and 126 dimensional Higgs embedded with SO(10) GUTs in the framework of type II seesaw mechanism agrees well with the latest global fit data of $\nu$ oscillation parameters. Our analysis of $\sin^2 \theta_{23} \in [0.443, 0.599]$, $\Delta m^2_{12} \in [7.59, 7.79] \times 10^{-5} eV^2$ and $\sin^2 \theta_{12} \in [0.307, 0.339]$, is in consistent with the recent global fits of world $\nu$ oscillation data. Though we have used $\tan \beta = 55$, in this work, same calculations can be done for other values of $\tan \beta = 10$ as well.

- In chapter 3 we studied the branching ratios of rare cLFV decays in mSUGRA, NUHM, NUGM and NUSM theories in type I SUSY SO(10) theories to perceive the masses of new SUSY particles that could be accessible at LHC’s second run. Here we find that in mSUGRA model, the parameter space $M_{1/2} \geq 1$ TeV is allowed by present MEG bounds on $BR(\mu \rightarrow e\gamma)$, while the future MEG limit excludes small $M_{1/2}$ space $\leq 3.5$ TeV. As compared to mSUGRA, in NUHM, a wider SUSY space is allowed. In NUGM model, $BR(\mu \rightarrow e\gamma)$
6.1 New Results Presented In The Thesis (Highlights)

is found to increase with increase in $m_0$. In NUGM, the allowed values of $|A_0|$ are shifted towards the heavier side (compared to mSUGRA and NUHM). In NUSM, for a Higgs boson mass around 125 GeV, $M_{1/2}$ lies between $4 \text{ TeV} \leq M_{1/2} \leq 6 \text{ TeV}$ and $m_0 \geq 2 \text{ TeV}$ is favored. Here the branching ratio of $\mu \to e\gamma$ does not change significantly with variation of first and second generation sfermion masses at the GUT scale. We have used $\tan \beta = 10$ here and we found that for higher values of $\tan \beta$ the allowed parameter space becomes narrower.

- In chapter 4 we showed how baryogenesis via leptogenesis can resolve the entanglement of octant of atmospheric mixing angle $\theta_{23}$ and quadrant of leptonic CPV phase $\delta_{CP}$ (measured at LBNEs). The results of our analysis, $\delta_{CP} = 276.5^0$ or $1.536\pi$ in IH, LO of $\theta_{23}$ case, corresponding to $\theta_{13} \sim 9.1^0$ to $9.6^0$ and $\Delta m^2_{31} \in [-2.45 \times 10^{-3}, -2.21 \times 10^{-3}] eV^2$ is near to the preferred data $\delta_{CP} = \frac{3}{2}\pi$ reported in [208–210]. Also our calculated value of $\delta_{CP} = 1.43\pi$ of IH, HO of $\theta_{23}$ case is close to the best fit value of $\delta_{CP} = 1.48\pi$ for inverted ordering from recent global fits. We also predicted some more values of $\delta_{CP}$ (consistent with current BAU limits) which could be tested in future.

- In chapter 5, we calculated new values of non–unitarity parameters of $U_{PMNS}$ matrix from the bounds on rare cLFV decays and hence predicted the absolute value of lightest $\nu$ mass in this regard. We consider here a model where see-saw is extended by an additional singlet $S$ which is very light, but can give rise to non-unitarity effects without affecting the form of see-saw formula. The values of lightest $\nu$ mass lies in the range of $0.0018 \text{ eV}$ to $0.0023 \text{ eV}$, $0.048 \text{ eV}$ to $0.056 \text{ eV}$, $0.05 \text{ eV}$ to $0.054 \text{ eV}$, $0.053 \text{ eV}$ to $0.062 \text{ eV}$ in one flavored leptogenesis regime, $0.023 \text{ eV}$ to $0.03 \text{ eV}$, $0.058 \text{ eV}$ to $0.06 \text{ eV}$, $0.023 \text{ eV}$ to $0.037 \text{ eV}$, $0.062 \text{ eV}$ to $0.07 \text{ eV}$, $0.063 \text{ eV}$ in two flavored leptogenesis regime and $0.065 \text{ eV}$ to $0.07 \text{ eV}$, $0.08 \text{ eV}$ to $0.085 \text{ eV}$ in three flavored leptogenesis regime. All these values satisfy the constraint, $\sum_i m(\nu_i) < 0.23 \text{ eV}$. The lightest $\nu$ mass range from $0.08 \text{ eV}$ to $0.085 \text{ eV}$ is consistent with the absolute $\nu$ mass bounds from tritium beta decay $\sum_i m(\nu_i) < 2\text{ eV}$. We found that in case
of three flavored leptogenesis no value of $m_{\text{lightest}}$ satisfy the constraint on $Y_B$ in NH, $U_{\text{PMNS}}$ unitary case.

6.2 Importance Of The Results

- The results presented in chapter 2 are important for model building to explain neutrino (neutral lepton) oscillations.
- In third chapter we predicted the masses of sparticles that comply with the present constraints on branching ratio of rare cLFV decays. If new particles are detected at the second run of LHC it could help discriminate various BSM theories (mSUGRA, NUHM, NUSM and NUGM).
- The results of fourth chapter shed light on the challenging issue of resolving the entanglement of octant of $\theta_{23}$ — quadrant of $\delta_{CP}$ present in long baseline neutrino oscillation experiments. We predicted values of leptonic CPV phase for both the octants and hierarchies.
- In fifth chapter, we calculated new limits on non-unitarity parameters from cLFV decays and predicted masses of lightest neutrino mass, by considering the possibility that $U_{\text{PMNS}}$ could be non-unitary.

All the above results are testable in future experiments.