5

Effects Of Leptonic Non-Unitarity on LFV,
Neutrino Oscillation, Leptogenesis and
Lightest ν Mass

5.1 Introduction

In last three chapters, we discussed about neutrino oscillation parameters (mass square
difference, mixing angles, octant of $\theta_{23}$, and quadrant of $\delta_{CP}$) and how neutrino oscillation
can affect rare cLFV decays. Now, we shall discuss about another important aspect of lepton (neutral) mixing — i.e the non-unitarity of the mixing matrix $U_{PMNS}$. We shall calculate in this chapter, new values of non-unitarity parameters of $U_{PMNS}$ using latest bounds on branching ratio of rare cLFV decays, and shall also predict values of lightest neutrino mass (still unknown).

Neutrinos have non zero masses. There are 3 known flavors of neutrinos, $\nu_e, \nu_\mu, \text{and } \nu_\tau$, each of which couples only to the charged lepton of the same flavor, $\nu_e, \nu_\mu, \text{and } \nu_\tau$ are superpositions of three mass eigenstates, $|\nu_\alpha> = \sum U_{\alpha i}^* |\nu_i>$, where $\alpha = e, \mu, \tau$ and $\nu_i$ is the neutrino of definite mass $m_i$. The cosmological constraints of the sum of the $\nu$ masses bound is $\sum_i m(\nu_i) < 0.23 \text{ eV}$ from CMB, Planck 2015 data (CMB15+ LRG+ lensing + $H_0$) [37]. We note that the lepton mixing matrix $U$ has a big mixing and we know almost nothing about the phases. The discoveries of neutrino mass and leptonic mixing have come from the observation of neutrino flavor change, $\nu_\alpha \rightarrow \nu_\beta$. CP violation interchanges every particle in a process by its antiparticle. This CP violation can be produced by the phase $\delta_{CP}$ in $U$. Neutrinos can have two types of mass term in the lagrangian — dirac and Majorana mass terms. To determine whether Majorana masses occur in nature, so that $\bar{\nu}_i = \nu_i$, the favorable approach to seek is Neutrinoless Double Beta Decay [0$\nu\beta\beta$]. Whatever necessary processes cause $0\nu\beta\beta$, its observation would intend the existence of a Majorana mass term. We consider here a model where see-saw is extended by an additional singlet $S$ which is very light, but can give rise to non-unitarity effects without affecting the form of see-saw formula [30].

In the conventional type I see-saw framework there are Dirac and Majorana mass matrices $m_D$ and $M_R$ in the Lagrangian,

$$L = \frac{1}{2} N_R M_R N_R^c + N_R m_D \nu_L + h.c$$ (5.1)
The low energy mass matrix is given by

\[ m_\nu = -m_D^T M_R^{-1} m_D \]  

(5.2)

In the usual unitarity scenario, the three active neutrinos, the flavor eigen states \( \nu_e, \nu_\mu, \nu_\tau \) are connected to the mass eigen states \( \nu_1, \nu_2, \nu_3 \) via \( \nu_\alpha = N_\alpha \nu_i \). Here \( N \) is the the generalised \( \nu \) mixing matrix which could be both unitary and nonunitary. In the diagonal charged lepton basis, \( m_\nu \) is diagonalised by a unitary matrix as

\[ U^* P^* m_\nu P U^T = m_D^\nu \]  

(5.3)

The Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix is \( U \), where \( U \) is

\[
U = \begin{pmatrix}
  c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\
-s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i \delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i \delta} & s_{23} c_{13} \\
s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i \delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i \delta} & c_{23} c_{13}
\end{pmatrix}
\]

(5.4)

where, \( \theta_{12} = 33^0, \theta_{23} = 38^0 - 53^0, \theta_{13} = 8^0 \) [15–20] are the solar, atmospheric and reactor angles respectively. The Majorana phases reside in \( P \), where

\[
P = \text{diag} \left( 1, e^{i \alpha}, e^{i(\beta + \delta)} \right)
\]

(5.5)

Cosmologists suggest that just after the Big Bang, the universe contained equal amounts of matter and antimatter. Today the universe contains matter but almost no antimatter. This change need that matter and antimatter act differently (CP violation). The CP-violating scenario to explain this change is Leptogenesis. Leptogenesis is a natural outcome of the See-Saw Mechanism. In the seesaw picture, we assume that, just as there are 3 light neutrinos \( \nu_1, \nu_2, \nu_3 \), there are 3 heavy right handed neutrinos \( M_1, M_2, M_3 \), where \( M_R \sim 10^{9-14} \text{ GeV} \).
$M_R \sim M_1, M_2, M_3$ which were there in the Hot Big Bang. The $M_R$ decays modes are:

$$M \rightarrow l^- + H^+, M \rightarrow l^+ + H^-, M \rightarrow \nu + H^0, M \rightarrow \bar{\nu} + \bar{H}^0$$  \hspace{1cm} (5.6)$$

where, $l^-$ are $e^-, \mu^-, \tau^-$ and $H^+, H^-, H^0$ are the Higgs fields. CP violation effects in the $M_R$ decays, may result from phases in the decay coupling constants. This leads to unequal numbers of leptons ($l^-$ and $\nu$) and antileptons ($l^+$ and $\bar{\nu}$) in the Universe.

$$\Gamma(M \rightarrow l^+ + H^-) \neq \Gamma(M \rightarrow l^- + H^+)$$  \hspace{1cm} (5.7)$$

In leptogenesis, CP violating decays of heavy Majorana neutrinos creates a lepton – antilepton symmetry [32] and then B+L violating sphaleron processes [34] at and above the electroweak symmetry breaking scale converts part of this asymmetry into the observed baryon-antibaryon asymmetry. The heavy neutrinos are seesaw partners of the observed light ones.

For unflavored leptogenesis, valid for $M_1 \geq 10^{11}$ GeV, we have taken here $M_1 \sim 10^{12}$ GeV where the flavor of the final state leptons plays no role. It can be shown that for lower values of $M_1$ it depends on the flavor of the final state leptons, and hence is called flavored leptogenesis [35, 36]. Here we consider both flavored and unflavored leptogenesis. For unflavored leptogenesis, the decay asymmetry in case of hierarchical heavy neutrinos is given by [30]

$$\varepsilon_1 = \frac{1}{8\pi v^2} \frac{1}{(m_D m_D^\dagger)_{11}} \sum_{2,3} \text{Im}(m_D m_D^\dagger)^2_{1j} f(M_j^2/M_1^2)$$  \hspace{1cm} (5.8)$$

where, $f(x) = -\frac{3}{2\sqrt{x}}$ for $x > 1$, i.e., for hierarchical heavy $\nu$. The baryon asymmetry of the Universe is proportional to the decay asymmetry $\varepsilon_1$. The Dirac mass matrix $m_D$ in terms of an orthogonal matrix $R$ is [229]

$$m_D = i\sqrt{M_R R} \sqrt{m^\text{diag}_\nu} U^\dagger$$  \hspace{1cm} (5.9)$$
And, compatible quantity for leptogenesis is then,

$$m_Dm_D^\dagger = \sqrt{M_R}R\sqrt{m_{\nu}^{\text{diag}}}U^\dagger U\sqrt{m_{\nu}^{\text{diag}}}R^\dagger \sqrt{M_R} = \sqrt{M_R}Rm_{\nu}^{\text{diag}}R^\dagger \sqrt{M_R}$$

(5.10)

If $R$ is real then there is no leptogenesis at all. Here, we have taken an adhoc assumption, that $R = U$ (see Eq. 5.4). $R$ consists of the low energy mixing elements and the CP phases.

Since unitarity of $\nu$ mixing matrix has not been proved yet, if it is non unitary, then for the $\nu$ mixing matrix $N$ to be non unitary, we have

$$\nu_\alpha = N_{\alpha i}\nu_i$$

(5.11)

fusing the flavor and mass states. The non-unitary matrix $N$ is now assigned as

$$N = (1 + \eta)U_0$$

(5.12)

where $U_0 = U * P$. If $m_\nu$, which is diagonalized by a non-unitary mixing matrix, originates from the see-saw mechanism, we have

$$m_D = i\sqrt{M_R}R\sqrt{m_{\nu}^{\text{diag}}}N^\dagger$$

(5.13)

And thereupon, we have

$$m_Dm_D^\dagger = \sqrt{M_R}R\sqrt{m_{\nu}^{\text{diag}}}N^\dagger N\sqrt{m_{\nu}^{\text{diag}}}R^\dagger \sqrt{M_R} = \sqrt{M_R}Rm_{\nu}^{\text{diag}}R^\dagger \sqrt{M_R}$$

(5.14)

since, $1 + 2U_0^\dagger \eta U_0 \neq 1$. Leptogenesis is no longer independent of the low-energy phases. It depends on the phases in $U_0$ as well as to the phases in $\eta$. Leptogenesis [32] is one of the exceedingly well inspired framework which produces baryon asymmetry of the Universe through $B + L$ violating electroweak sphaleron process [34]. As directed by Sakharov the
three basic requirements that produces baryon asymmetry in this Universe are [119]
(i) Baryon number violation,
(ii) C and CP violation and
(iii) Out of equilibrium decay

We consider here, that lepton asymmetry is generated by out of equilibrium decay of heavy
right handed Majorana neutrinos into Higgs and lepton within the framework of type I seesaw
mechanism. In a hierarchical case of three right handed heavy Majorana neutrinos $M_{2,3} > M_1$, the
lepton asymmetry created by the decay of $M_1$, the lightest of three heavy right handed
neutrinos is [36, 241] (for both flavored and unflavored leptogenesis)

$$\varepsilon_1^a = \frac{1}{8\pi v^2} \frac{1}{(m_D^a m_D^a)_{11}} \left[ \sum_{2,3} \text{Im}[(m_D^a)_{\alpha 1} (m_D^j m_D^j)_{\alpha j}] g(x_j) \right]$$

$$+ \left[ \sum_{2,3} \text{Im}[(m_D^a)_{\alpha 1} (m_D^a m_D^j)_{\alpha j}] \frac{1}{1-x_j} \right]$$

At temperatures, $T \geq 10^{12}$ GeV all charged lepton flavors come out of equilibrium and thus
all of them behave in the same way which results in the one flavor regime. At moderate
temperatures $T < 10^{12}$ GeV ($T < 10^9$ GeV), tau (muon) yukawa coupling interactions come
into equilibrium and hence flavor effects play an important role in the calculation of lepton
asymmetry [120–127, 242]. The region of temperatures belonging to $10^9 < T/GeV < 10^{12}$
and $T/GeV < 10^9$ are respectively denoted as two and three flavor regimes of leptogenesis.

$Y_B$ in the two and three flavor regimes are designated as [36]

$$Y_B^{2\text{flavor}} = \frac{-12}{37 g^2} [\varepsilon_2 \rho \left( \frac{417}{589} \bar{n}_\tau \right) + \varepsilon_1^\tau \rho \left( \frac{390}{589} \bar{n}_\tau \right)]$$
Y^{3\text{flavor}}_B = \frac{-12}{37g^*} [\varepsilon_1^e \rho \left(\frac{151}{179} \bar{n}_e\right) + \varepsilon_1^\mu \rho \left(\frac{344}{537} \bar{n}_\mu\right) + \varepsilon_1^\tau \rho \left(\frac{344}{537} \bar{n}_\tau\right)] \quad (5.18)

\begin{align*}
g^* \sim 110 \text{ is the effective number of relativistic degrees of freedom at } T = M_1,

\rho(n_\alpha) &= \left[\left(\frac{n_\alpha}{8.25 \times 10^{-3} \text{eV}}\right)^{-1} + \left(\frac{0.2 \times 10^{-3} \text{eV}}{n_\alpha}\right)^{-1.16}\right]^{-1}

\varepsilon_2 &= \varepsilon_1^e + \varepsilon_1^\mu \quad ,

\bar{n}_2 &= \bar{n}_e + \bar{n}_\mu

\bar{n}_\alpha &= \frac{m^*_{D\alpha_1} m_{D\alpha_1}}{M_1}
\end{align*}

where \varepsilon_1^e, \varepsilon_1^\mu, \varepsilon_1^\tau \text{ are the electron type, muon type and tau type lepton asymmetry in the unflavored leptogenesis regime.}

The building blocks of matter are the quarks, the charged leptons, and the neutrinos. The discovery and study of the Higgs boson at the Large Hadron Collider (LHC) has provided strong evidence that the quarks and charged leptons derive their masses from a coupling to the Higgs field. Most theorists strongly suspect that the origin of the neutrino masses is different from the origin of the quark and charged lepton masses. Neutrino oscillation has proved that neutrinos have nonzero masses. These masses may have a quite different origin than the quark and charged lepton masses. We, and all matter, may be descended from heavy neutrinos. We list the values of \(m_{\text{lightest}}\) for each one flavor, two flavor and three flavor regime for different hierarchies and unitarity, non-unitarity of \(U_{PMNS}\) in Table 5.2, 5.3, 5.4 respectively and check whether our values of \(m_{\text{lightest}}\) are in consistent with the constraints on the absolute scale of \(\nu\) masses.

The plan of this chapter is as follows. In Section 5.2, we show the effect of low energy phenomenology of non unitarity on charged lepton flavor violating decays in type I seesaw
theories and present the values of various parameters used in our analysis for the generation of baryon asymmetry of the Universe through the mechanism of leptogenesis. Section 5.3 contains our calculations and results. Section 5.4 summarizes the work.

5.2 Low Energy Phenomonology of Non-Unitarity

One interesting feature of non-unitarity of the PMNS matrix lies in Lepton Flavor Violation (LFV). In the light of unitarity violation in decays such as $\alpha \rightarrow \beta \gamma, (\alpha, \beta) = (\tau, \mu), (\tau, e)$ or $(\mu, e)$, the branching ratio is [30]

$$\frac{BR(\alpha \rightarrow \beta + \gamma)}{BR(\alpha \rightarrow \beta + \nu \bar{\nu})} = \frac{100\alpha}{96\pi} |(NN^\dagger)_{\alpha\beta}|^2 \quad (5.19)$$

Also

$$\frac{BR(\tau \rightarrow \mu + \gamma)}{BR(\tau \rightarrow \mu + \nu \bar{\nu})} \approx \frac{25\alpha}{6\pi} |\eta_{\mu\tau}|^2; \quad \frac{BR(\tau \rightarrow \mu + \gamma)}{4.2 \times 10^{-10}} \approx \frac{|\eta_{\mu\tau}|^2}{25 \times 10^{-8}} \quad (5.20)$$

Using the latest updated constraint on $BR(\tau \rightarrow \mu + \gamma) = 4.4 \times 10^{-8}$ [99], one can derive bounds on $|\eta_{\mu\tau}|$ from Eq. 5.20. Now we have,

$$\frac{BR(\tau \rightarrow \mu + \gamma)}{BR(\mu \rightarrow e + \gamma)} = \frac{BR(\tau \rightarrow \mu + \nu_\tau \bar{\nu}_\mu)}{BR(\mu \rightarrow e + \nu_\mu \bar{\nu}_e)} \times \frac{|\eta_{\mu\tau}|^2}{|\eta_{\tau e}|^2} \quad (5.21)$$

Now, we calculate the ratio,

$$\frac{BR(\tau \rightarrow \mu + \nu_\tau \bar{\nu}_\mu)}{BR(\mu \rightarrow e + \nu_\mu \bar{\nu}_e)} = 0.176745$$

Thus we find constraints on $|\eta_{\mu e}|$ from Eq. 5.21, using the latest constraint on $BR(\mu \rightarrow e + \gamma), BR(\mu \rightarrow e + \gamma) = 4.2 \times 10^{-13}$ [99]. Again we have

$$\frac{BR(\tau \rightarrow \mu + \gamma)}{BR(\tau \rightarrow e + \gamma)} = \frac{BR(\tau \rightarrow \mu + \nu_\tau \bar{\nu}_\mu)}{BR(\tau \rightarrow e + \nu_\mu \bar{\nu}_e)} \times \frac{|\eta_{\mu\tau}|^2}{|\eta_{\tau e}|^2} \quad (5.22)$$
From our calculation, the ratio \( \frac{BR(\tau \rightarrow \mu + \nu \bar{\nu})}{BR(\tau \rightarrow e + \nu \bar{\nu})} \) is,

\[
\frac{BR(\tau \rightarrow \mu + \nu \bar{\nu})}{BR(\tau \rightarrow e + \nu \bar{\nu})} = 2.509
\] (5.23)

And then we calculate latest updated bounds on \( |\eta_{\tau e}| \). The calculations are summarised in Table 5.1.

| Latest updated Branching Ratios on cLFV Decays | Calculated bounds on \( |\eta_{\alpha\beta}| \) |
|-----------------------------------------------|-----------------------------------------------|
| \( BR(\mu \rightarrow e + \gamma) = 4.2 \times 10^{-13} \) [99] | \( |\eta_{\mu e}| = 6.64733013 \times 10^{-6} \) |
| \( BR(\tau \rightarrow \mu + \gamma) = 4.4 \times 10^{-8} \) [99] | \( |\eta_{\tau \mu}| = 5.11766 \times 10^{-3} \) |
| \( BR(\tau \rightarrow e + \gamma) = 3.3 \times 10^{-8} \) [99] | \( |\eta_{\tau e}| = 7.021 \times 10^{-3} \) |

**Table 5.1**: Our calculated constraints on non-unitarity parameter \( \eta_{\tau e}, \eta_{\tau \mu}, \eta_{\mu e} \) from latest cLFV decays.

Returning to leptogenesis, the baryon asymmetry should lie in the interval, \( 5.8 \times 10^{-10} < Y_B < 6.6 \times 10^{-10} \) [243]. In general we have taken complex and orthogonal matrix \( R = U_{PMNS} \), as an adhoc assumption, but \( R \) can be taken as \( R = V_{CKM} \times U_{PMNS} \) in non susy SO(10) models [242] as studied in the context of breaking entanglement of octant of \( \theta_{23} \) and \( \delta_{CP} \) in the light of baryon asymmetry of the universe through the mechanism of leptogenesis.

For the Normally ordered light \( \nu \) masses, we have

\[
M_{R}^{\text{diag}} = \text{diag}(M_1,M_2,M_3) = M_1 \text{diag}(1, \frac{M_2}{M_1}, \frac{M_3}{M_1}) = M_1 \text{diag}(1, \frac{m_1}{m_2}, \frac{m_1}{m_3})
\] (5.24)

With \( m_1 \in [10^{-6} \text{eV}, 10^{-1} \text{eV}], and, m_2^2 - m_1^2 = 7.60 \times 10^{-5} \text{eV}^2, m_3^2 - m_1^2 = 2.48 \times 10^{-3} \text{eV}^2 \) as is evident from the \( \nu \) oscillation data [15–20], \( m_1 \) being the lightest of three \( \nu \) masses. For the inverted ordered light \( \nu \) masses, we have

\[
M_{R}^{\text{diag}} = \text{diag}(M_1,M_2,M_3) = M_1 \text{diag}(1, \frac{M_2}{M_1}, \frac{M_3}{M_1}) = M_1 \text{diag}(1, \frac{m_1 m_3}{m_2^2}, \frac{m_1}{m_2})
\] (5.25)
with $m_3$ being the lightest of three $\nu$ masses. For flavored leptogenesis regime, we take $M_1 \sim 10^{10}$ GeV. Next we do the parameter scan for both flavored and unflavored leptogenesis of a minimal seesaw model satisfying the Planck data on baryon to photon ratio of the universe for four cases:

1) Normal Hierarchical structure neutrino masses, Non unitarity of PMNS matrix.
2) Normal Hierarchical structure neutrino masses, unitarity of PMNS matrix.
3) Inverted Hierarchical structure of neutrino masses, Non unitarity of PMNS matrix.
4) Inverted Hierarchical structure neutrino masses, unitarity of PMNS matrix.

We perform random scan of the parameter space for NH, IH in the light of recent ratio of the baryon to photon density bounds $5.8 \times 10^{-10} < Y_B < 6.6 \times 10^{-10}$ in the following ranges:

$$m_1(m_3) \in [10^{-6} \text{ eV}, 0.1 \text{ eV}] \ ( [10^{-6} \text{ eV}, 0.1 \text{ eV}])$$

$$\delta_{CP} \in [0, 2\pi]$$

$$\alpha \in [0, 2\pi]$$

$$\beta \in [0, 2\pi] \quad (5.26)$$

While doing parameter scan, we find values of lightest $\nu$ mass, Majorana phases $\alpha, \beta$ and dirac CPV phase $\delta_{CP}$, for which baryon to photon ratio $Y_B$ lies in the given range, for above four cases. This is done for unflavored, two flavor and three flavor leptogenesis regimes.

### 5.3 Calculations and Results

Results of our analysis have been presented in Fig. 5.1 – Fig. 5.19. It can be seen from Fig. 5.1 that in the one-flavor regime, NH structure of neutrino masses, non-unitarity textures of PMNS matrix can give rise to correct baryon asymmetry of the Universe, $5.8 \times 10^{-10} < Y_B < 6.6 \times 10^{-10}$, if the lightest $\nu$ mass lies around 0.0018 eV to 0.0023 eV. A close numerical
inspection of the situation reveals that for a lightest neutrino mass of 0.0024 eV-0.005 eV, one can exceed the upper bound on $Y_B$. The dependence of lightest neutrino mass $m_1$ on

![](image1.png)

**Fig. 5.1:** Scatter plot of the lightest neutrino ($\nu$) mass $m_1$ against the baryon asymmetry of the Universe with normal hierarchy (NH), non-unitarity case in one flavored leptogenesis regime.

Majorana phases $\alpha, \beta$ is shown in the left, right panel of Fig. 5.2 respectively, $Y_B$ being constrained in the order $10^{-10}$. It may be noted that $M_1 = 10^{12}$ GeV is favored in the light of baryon asymmetry of the Universe for 1 flavor regime. Figure 5.3 shows the scatter

![](image2.png)

**Fig. 5.2:** Variation of lightest $\nu$ mass $m_1$ with Majorana phases $\alpha$ and $\beta$ in case of NH, non-unitarity case in one flavored leptogenesis regime.

plot of the lightest neutrino mass $m_1$ against the baryon asymmetry of the Universe with
Normal hierarchy, and unitary $U_{PMNS}$ in one flavor regime. For $Y_B$ to be in the range, $5.8 \times 10^{-10} < Y_B < 6.6 \times 10^{-10}$, $m_1$ lies between 0.048 eV to 0.056 eV. For $Y_B$ in the order $10^{-10}$, the lightest $\nu$ mass $m_1$ is mostly concentrated in the region 0.043eV to 0.006eV. Figure 5.4 shows the variation of lightest neutrino mass $m_1$ with Dirac CP phase $\delta_{CP}$ and Majorana phase $\alpha$ for NH (normal hierarchy), unitarity texture. For $Y_B$ to be in the consistent BAU range $5.8 \times 10^{-10} < Y_B < 6.6 \times 10^{-10}$, $\delta_{CP}$ is centered around 253$^0$ and 290$^0$ with $m_1$ constrained in the region from 0.003 eV to 0.065 eV. $\delta_{CP} = 1.405\pi$ is favored with the recent hint of $\delta_{CP} = 1.41\pi$ for normal hierarchy. It can be seen from Fig. 5.5 that in the one-flavor regime, IH structure of neutrino masses, non-unitarity textures of PMNS matrix can give rise to baryon asymmetry of the Universe, of the order of $10^{-10}$, if the lightest $\nu$ mass lies around 0.05 eV to 0.054 eV. Few Points lie in the region, $5.8 \times 10^{-10} < Y_B < 6.6 \times 10^{-10}$. Figure 5.6 shows the scattered plot of lightest $\nu$ mass $m_3$ against Dirac CPV phase, $\delta_{CP}$ and Majorana phase $\alpha$ in inverted hierarchy of $\nu$ masses non unitarity case in one flavored leptogenesis regime. Figure 5.7 shows the variation of the lightest neutrino mass against the baryon asymmetry of the Universe and Dirac CP phase $\delta_{CP}$ with Inverted hierarchy, unitarity case in one flavored lepogenesis regime. For $Y_B$ in the order $10^{-10}$, the lightest $\nu$ mass $m_3$ is
5.3 Calculations and Results

Fig. 5.4: Variation of lightest $\nu$ mass $m_1$ with Dirac CP phase $\delta_{\text{CP}}$ and Majorana phases $\alpha$ in case of NH unitarity case in one flavored leptogenesis regime.

concentrated in the region 0.053eV to 0.062 eV. Figure 5.8 depicts the scatter plot of $m_3$ with Majorana phase $\alpha$ and $\beta$ for IH, Unitarity texture of $U_{PMNS}$ in one flavor leptogenesis regime. From Fig. 5.9 we find that in the two flavor regime of leptogenesis, normal hierarchical structure of neutrino masses, non-unitarity texture of PMNS matrix gives rise to correct baryon asymmetry of the Universe, $5.8 \times 10^{-10} < Y_B < 6.6 \times 10^{-10}$, if the lightest $\nu$ mass lies around 0.023 eV to 0.03 eV, 0.034 eV to 0.038 eV and also around 0.058 eV to 0.06 eV. If $M_1 \sim 10^{10}$ GeV, then 2 flavored leptogenesis is favored in the light of baryon asymmetry.

We show in Fig. 5.11, the scatter plot of the lightest neutrino mass $m_1$ against the baryon asymmetry of the Universe with normal hierarchy, unitarity of $U_{PMNS}$ in two flavor regime. For $Y_B$ to be in the region, $5.8 \times 10^{-10} < Y_B < 6.6 \times 10^{-10}$, $m_1$ lies between 0.023 eV to 0.037 eV and 0.06 eV. For $Y_B$ to be in the order $10^{-10}$, $m_1$ is mostly concentrated in the region 0.01eV to 0.065eV. In Fig. 5.13, for $Y_B$ in the range, $5.8 \times 10^{-10} < Y_B < 6.6 \times 10^{-10}$, $m_3$, lies between 0.062 eV to 0.07 eV. For $Y_B$ in the order $10^{-10}$, the lightest $\nu$ mass $m_3$ is mostly concentrated in the region 0.01eV to 0.07eV. From Fig. 5.15 we find that in the 2-flavor regime, IH structure of neutrino masses, unitarity textures of PMNS matrix gives rise to correct BAU (baryon asymmetry of the Universe), if the lightest $\nu$ mass $m_3$ lies around 0.063 eV. Here $M_1 = 10^{10}$ GeV, then IH unitarity texture of PMNS matrix, in 2 flavor regime is
Fig. 5.5: Scatter plot of the lightest neutrino mass $m_3$ against the baryon asymmetry of the Universe with inverted hierarchy, non-unitarity case in one flavored leptogenesis regime.

favored in the light of baryon asymmetry. In the three-flavor regime, from Fig. 5.17 we find that NH structure of neutrino masses, non-unitarity textures of PMNS matrix can give rise to correct baryon asymmetry of the Universe, $5.8 \times 10^{-10} < Y_B < 6.6 \times 10^{-10}$, if the lightest $\nu$ mass $m_1$ lies around 0.065 eV to 0.07 eV, 0.08 eV to 0.085 eV, Here $M_1 = 10^8$ GeV. It can be noted from Fig. 5.19 that in the three-flavor regime, NH structure of neutrino masses, unitarity Textures of PMNS matrix one cannot give rise to correct BAU, for $M_1 = 10^8$ GeV. Here $Y_B$ is in the order of $10^{-20}$ which is very small compared to the allowed range as set by the Planck data [243].

5.4 Conclusions

To conclude, in this work, we have considered the possibility that the neutrino mixing matrix (considering charged lepton mass matrix to be diagonal), $U_{PMNS}$ could be non unitary, and then calculated the limits on non unitary parameters $\eta_{\mu e}$, $\eta_{\tau e}$ and $\eta_{\tau \mu}$ [Table 5.1] from latest constraints on branching ratios of cLFV decays. Baryogenesis through leptogenesis is believed to be responsible for producing the matter − antimatter asymmetry present in
5.4 Conclusions

Fig. 5.6: Variation of lightest \( \nu \) mass \( m_3 \) against Dirac CP phase \( \delta_{CP} \) and Majorana phase \( \alpha \) in case of IH, non-unitarity case in one flavored leptogenesis regime.

the present day universe, which can be expressed through parameter \( Y_B \) (baryon to photon ratio). We then analysed how the non unitarity of \( U_{PMNS} \) can affect leptogenesis, and hence calculate the values of lightest \( \nu \) mass, dirac CPV phase \( \delta_{CP} \) and Majorana phases \( \alpha \) and \( \beta \), such that \( Y_B \) lies in the present day constraints \( (5.8 \times 10^{-10} < Y_B < 6.6 \times 10^{-10}) \). This was done using type I see saw mechanisms for producing light \( \nu \) masses. Above analysis was done for different cases — NH \( \nu \) masses, unitary \( U_{PMNS} \); NH \( \nu \) masses, non-unitary \( U_{PMNS} \); IH \( \nu \) masses, unitary \( U_{PMNS} \); IH \( \nu \) masses, non unitary \( U_{PMNS} \). We discussed these issues for unflavored, two flavor and three flavor leptogenesis regimes, for which \( M_1 \geq 10^{12} \) GeV, \( M_1 < 10^{12} \) GeV and \( M_1 < 10^{9} \) GeV respectively where \( M_1 \) is the lightest of the three heavy right handed Majorana neutrinos, whose out of equilibrium decays produces lepton asymmetry, which in turn can be converted to BAU. We found that except the NH \( \nu \) masses, unitary \( U_{PMNS} \) in three flavored leptogenesis case, in all other cases, we get values of lightest \( \nu \) mass satisfying \( \sum_i m(\nu_i) < 0.23 \) eV, such that BAU also lies in presently allowed values \( 5.8 \times 10^{-10} < Y_B < 6.6 \times 10^{-10} \). Thus we have predicted values of lightest \( \nu \) mass, (Tables 5.2-5.4), for both the hierarchies, which is still unknown experimentally.
5.4 Conclusions

Fig. 5.7: Scatter plot of the lightest neutrino mass against the baryon asymmetry of the Universe and Dirac CP phase $\delta_{CP}$ with inverted hierarchy, unitarity case in one flavored leptogenesis regime. For $Y_B$ in the order $10^{-10}$, the lightest $\nu$ mass $m_3$ is concentrated in the region 0.053 eV to 0.062 eV.

Fig. 5.8: Variation of lightest neutrino mass $m_3$ with Majorana phase $\alpha$ and $\beta$ for IH, Unitarity texture of $U_{PMNS}$ in one flavor leptogenesis regime.
Fig. 5.9: Scatter plot of the lightest neutrino mass $m_1$ against the baryon asymmetry of the Universe with normal hierarchy, non-unitarity case in two flavored leptogenesis regime.

Fig. 5.10: Variation of lightest neutrino mass $m_1$ against Dirac CP phase $\delta_{CP}$ and Majorana phase $\alpha$ for normal hierarchy, non-unitarity texture of $U_{PMNS}$ in the two flavored leptogenesis regime.
5.4 Conclusions

Fig. 5.11: Variation of the lightest neutrino mass $m_1$ against the baryon asymmetry of the Universe with normal hierarchy, unitarity in two flavor regime of leptogenesis.

Fig. 5.12: Variation of lightest neutrino mass $m_1$ against Dirac CP phase $\delta_{CP}$ and Majorana phase $\alpha$ for normal hierarchy, unitarity case, in the two flavored leptogenesis regime.
Fig. 5.13: Variation of the lightest neutrino mass $m_3$ against the baryon asymmetry of the Universe with inverted hierarchy, non-unitarity case in two flavor regime of leptogenesis.

Fig. 5.14: Variation of lightest neutrino mass $m_3$ against Dirac CP phase $\delta_{CP}$ and Majorana phase $\alpha$ for inverted hierarchy, non-unitarity case, in the two flavored leptogenesis regime.
Fig. 5.15: Variation of the lightest neutrino mass $m_3$ against the baryon asymmetry of the Universe with inverted hierarchy, unitarity case in two flavor regime of leptogenesis.

Fig. 5.16: Variation of lightest neutrino mass $m_3$ against Dirac CP phase $\delta_{CP}$ and Majorana phase $\alpha$ for inverted hierarchy, unitarity case, in the two flavored leptogenesis regime.
Fig. 5.17: Variation of the lightest neutrino mass $m_1$ against the baryon asymmetry of the Universe with normal hierarchy, non-unitarity case in three flavor regime of leptogenesis.

Fig. 5.18: Variation of lightest neutrino mass $m_1$ against Dirac CP phase $\delta_{CP}$ and Majorana phase $\alpha$ for normal hierarchy, non-unitarity texture, in the three flavored regime, in the light of recent baryon to photon ratio of the Universe.
Fig. 5.19: Scatter plot of the lightest neutrino mass $m_1$ against the baryon asymmetry of the Universe with normal hierarchy, unitarity case in three flavor regime of leptogenesis.
5.4 Conclusions

### Table 5.2

<table>
<thead>
<tr>
<th>Case</th>
<th>$m_{\text{lightest}}$</th>
<th>OneFlavor</th>
</tr>
</thead>
<tbody>
<tr>
<td>NH, non-unitarity</td>
<td>0.0018 eV to 0.0023 eV</td>
<td>✓</td>
</tr>
<tr>
<td>NH, unitarity</td>
<td>0.048 eV to 0.056 eV</td>
<td>✓</td>
</tr>
<tr>
<td>IH, non-unitarity</td>
<td>0.05 eV to 0.054 eV</td>
<td>✓</td>
</tr>
<tr>
<td>IH, unitarity</td>
<td>0.053 eV to 0.062 eV</td>
<td>✓</td>
</tr>
</tbody>
</table>

We also have predicted values of CPV phase $- \delta_{CP}$ (Dirac phase) and $\alpha$ and $\beta$ (Majorana phases), which are also unknown so far. Though Majorana phases do not affect $\nu$ oscillation probability, they may affect $\nu$ mass measurements in $0\nu\beta\beta$ experiments. Hence the results presented in this work are important, keeping in view that in future, experiments will be endeavoring to measure the values of absolute value of neutrino mass, and CP violating phase $\delta_{CP}$ and $\alpha, \beta$, Majorana phases. In the pattern normal hierarchy $\nu$ masses, non-unitarity case in three flavored leptogenesis regime, we find that $m_{\text{lightest}}$ in the region 0.08 eV to 0.085 eV is however disfavored by the constraints on the absolute scale of $\nu$ masses, which is $\sum_i m(\nu_i) < 0.23\text{eV}$ [37] from cosmology, WMAP, Planck data. Nevertheless this region is

### Table 5.3

<table>
<thead>
<tr>
<th>Case</th>
<th>$m_{\text{lightest}}$</th>
<th>TwoFlavor</th>
</tr>
</thead>
<tbody>
<tr>
<td>NH, non-unitarity</td>
<td>0.023 eV to 0.03 eV, 0.034 eV to 0.038 eV, 0.058 eV to 0.06 eV</td>
<td>✓</td>
</tr>
<tr>
<td>NH, unitarity</td>
<td>0.023 eV to 0.037 eV , 0.06 eV</td>
<td>✓</td>
</tr>
<tr>
<td>IH, non-unitarity</td>
<td>0.062 eV to 0.07 eV</td>
<td>✓</td>
</tr>
<tr>
<td>IH, unitarity</td>
<td>0.063 eV</td>
<td>✓</td>
</tr>
</tbody>
</table>
favored when one considers bounds on sum of the absolute $\nu$ mass from tritium beta decay
$\sum_i m(\nu_i) < 2\text{eV}$ [37].
5.4 Conclusions

<table>
<thead>
<tr>
<th>Case</th>
<th>$m_{\text{lightest}}$</th>
<th>ThreeFlavor</th>
</tr>
</thead>
<tbody>
<tr>
<td>NH, non-unitarity</td>
<td>0.065 eV to 0.07 eV, 0.08 eV to 0.085 eV</td>
<td>✓</td>
</tr>
<tr>
<td>NH, unitarity</td>
<td>0.01 eV to 0.1 eV for $Y_B$ in the range $10^{-20}$</td>
<td>×</td>
</tr>
</tbody>
</table>

Table 5.4: Our calculated results for $m_{\text{lightest}}$ with inverted hierarchy, normal hierarchy and three flavored leptogenesis. The symbol ✓ (×) is used when $Y_B$ is within (not within) updated BAU range.

Future measurements related to Dirac CPV phase in neutrino experiments will validate or contradict some of the results presented here. Our analysis in this work only provides a benchmark for consistent works affiliated to model building.