Neutrino masses and mixings using updated values of running quark and lepton masses

2.1 Introduction

Current research in experimental and theoretical High Energy Physics aims at explaining the origin of all fermions masses and mixings including those of neutrinos. Neutrinos have masses and mix with each other and oscillate to other flavours. Grand Unified theories (GUTs) like SO(10) which unify strong and electroweak interactions are considered as prime
candidates for describing neutrino masses [128, 129]. In SO(10) all the known fermions of a given generation including the right handed neutrino are present in a single sixteen dimensional spinorial representation of the group. These theories require running masses and mixings of quarks and charged leptons at GUT scales for calculating neutrino masses. SO(10) GUT has a number of desirable features which helps in describing physics beyond the Standard Model. Non zero neutrino masses, the see-saw scale and a high B-L scale fits naturally in a grand unified model group based on the gauge group SO(10). SO(10) model contains the L-R symmetric unification group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ which provides the origin of parity violation in nature. In a class of minimal SO(10) model both the Dirac neutrino masses and Majorana neutrino masses are related to observables in the charged fermion sector. We use a simple Higgs system with one 10 and 126 which have Yukawa couplings to Fermions $16_F (Y_{10} 10_H + Y_{126} 126_H)$ [130]. The 10 is for quark and lepton masses, 126 is required for seesaw mechanism. The breaking of SO(10) via $G_{224}$ is achieved by either 54 or 210 dimensional Higgs. The Fermion Higgs coupling of the SO(10) model can be written down as in Ref [129]

$$L_Y = h_{ab} \psi_a \psi_b H + f_{ab} \psi_a \psi_b \bar{\Delta} + H.C$$

(2.1)

Where $H$ is the 10 dimensional Higgs of SO(10) and $\bar{\Delta}$ is the 126-plet of Higgs. Here $\psi_a$, $a=1-3$ denote the three families of fermions belonging to the 16 dimensional representation of SO(10). Under $G_{224}$,

$$\begin{align*}
126 &= (1,1,6) + (1,3,10) + (3,1,\overline{10}) + (2,2,15) \\
10 &= (1,1,6) + (2,2,1)
\end{align*}$$

(2.2) (2.3)
where (1,3,10) and (2,2,15) component is denoted by $\Delta_R$ and $\Sigma$ respectively. $<\Delta_R^\nu> = \nu_R = 10^{12}$ GeV breaks the intermediate symmetry down to the SM and generates Majorana neutrino mass. The scalar potential contains a crucial term [129]

$$V = \lambda \Delta \bar{\Delta} H$$  \hspace{1cm} (2.4)

which induces vev’s for the standard doublets in the $\Sigma$ multiplet of 126. In above SO(10) model , the neutrino mass matrix is given by the type II seesaw formula, that contains the induced triplet vev which dominates the neutrino mass matrix. From Eq. 2.1 it can be shown that [94]

$$M_\nu = a (m_l - m_d)$$  \hspace{1cm} (2.5)

Here parameter "a" contains various Yukawa couplings and the parameter tan $\beta$. The chapter is organized as follows. In the Section 2.2 we give formula for type II seesaw mechanism using induced vev. In the Section 2.3 we calculate neutrino oscillation parameters using Eq. 2.5. In the Section 2.4 we give our conclusions.

### 2.2 Type II Seesaw Mechanism

The $SO(10)$ invariant Superpotential giving the Yukawa Couplings of the 16 dimensional matter spinor $\psi_i$ with the Higgs set $H_{10} = 10$ and $\Delta = \overline{126}$ is [94]

$$W_Y = h_{ij} \psi_i \psi_j H_{10} + f_{ij} \overline{\psi_i} \psi_j \overline{\Delta}$$  \hspace{1cm} (2.6)

where h and f are symmetric matrices. In the type I seesaw mechanism the neutrino mass matrix is given by

$$M_\nu = -M_D^T M_{NR}^{-1} M_{DR}$$  \hspace{1cm} (2.7)
where $M_{NR} = f \nu_{B-L}$ and $\nu_{B-L}$ is the see saw scale. In SO(10) the true seesaw formula has a second term which comes from an induced $SU(2)_L$ triplet vev. The type II see saw mechanism has the following form [94]

$$M_\nu = f \sigma_L - M^D_\nu M^{-1}_{NR} M^T_D$$

(2.8)

$$\sigma_L = \lambda \frac{\nu^2}{\nu_{B-L}}$$

(2.9)

$\nu$ is the $SU(2)_L$ breaking scale. $\lambda$ is a combination of parameters in Higgs potential. We note the decomposition of $\overline{126}$ under the group $SU(2)_L \times SU(2)_R \times SU(4)_C$

$$\overline{126} = (1, 1, 6) + (2, 2, 15) + (3, 1, \overline{10}) + (1, 3, 10)$$

(2.10)

The $SU(2)_L$ triplet $\Delta_L = (3, 1, \overline{10})$ couples to the left handed multiplet $\psi_L = (2, 1, 4)$ of the 16 dimensional SO(10) containing the matter spinor i.e $\psi_L \psi_L \Delta_L$. The mass of the right handed neutrino originates from the coupling of $\Delta_R = (1, 3, 10)$ of $\overline{126}$ to right handed fermion multiplet $\psi^C_R = (1, 2, 4)$ i.e $\psi_R \psi_R^C \Delta_R$ [131]. $\Delta_R$ produces the second term in the type II seesaw formula. The sumrule Eq. 2.5 holds when triplet vev contribution to the neutrino mass matrix dominates in the type II seesaw mechanism.

### 2.3 Calculation of Neutrino Oscillation Parameters

In our calculation of neutrino oscillation parameters, we use the updated values of running quark and lepton masses from Ref. [95] and are given in Table 2.1. Though we have used $\tan \beta = 55$, in this work, same calculations can be done for other values of $\tan \beta = 10$ as well. Then using Eq. 2.5 we calculate neutrino oscillation parameters, which are presented in Fig. 2.1. In Fig. 2.1 in the first figure we have shown our results for $\sin^2 \theta_{23}$ and $\Delta m^2_{23}$. In the second figure we have shown the results for $\sin^2 \theta_{12}$ and $\Delta m^2_{12}$ and in the third figure the
2.3 Calculation of Neutrino Oscillation Parameters

<table>
<thead>
<tr>
<th>Fermion</th>
<th>Mass</th>
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<tbody>
<tr>
<td>$m_u$</td>
<td>$0.3963^{+0.1506}_{-0.1284}$ MeV</td>
</tr>
<tr>
<td>$m_c$</td>
<td>$0.1932^{+0.0240}_{-0.0246}$ GeV</td>
</tr>
<tr>
<td>$m_t$</td>
<td>$80.4472^{+2.9128}_{-2.6158}$ GeV</td>
</tr>
<tr>
<td>$m_d$</td>
<td>$0.9284^{+0.3838}_{-0.3754}$ MeV</td>
</tr>
<tr>
<td>$m_s$</td>
<td>$17.6097^{+4.8972}_{-4.6737}$ MeV</td>
</tr>
<tr>
<td>$m_b$</td>
<td>$1.2424^{+0.0626}_{-0.0572}$ GeV</td>
</tr>
<tr>
<td>$m_e$</td>
<td>$0.3569^{+0.0001}_{-0.0001}$ MeV</td>
</tr>
<tr>
<td>$m_\mu$</td>
<td>$75.3570^{+0.0744}_{-0.0682}$ MeV</td>
</tr>
<tr>
<td>$m_\tau$</td>
<td>$1.6459^{+0.0114}_{-0.0206}$ GeV</td>
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Table 2.1: Values of Masses in MSSM, 2 Loop, $\tan \beta = 55$ [UPDATED] fermion masses used from [95] at GUT scale = $2 \times 10^{16}$ GeV

results are given for $\Delta m^2_{23}$ and $\theta_{13}$. We know the latest values of global fit values of neutrino oscillation parameters are [15–20]

$$\Delta m^2_{21}[10^{-5}eV^2] = 7.60^{+0.19}_{-0.18}$$

$$|\Delta m^2_{31}|[10^{-3}eV^2] = 2.48^{+0.05}_{-0.05}(2.38^{+0.05}_{-0.06})$$

$$\sin^2 \theta_{12} = 0.323 \pm 0.016$$

$$\sin^2 \theta_{23} = 0.567^{+0.032}_{-0.124}(0.573^{+0.025}_{-0.039})$$

$$\sin^2 \theta_{13} = 0.0226 \pm 0.0012(0.0229 \pm 0.0012)$$

(2.11)

We find that values of oscillation parameters as calculated by us in Fig. 2.1 agree well with latest global fit values.
Fig. 2.1: Curves are plotted within 1σ errors of the best fit values of $\Delta m^2_{23}$ and $\sin^2 \theta_{23}$, $\Delta m^2_{12}$ and $\sin^2 \theta_{12}$, $\Delta m^2_{23}$ and $\theta_{13}$ respectively.
2.4 Conclusion

Running values of quark and lepton masses at higher scales in Grand Unified Theories are very important, for building models of fermion masses, and to calculate neutrino masses and mixings. To conclude, in this chapter we have calculated neutrino oscillation parameters in minimal SO(10) theory, using updated values of running quark and lepton masses. We find that these values agree with latest global fit values as given in Ref. [15–20] of these parameters. We would like to do Renormalisation Group Evolution study of the neutrino oscillation parameter in our future work.