Intrinsic Localized Spin Modes in a 2-Dimensional FM Spin System with Biquadratic Interactions

6.1 Introduction

In chapter 4, we have studied intrinsic localised spin modes associated with a 2D model of Heisenberg ferromagnet with bilinear interactions. We observed interesting results which motivated us to investigate whether the addition of biquadratic interactions can affect the spin wave propagation. Hence in this chapter, we investigate the formation of intrinsic localized spin modes in a square lattice model of ferromagnetic spin system incorporating biquadratic interactions. Spin wave pattern is analysed for different values of interaction parameters. Also we find the conditions for the soliton stability using linear stability analysis. We
obtain the characteristic of MI in the form of typical dependence of the instability growth rate on the perturbation wave numbers and the system.

6.2 Dynamical Equation

The Hamiltonian for a square lattice model of ferromagnetic spin system with varying bilinear and biquadratic exchange interactions is given by Eq.(5.1).

Following the same procedure as in chapter 4, we recast the Hamiltonian in to the following dimensionless form

\[
H = - \sum_{n,m} \left[ \frac{J}{2} (S_{n,m}^+ S_{n+1,m}^- + S_{n,m}^- S_{n+1,m}^+ + 2S_{n,m}^z S_{n+1,m}^z) 
\right. \\
+ \frac{J_1}{2} (S_{n,m}^+ S_{n,m+1}^- + S_{n,m}^- S_{n,m+1}^+ + 2S_{n,m}^z S_{n,m+1}^z) + \\
\left. \frac{J_2}{2} (S_{n,m}^+ S_{n+1,m+1}^- + S_{n,m}^- S_{n+1,m+1}^+ + 2S_{n,m}^z S_{n+1,m+1}^z) + \\
\frac{J'}{4} (S_{n,m}^+ S_{n+1,m}^- S_{n,m}^+ S_{n+1,m}^- + S_{n,m}^- S_{n+1,m}^+ S_{n,m}^- S_{n+1,m}^+) + \\
4S_{n,m}^z S_{n+1,m}^- S_{n,m}^z S_{n+1,m}^+ + 4S_{n,m}^- S_{n+1,m}^z S_{n,m}^z S_{n+1,m}^+ + \\
4S_{n,m}^z S_{n+1,m}^- S_{n,m}^z S_{n+1,m}^+ \right]
\]
The nonlinear equation of motion for $S^+_{n,m}$ is then

$$i\hbar \frac{\partial S^+_{n,m}}{\partial t} = -\frac{1}{2}[(J + 2J')S^z_{n,m}(S^+_{n+1,m} + S^+_{n-1,m}) + S^+_{n,m}(S^+_{n+1,m} + S^+_{n-1,m})]$$

$$+(J_1 + 2J'_1)S^z_{n,m}(S^+_{n,m+1} + S^+_{n,m-1}) + S^+_{n,m}(S^+_{n,m+1} + S^+_{n,m-1}) +$$

$$+(J_2 + 2J'_2)S^z_{n,m}(S^+_{n+1,m} + S^+_{n-1,m}) + S^+_{n,m}(S^+_{n+1,m})$$

$$+S^z_{n-1,m-1}] - (A + 2A')S^z_{n,m}S^+_{n,m}.$$  \hspace{1cm} (6.2)

Using Eqs.(4.3)-(4.5), we write

$$\Omega_{S,n,m} = (J + 2J')S\sqrt{1 - S^2_{n,m}}(S_{n+1,m} + S_{n-1,m}) + S_{n,m}(\sqrt{1 - S^2_{n+1,m}} +$$

$$\sqrt{1 - S^2_{n-1,m}}] + (J_1 + 2J'_1)S\sqrt{1 - S^2_{n,m}}(S_{n,m+1} + S_{n,m-1}) +$$

$$S_{n,m}(\sqrt{1 - S^2_{n,m+1}} + \sqrt{1 - S^2_{n,m-1}}] + (J_2 + 2J'_2)S\sqrt{1 - S^2_{n,m}}$$

$$(S_{n+1,m} + S_{n-1,m}) + S_{n,m}(\sqrt{1 - S^2_{n+1,m}} +$$

$$\sqrt{1 - S^2_{n-1,m}}] - (A + 2A')SS_{n,m}\sqrt{1 - S^2_{n,m}}.$$$  \hspace{1cm} (6.3)

Eq. (6.3) determines the envelope of the spin modes.

The spin wave pattern observed for different values of $\Omega$, $J$, $J_1$ and $J_2$ is found to be similar to that in the case of Heisenberg spin system with only bilinear interaction. The parameter $J'_2$ has an interesting effect in the localization process.
Figure 6.1: Spin deviation $S_{n,m}$ versus site number for $J = J_1 = 12$, $J_2 = 8.5$, $J' = J'_1 = 6$, $A = 0.1$, $A' = 1$, $\Omega = -3.95$, $S = 1$ (a) $J'_2 = 6$ (b) $J'_2 = 7$ (c) $J'_2 = 10$ (d) $J'_2 = 12$. 
of spin wave. As $J'_2$ increases the stability increases and the spin wave gets completely localized for $J'_2 = 12$. This behaviour is described in Figure 6.1.

6.3 Linear Stability Analysis

To study the linear stability of spin waves, we consider the continuum equation given by Eq.(5.10).

6.3.1 Analytical results

Using Eq.(4.8) in Eq. (5.10) we obtain the nonlinear dispersion relation

$$\omega = k_1^2 \alpha_1 + k_2^2 \alpha_2 + k_1 k_2 \alpha_3 + \alpha_4 U_0^2 - \alpha_5 (k_1^4 + 6k_1^2 U_0) - \alpha_6 (k_2^4 + 6k_2^2 U_0) + \alpha_7 (2k_1^2 k_2 + 3k_1 k_2^2) + \alpha_8 U_0^2 k_1^2 + \alpha_9 k_1 k_2 U_0^2 - \alpha_{10} k_1 k_2 U_0^2 + \alpha_{11} U_0^2 k_2^2 + \alpha_{12} U_0^4 - \alpha_{13} U_0^2 k_1^2 - \alpha_{14} k_1 k_2 U_0^2 + \alpha_{15} U_0^2 k_1^2 - \alpha_{16} U_0^2 k_2^2 + \alpha_{17} U_0 k_2^2. \quad (6.4)$$

To analyze the MI of plane wave solution to Eq.(6.4), we perturb by introducing Eq.(4.9) and assuming $u_1(x, y, t) = ae^{i(Qx+Qy-\Omega t)}$, $u_2(x, y, t) = be^{i(Qx+Qy-\Omega t)}$ in Eq.(6.4). Using Eq.(4.10) and neglecting nonlinear terms, we obtain

$$i e u_0 b \Omega + e a \Omega + \alpha_1 (-2i e k_1 u_0 b Q - i e u_0 Q^2 b - 2e a k_1 Q - Q^2 e a) + \alpha_2 (-2i e k_2 u_0 b Q - i e u_0 Q^2 b - 2e a k_2 Q - Q^2 e a) + \alpha_3 (-e u_0 k_1 b Q - e k_1 Q a - e Q^2 a - i e u_0 k_2 Q b) - 2\alpha_4 e u_0^2 a + \alpha_5$$
\begin{align*}
&(4ieu_0 k_1^3 bQ + 6ie k_1^2 u_0 Q^2 b + 4ie k_1 u_0 Q^3 b + ieu_0 Q^4 b + 3\epsilon ak_1^3 Q + \\
&5\epsilon k_1^2 Q^2 a + 4\epsilon Q^3 ak_1 + \epsilon Q^4 a + 18\epsilon u_0^2 k_1^2 a + 12ieu_0^3 k_1Q + 12eu_0^3 k_1 a Q) \\
&+ \alpha_6 (4ieu_0 k_2^3 bQ + 6ie k_2^2 u_0 Q^2 b + 4ie k_2 u_0 Q^3 b + ieu_0 Q^4 b + 3\epsilon ak_2^3 Q + \\
&5\epsilon k_2^2 Q^2 a + 4\epsilon Q^3 ak_2 + \epsilon Q^4 a + 18\epsilon u_0^2 k_2^2 a + 12ieu_0^3 k_2 bQ + 12eu_0^3 k_2 a Q) \\
&+ \alpha_7 (-6ieu_0 k_1^2 k_2 bQ - 6ie k_1 k_2 u_0 Q^2 b - 2ie k_2 u_0 Q^3 b - 6ie k_1^2 u_0 Q^3 b \\
&- 2ieu_0 k_1^3 bQ - 6ieu_0 k_1 Q^3 b - 2ieu_0 Q^4 b - 6ek_1^2 k_2 a Q - 6ek_1 Q^2 a \\
&- 2ek_2 Q^3 a - 2ek_2^3 a Q - 6ek_1 Q^3 a - 6ek_1 Q^3 a - 2\epsilon Q^4 a - 6ieu_0 k_1 k_2^2 bQ \\
&- 3ieu_0 k_2^2 Q^2 b - 6ieu_0 k_2^2 k_2 bQ - 12ieu_0 k_1 k_2 Q^3 b - 6ieu_0 k_2 Q^3 b \\
&- 3ieu_0 k_1^2 Q^2 b - 6ieu_0 k_1 Q^3 b - 3ieu_0 Q^4 b - 3ek_1 k_2^2 a Q - 3ek_2^2 Q^2 a \\
&- 3ek_1 k_2 a Q - 6ek_1^2 k_2 a Q - 12ek_1 k_2 Q^3 a - 6ek_2 Q^3 a - 3iek_1^2 k_2 Q^2 a \\
&- 6ieu_0 k_2^2 Q^3 b - 6ieu_0 k_1 k_2 Q^2 b - 6ieu_0 k_2 Q^3 - 6ek_2 Q^3 a - 2ek_1 Q^3 a \\
&- 2\epsilon Q^4 a) \quad (6.5)
\end{align*}

Finally we obtain the dispersion relation of the amplitude modulation of the plane wave as

\begin{equation}
\Omega^2 U_0 + \Omega (M + U_0 N) + MN = 0 \quad (6.6)
\end{equation}

with

\[ M = \alpha_1 (-2k_1 Q - Q^2) + \alpha_2 (-2k_2 Q - Q^2) + \alpha_3 Q(-k_1 - \]
\[ N = \alpha_1 Q U_0(2k_1 - Q) + \alpha_2 Q U_0(2k_2 - Q) + \alpha_3 Q U_0(k_1 - k_2 - Q) \]

\[ \alpha_5 Q U_0(k_1^3 + 6k_1^2 Q + 4k_1 Q^2 + Q^3 + 12U_0^2 k_1) + \alpha_6 Q U_0(k_2^3 + 6k_2^2 Q + 4k_2 Q^2 + Q^3 + 12U_0^2 k_2) + \alpha_7 Q U_0(-6k_1^2 k_2) \]

\[ -6k_1 k_2 Q - 2k_2 Q^2 - 6k_1^2 Q^2 - 2k_1^3 - 6k_1 Q^2 - 2Q^3 - 6k_1 k_2^2 \]

\[ -3k_2^3 Q - 6k_2^2 k_2 - 12k_1 k_2^2 Q^2 - 6k_2 Q^2 - 3k_2^3 Q - 6k_1 k_2^2 Q \]

\[ -3Q^3 - 3k_1^2 k_2 Q - 6k_1 k_2 Q^2 - 2k_2^3 - 6k_1 k_2^2 - 6k_2^2 Q \]

\[ -6k_1 k_2 Q - 6k_2 Q^2 + \alpha_8 Q k_1 U_0^3(-2 - Q) + \alpha_9 Q k_1 U_0^3(2k_1 + k_2 + Q) + \alpha_{10} Q U_0^3(2k_1 + k_2 + Q) + \alpha_{11} Q k_2 U_0^3(-2 - Q) - 2\alpha_{13} U_0^3 k_1 \]

\[ \alpha_{12}(-4U_0^4 - 3U_0^3 - 6U_0^2) + \alpha_{13} U_0(k_1^2 U_0 - 2k_1) + \alpha_{14} U_0^2(k_1 k_2 - 2k_1 - k_2 + Q) - 2\alpha_{15} U_0^2 k_1 Q \]

\[ + \alpha_{16} U_0^2 Q(-3k_2^2 - 2) - 2\alpha_{17} U_0^2 Q k_2, \quad (6.7) \]
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Figure 6.2: Regions of modulational instability (a) for different values of \( k_1 \) (i) \( k_1 = -0.2 \) (ii) \( k_1 = -0.8 \) (iii) \( k_1 = -1 \) (iv) \( k_1 = -1.4 \) (v) \( k_1 = -2 \), (b) for different values of \( k_2 \) (i) \( k_2 = -2 \) (ii) \( k_2 = -1 \) (iii) \( k_2 = -0.5 \) (iv) \( k_2 = 0.1 \).

where \( Q \) and \( \Omega \) are respectively an arbitrary wavenumber and the corresponding frequency of the perturbation. Solving Eq.(6.6) yields

\[
\Omega = \frac{-(M + U_0 N) \pm \sqrt{(M + U_0 N)^2 - 4U_0 MN}}{2U_0} \tag{6.9}
\]

6.3.2 Graphical results

Figure 6.2 represents the characteristic feature that the growth rate is being improved as the characteristic band width increased for the choice of parameters

\( J = J_1 = 12, J_2 = 8.5, J' = J'_1 = 6, J'_2 = 4.25, A = 0.1, A' = 1, \gamma = 0.1, U_0 = 0.1, k_2 = 0.001 \) and (i) \( k_1 = -0.2 \) (ii) \( k_1 = -0.8 \) (iii) \( k_1 = -1 \) (iv) \( k_1 = -1.4 \) and (v) \( k_1 = -2 \). Figure 6.3 shows the MI growth rate spectra for (i) \( k_2 = -2 \) (ii) \( k_2 = -1 \) (iii) \( k_2 = -0.5 \) and (iv) \( k_2 = 0.1 \) keeping \( k_1 = -1 \). It is observed
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Figure 6.3: Regions of modulational instability for different values of $k_2$ (i) $k_2 = -2$ (ii) $k_2 = -1$ (iii) $k_2 = -0.5$ (iv) $k_2 = 0.1$.

Figure 6.4: Variation of growth rate for different values of interaction parameter $J'$ (i) $J' = 0.1$ (ii) $J' = 3$ (iii) $J' = 9$ (iv) $J' = 15$. 
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Figure 6.5: Variation of growth rate for different values of interaction parameter $J'_2$ (i) $J'_2 = 0.1$ (ii) $J'_2 = 3$ (iii) $J'_2 = 5$ and (iv) $J'_2 = 7$.

Figure 6.6: Variation of growth rate for different values of lattice parameter $\gamma$ (i) $\gamma = -0.04$ (ii) $\gamma = -0.7$ (iii) $\gamma = 0.04$ (iv) $\gamma = 0.1$. 
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Figure 6.7: Variation of growth rate for different values of amplitude $u_0$ (i) $u_0 = 0.2$, (ii) $u_0 = 0.6$, (iii) $u_0 = 0.9$, (iv) $u_0 = 1$. 

From the figure that the growth rate is suppressed and the band width is shifted to the lower Q value for increasing values of $k_2$. This effect is inverse to the case with bilinear interaction. It predicts that the biquadratic interaction drastically changes the behaviour of the spin dynamics.

Next, we analyse the effect of biquadratic interaction parameters $J', J'_1$, $J'_2$. Figure 6.4 displays the variation of growth rate with respect to $J'$ i.e for (i) $J' = 0.1$ (ii) $J' = 3$ (iii) $J' = 9$ and (iv) $J' = 15$. The magnitude of growth rate and the band width increase as the strength of $J'$ increases. The interaction parameter $J'_1$ also exhibits similar effect and we do not present the details here. The variation of growth rate with various values of interaction parameter $J'_2$ is illustrated in Figure 6.5. The system parameters are (i) $J'_2 = 0.1$ (ii) $J'_2 = 3$ (iii) $J'_2 = 5$ and (iv) $J'_2 = 7$. This parameter also plays an important role in
modulating the MI growth rate similar to $J'$ but the width of the regime remains same.

Figure 6.6 depicts the variation of growth rate with the lattice parameter $\gamma$
(i) $\gamma = -0.04$ (ii) $\gamma = -0.7$ (iii) $\gamma = 0.04$ and (iv) $\gamma = 0.1$. We noticed that the increasing values of lattice parameter $\gamma$ increases the amplitude of MI growth rate which illustrates that addition of biquadratic interaction improves stability of soliton in spin dynamics.

The growth rate with perturbation amplitude (i) $u_0 = 0.2$, (ii) $u_0 = 0.6$, (iii) $u_0 = 0.9$, (iv) $u_0 = 1$ is shown in Figure 6.7. In this case the bandwidth increases with increase in perturbation amplitude unlike the case represented by Figure 4.10.

6.4 Conclusion

In this chapter, we investigate the existence of intrinsic localised modes in a square lattice model of FM spin system with biquadratic interactions. It is found that the characteristic feature of the dynamics of the lattice with biquadratic interaction depends on the values of the spin wave parameters. It is also noticed from the spin deviation curve that the addition of biquadratic interactions enhances localization of spin wave. Also we find the conditions for the soliton stability using linear stability analysis. We obtain the characteristics of MI in
the form of typical dependence of the instability growth rate on the perturbation wave number and the system parameters. In this case of FM spin system with biquadratic interactions, the analytical result displays that the instability growth rate decreases with the amplitude and the system parameters. We observe that addition of biquadratic interaction improves stability of soliton in spin dynamics.