Chapter- 3
Research Methodology

The present chapter has five sections. Section 3.1 presents the objectives of the study; section 3.2 discusses the hypotheses to be tested; section 3.3 describes data used; section 3.4 describes methodology employed for achieving the stated objectives; and section 3.5 concludes.

3.1 OBJECTIVES OF THE STUDY

In the last few decades, enormous literature has been produced concerning relationships between futures and spot markets. Chapter 2 of the present study reviews an extensive literature survey of these studies. Most of these studies have examined lead-lag relationship between a market index and its associated futures contracts. Some studies have also investigated volatility linkages between the two markets viz., futures and spot markets. However, only a few studies have investigated lead-lag relationship and volatility spillovers at the level of individual stocks. The present study is an attempt to fill this gap to some extent.
The study is an attempt to examine first and second moments relationships between spot and futures markets in India. More specifically, the main objectives of the study are as follows:

1. To examine long-term relationship between futures and spot markets.
2. To examine lead-lag relationship for returns between futures and spot markets.
3. To examine volatility spillovers between futures and spot markets.
4. To examine the nature and strength of relationship between futures and spot markets.

3.2 HYPOTHESES OF THE STUDY

The present study requires a number of hypotheses to be tested. The major hypotheses to be tested are given below in the statistical form:

Long-term Relationship between Futures and Spot Markets-

H₀: There is no cointegration between futures and spot markets.
H₁: Futures and spot markets are cointegrated.

Lead-lag Relationships between Futures and Spot Markets-

H₀: Spot market (returns) does not granger cause futures market (returns).
H₁: Spot market (returns) granger causes futures market (returns).
H₀: Futures market (returns) does not granger cause spot market (returns).
H₁: Futures market (returns) granger causes spot market (returns).

Volatility Spillovers between Futures and Spot Markets-

H₀: Volatility from one market does not spill over to the other market.
H₁: Volatility from one market spills over to the other market.
The above stated hypotheses are tested for CNX Nifty and CNX Nifty Futures as well as for all of their 50 constituent stocks.

3.3 DATA

One of the primary objectives of the present study is to examine relationships between spot and futures markets in India. Generally, spot and futures markets relationships are studied at returns and volatility levels. For returns relationships, the primary concern is to investigate which market reacts to restore long-term equilibrium relationship and whether there exists any lead-lag relationship between the two markets. It is commonly known that lead-lag relationship between spot and futures markets which does not last for more than half an hour. Therefore, to study lead-lag relations, it is necessary that high frequency data should be used. The present study has used 5-min transaction prices data for CNX Nifty and all of its fifty constituent stocks. The data has been obtained from National Stock Exchange's data vending partner Dotex International Ltd. For the present study, one year data from 1\textsuperscript{st} June 2012 to 31\textsuperscript{st} May 2013 has been used. The reason for choosing this time period is that when the data was obtained then it was the latest data available with the data provider. In the time period considered there were a total of 250 trading days. Out of these days, 3 special trading sessions (2 on Sunday and 1 on Diwali) and one unusual\(^1\) day has been removed. Thus, after excluding the four special/unusual days, data for 246 days have been used for carrying out the study.

The stocks used in the study are given in Table 3.1. NSE maintains a database of every single trade that takes place both in the cash market as well as in the derivatives market. From this database, 5-minutes transactions data has been filtered using R-

\(^1\) September 08, 2012 (Saturday), November 13, 2012 (Diwali) and May 11, 2013 (Saturday) were special trading sessions.
software. For futures market, data for three different time periods is available viz. 1-month (nearby month), 2-month (mid-month), and 3-month (far-month). Since the nearby contracts are more liquid and usually most actively traded, only data for the nearby contracts has been used.

Table 3.1: Components of CNX Nifty as on 1st June 2012-

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Symbol</th>
<th>Stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ACC</td>
<td>ACC Ltd.</td>
</tr>
<tr>
<td>2</td>
<td>AMBUJACEM</td>
<td>Ambuja Cements Ltd.</td>
</tr>
<tr>
<td>3</td>
<td>AXISBANK</td>
<td>Axis Bank Ltd.</td>
</tr>
<tr>
<td>4</td>
<td>BAJAJ-AUTO</td>
<td>Bajaj Auto Ltd.</td>
</tr>
<tr>
<td>5</td>
<td>BHARTIARTL</td>
<td>Bharti Airtel Ltd.</td>
</tr>
<tr>
<td>6</td>
<td>BHEL</td>
<td>Bharat Heavy Electricals Ltd.</td>
</tr>
<tr>
<td>7</td>
<td>BPCL</td>
<td>Bharat Petroleum Corporation Ltd.</td>
</tr>
<tr>
<td>8</td>
<td>CAIRN</td>
<td>Cairn India Ltd.</td>
</tr>
<tr>
<td>9</td>
<td>CIPLA</td>
<td>Cipla Ltd.</td>
</tr>
<tr>
<td>10</td>
<td>COALINDIA</td>
<td>Coal India Ltd.</td>
</tr>
<tr>
<td>11</td>
<td>DLF</td>
<td>DLF Ltd.</td>
</tr>
<tr>
<td>12</td>
<td>DRREDDY</td>
<td>Dr. Reddy's Laboratories Ltd.</td>
</tr>
<tr>
<td>13</td>
<td>GAIL</td>
<td>GAIL (India) Ltd.</td>
</tr>
<tr>
<td>14</td>
<td>GRASIM</td>
<td>Grasim Industries Ltd.</td>
</tr>
<tr>
<td>15</td>
<td>HCLTECH</td>
<td>HCL Technologies Ltd.</td>
</tr>
<tr>
<td>16</td>
<td>HDFC</td>
<td>Housing Development Finance Corporation Ltd.</td>
</tr>
<tr>
<td>17</td>
<td>HDFCBANK</td>
<td>HDFC Bank Ltd.</td>
</tr>
<tr>
<td>18</td>
<td>HEROMOTOCO</td>
<td>Hero MotoCorp Ltd.</td>
</tr>
<tr>
<td>19</td>
<td>HINDALCO</td>
<td>Hindalco Industries Ltd.</td>
</tr>
<tr>
<td>20</td>
<td>HINDUNILVR</td>
<td>Hindustan Unilever Ltd.</td>
</tr>
<tr>
<td>21</td>
<td>ICICIBANK</td>
<td>ICICI Bank Ltd.</td>
</tr>
<tr>
<td>22</td>
<td>IDFC</td>
<td>Infrastructure Development Finance Co. Ltd.</td>
</tr>
<tr>
<td>23</td>
<td>INFY</td>
<td>Infosys Ltd.</td>
</tr>
<tr>
<td>24</td>
<td>ITC</td>
<td>ITC Ltd.</td>
</tr>
<tr>
<td>25</td>
<td>JINDALSTEL</td>
<td>Jindal Steel &amp; Power Ltd.</td>
</tr>
<tr>
<td>26</td>
<td>JPASSOCIAT</td>
<td>Jaiprakash Associates Ltd.</td>
</tr>
<tr>
<td>27</td>
<td>KOTAKBANK</td>
<td>Kotak Mahindra Bank Ltd.</td>
</tr>
<tr>
<td>28</td>
<td>LT</td>
<td>Larsen &amp; Toubro Ltd.</td>
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<td>29</td>
<td>M&amp;M</td>
<td>Mahindra &amp; Mahindra Ltd.</td>
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<td>30</td>
<td>MARUTI</td>
<td>Maruti Suzuki India Ltd.</td>
</tr>
<tr>
<td>31</td>
<td>NTPC</td>
<td>NTPC Ltd.</td>
</tr>
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<td>32</td>
<td>ONGC</td>
<td>Oil &amp; Natural Gas Corporation Ltd.</td>
</tr>
<tr>
<td>33</td>
<td>PNB</td>
<td>Punjab National Bank</td>
</tr>
<tr>
<td>34</td>
<td>POWERGRID</td>
<td>Power Grid Corporation of India Ltd.</td>
</tr>
<tr>
<td>35</td>
<td>RANBAXY</td>
<td>Ranbaxy Laboratories Ltd.</td>
</tr>
</tbody>
</table>
3.4 METHODOLOGY

This section describes the methodology used to achieve the stated objectives of the study.

3.4.1 Methodology for Examining Returns Relationship between Spot and Futures Markets-

In this section methodology for studying returns relationship between spot and futures markets of CNX Nifty and its component stocks has been described.

Testing for Unit root-

Most of the economic and financial time series are unit root non-stationary. Further, most of these series have a single unit root. Series which have a single unit root become stationary after differencing once. Non-stationary series which become stationary after differencing once are called integrated of order one, i.e., I(1). Generally, a linear combination of integrated series is also integrated of the same order. However, the Granger representation theorem (1987) states that linear
combination of non-stationary I(1) series may be stationary i.e., I(0). Hence, for studying cointegration, the first step is to test the time series under consideration for their order of integration. For this purpose Augmented Dickey-Fuller (ADF) test (Dickey & Fuller 1979) has been used. The test equation for ADF is given below:

$$\Delta y_t = \alpha_0 + \psi y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-i} + \varepsilon_t \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.1)$$

In the above eq. if \( \psi = 0 \), then the series \( y_t \) is unit root non-stationary. As mentioned in Brooks (2008), owing to inherent instability of a non-stationary process, the usual t-statistics are no longer valid, therefore, to test whether \( \psi = 0 \), the critical values are derived from simulation. The critical values for ADF test based on simulation studies were tabulated by Dickey and Fuller (1979).

**Cointegration and error correction model**-

In a no arbitrage world there should be a long-term equilibrium relationship between spot and futures markets for any asset. If the series under consideration have a long-term equilibrium relationship which negates any arbitrage opportunity then it is said that such series are cointegrated. This long-run relationship or so called cointegration binds the series together and prevents them from wandering apart too far away. Though in the short-run, the series may deviate from each other, but arbitrage forces ensure that this deviation is corrected and the long-run equilibrium relationship is restored. Absence of cointegration implies that there is no long-term relationship between the series and they can very well wander apart without any bound. Since spot and futures prices represent prices of the same asset at two different points in time, they are expected to be cointegrated.
Let there be an $m$-dimensional vector $y_t, (y_{1t}, y_{2t}, \ldots, y_{mt})'$. If all the elements in $y_t$ are stationary i.e., I(0), then a VAR could be set up containing $p$-lags of each of the variables in $y_t$ as follows—

$$y_t = \beta_1 y_{t-1} + \beta_2 y_{t-2} + \cdots + \beta_p y_{t-p} + \epsilon_t \quad \text{......(3.2)}$$

However, if all the elements in $y_t$ above are I(1), then the above VAR(p) model is meaningless. If the number of components in a system is more than the number of unit roots in the system, then the system is said to be cointegrated. Differencing each of the individual components of $y_t$ to induce stationarity would result in over-differencing which creates problem of non-invertibility due to unit root introduced in its MA structure which in turn may lead to difficulties in parameter estimation (Tsay, 2010, p. 431). To overcome the problem of over-differencing, Engle and Granger (1987) suggested a transformation of the above Vector Autoregressive (VAR) model into an Error Correction Model (ECM). ECM exists if all the elements in $y_t$ are I(1) and at least one linear combination of these elements is stationary i.e., I(0). The transformed model is given below—

$$\Delta y_t = \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \cdots + \Gamma_{p-1} \Delta y_{t-(p-1)} + \epsilon_t \quad \text{......(3.3)}$$

Where,

$$\Pi = \left( \sum_{i=1}^{p} \beta_i \right) - I_m \quad \text{and} \quad \Gamma_i = - \sum_{j=i+1}^{p} \beta_j \quad \text{[where} \ i = 1, 2, \ldots, p - 1 \]$$

$I_m$ is an $m$-dimensional square identity matrix.
Further, in the above specification $\Pi$ can be represented as the product of two matrices $\alpha$ and $\beta'$ i.e., $\Pi = \alpha \beta'$. Replacing $\Pi$ by $\alpha \beta'$ the above model can be written as follows:

$$\Delta y_t = \alpha \beta' \Delta y_{t-1} + \cdots + \Gamma_{p-1} \Delta y_{t-(p-1)} + \epsilon_t \ldots \ldots \ldots \ldots (3.4)$$

It is to be noted that in the above specification all the terms are stationary. $\Delta y_t$ and all $\Delta y_{t-i}$ are stationary, since $y_t$ is I(1) and hence its first difference is stationary. Besides, the expression $\Delta y_{t-1}$ is a linear combination of the components of $y_t$ which is stationary. If no linear combination of the elements of $y_t$ is stationary, then there is no cointegration. In that case, the usual first difference of each of the components of $y_t$ would be used in a VAR model and no Error Correction Model (ECM) exists. For a cointegrated m-dimensional vector time series $y_t$ with r cointegrating relations, $\alpha$ and $\beta$ are m $\times$ r matrices of full rank.

There are many ways in which an error correction model can be constructed. In fact, Tsay (2010) has mentioned that one can use any $\alpha \beta' y_{t-s}$ for $1 \leq s \leq p$ with some modifications made to the coefficient matrices $\Gamma_i$. For example, the above VECM can also be represented as follows—

$$\Delta y_t = \Pi y_{t-p} + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + \cdots + \Gamma_{p-1} \Delta y_{t-(p-1)} + \epsilon_t \ldots \ldots \ldots \ldots (3.5)$$

Where,

$$\Pi = \left( \sum_{i=1}^{p} \beta_i \right) - l_m \text{ and } \Gamma_i = \left( \sum_{j=1}^{i} \beta_j \right) - l_m$$

$$\Delta y_t = y_t - y_{t-1}$$

$\Pi$ and $\Gamma_i$ are m$\times$m coefficient matrices
Testing for cointegration-

Once it is found that the time series believed to be cointegrated are I(1), the next step is to test whether these series are cointegrated or not. For this purpose, Johansen-Juselius (1990) cointegration procedure has been employed. The J-J procedure is described below:

**Johansen-Juselius Procedure for Cointegration Analysis**

Let \( y_t \) be an m-dimensional time series whose components are I(1). If it is thought that the components of the \( y_t \) are cointegrated, then a Vector Error Correction Model (VECM) of the following form can be set up—

\[
\Delta y_t = \Pi y_{t-p} + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + \cdots + \Gamma_{p-1} \Delta y_{t-(p-1)} + \xi_t \quad \ldots \ldots \quad (3.6)
\]

Johansen test for cointegration among the elements of \( y_t \) is performed by examining the rank of the \( \Pi \) matrix in the VECM above. Brooks (2008) mentioned that rank of the \( \Pi \) matrix is examined by eigenvalues derived from rank-restricted moment matrices. Johansen (1988) has proposed two likelihood ratio test statistics, namely trace, and maximum eigenvalue for examining the rank of the \( \Pi \) matrix.

1. **Trace cointegration Test**-

Johansen (1988) proposed a joint test, called Trace test, to determine the number of cointegrating vectors. Under trace test the null and alternative hypotheses are as follows:

\[
H_0: \text{rank } (\Pi) = r; \quad H_a: \text{rank } (\Pi) > r
\]

Where, \( r \) = number of cointegrating vectors

The trace statistic is formulated as follows:
2. Maximum eigenvalue Test-

Johansen (1988) also suggested a sequential procedure, called maximum eigenvalue test, to identify the number of cointegrating relations where separate tests are conducted on each eigenvalue. The null and alternative hypotheses under maximum eigenvalue test are as follows

\[ H_0: \text{rank } (\Pi) = r; \quad H_a: \text{rank } (\Pi) = r+1 \]

Where, \( r \) = number of cointegrating vectors

The maximum eigenvalue test statistic is formulated as follows:

\[ \lambda_{\text{max}}(r, r + 1) = -T \ln(1 - \hat{\lambda}_{r+1}) \ldots \ldots \ldots \ldots (3.8) \]

Johansen and Juselius (1990) have provided critical values for the two likelihood ratio test statistics. However, the distribution of \( \lambda_{\text{trace}} \) and \( \lambda_{\text{max}} \) statistics is non-standard and does not follow the usual chi-squared distribution, hence critical values for the tests are obtained via simulation.

Granger Causality-

At times, one might be interested in knowing whether changes in one variable cause changes in another variable. This question is addressed with the help of causality tests. Let \( y_t \) be a two-dimensional vector \((y_{1t}, y_{2t})'\). If history of \( y_{1t} \) is helpful in predicting \( y_{2t} \), it is said that \( y_{1t} \) causes \( y_{2t} \). Similarly, if past information about \( y_{2t} \) is useful for predicting \( y_{1t} \), then \( y_{2t} \) is said to granger cause \( y_{1t} \).

\[ y_{1t} = \phi_{10} + \sum_{i=1}^{p} \alpha_i y_{1,t-i} + \sum_{i=1}^{p} \beta_i y_{2,t-i} + \epsilon_{1t} \ldots \ldots \ldots \ldots (3.9) \]
\[ y_{2t} = \phi_{20} + \sum_{i=1}^{p} \gamma_i y_{1,t-i} + \sum_{i=1}^{p} \delta_i y_{2,t-i} + \epsilon_{2t} \ldots \ldots \ldots \ldots \quad (3.10) \]

In the above VAR model, equation for \( y_{1t} \) states that \( y_{1t} \) depends on its own past values plus past values of \( y_{2t} \). However, if all \( \beta_i \) are jointly zero, then it would imply that \( y_2 \) does not cause \( y_1 \). Similarly, if all \( \gamma_i \) are jointly zero, then it would mean that \( y_1 \) does not cause \( y_2 \). If \( y_1 \) causes \( y_2 \) but not vice versa, then there is uni-directional flow of information from \( y_1 \) to \( y_2 \). If both \( y_1 \) and \( y_2 \) causes each other, then there is bidirectional flow of information and a feedback relationship is said to exist.

To perform Granger Causality test, two models are estimated:

1. Unrestricted model for \( y_1 \).

\[ y_{1t} = \phi_{10} + \sum_{i=1}^{p} \alpha_i y_{1,t-i} + \sum_{i=1}^{p} \beta_i y_{2,t-i} + \epsilon_{1t} \ldots \ldots \ldots \ldots \quad (3.11) \]

2. Restricted model for \( y_1 \).

\[ y_{1t} = \phi_{10} + \sum_{i=1}^{p} y_{1,t-i} + \epsilon_{1t} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (3.12) \]

In the second specification above, we have restricted all \( \beta_i \) i.e., lagged coefficients of \( y_2 = 0 \). This joint restriction that all \( \beta_i \) are simultaneously zero can be tested in the framework of an F-test as follows

\[ F \text{- test} = \frac{RRSS - URSS}{p} \left( \frac{URSS}{T-2p} \right) \ldots \ldots \ldots \ldots \quad (3.13) \]

Where,

\[ URSS = \text{residual sum of squares from unrestricted regression.} \]
RRSS = residual sum of squares from restricted regression.

p = number of restrictions; here equal to number of lags of $y_{2t}$.

T = total number of observations.

$2p =$ number of regressors in the unrestricted regression; here $p$ lags of $y_{1t}$ and $p$ lags for $y_{2t}$.

The above test-statistic follows an F-distribution with $p$ and $T−2p$ degrees of freedom in numerator and denominator respectively. Similar procedure is followed for $y_{2t}$.

3.4.2 Methodology for studying volatility spillover between spot and futures markets-

For studying volatility spillovers between different time series, two approaches are commonly used-

1. Multivariate GARCH models

2. Granger causality based on Vector Autoregressive (VAR) models of squared returns or squared residuals.

Multivariate GARCH Model-

There is numerous literature which indicate that financial assets returns exhibit features of volatility clustering and time-varying conditional heteroscedasticity which is well captured by Autoregressive Conditional Heteroscedasticity (ARCH) and Generalised Autoregressive Conditional Heteroscedasticity (GARCH) type models. In spirit, multivariate GARCH models are very similar to univariate volatility models, except that they also model covariances. Furthermore, multivariate formulations of ARCH/GARCH models allow for examination of inter-market volatility linkages.
including volatility spillovers. In the literature, several formulations of Multivariate GARCH models have been proposed. The present study has used two different bivariate GARCH family of models given below-

1. **Bivariate GARCH (1,1)-**

The following bivariate GARCH (1,1) with student-t innovations has been used to examine volatility spillovers between the spot and futures markets.

\[ \varepsilon_t = \begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{bmatrix} | \Omega_{t-1} \sim \text{Student} - t(0, H_t, \nu) \] \hspace{1cm} (3.14)

\[ H_t = \begin{bmatrix} \sigma_{s,t}^2 & \rho \sigma_{s,t} \sigma_{f,t} \\ \rho \sigma_{f,t} \sigma_{s,t} & \sigma_{f,t}^2 \end{bmatrix} \] \hspace{1cm} (3.15)

\[ \begin{bmatrix} \sigma_{s,t}^2 \\ \sigma_{f,t}^2 \end{bmatrix} = \begin{bmatrix} \omega_s + \alpha_s \varepsilon_{s,t-1}^2 + \gamma_s \varepsilon_{f,t-1}^2 + \beta_s \sigma_{s,t-1}^2 \\ \omega_f + \alpha_f \varepsilon_{s,t-1}^2 + \gamma_f \varepsilon_{f,t-1}^2 + \beta_f \sigma_{f,t-1}^2 \end{bmatrix} \] \hspace{1cm} (3.16)

\[ \sigma_{s,t}^2 = \omega_s + \alpha_s \varepsilon_{s,t-1}^2 + \gamma_s \varepsilon_{f,t-1}^2 + \beta_s \sigma_{s,t-1}^2 \] \hspace{1cm} (3.17)

\[ \sigma_{f,t}^2 = \omega_f + \gamma_s \varepsilon_{s,t-1}^2 + \alpha_f \varepsilon_{f,t-1}^2 + \beta_f \sigma_{f,t-1}^2 \] \hspace{1cm} (3.18)

Where,

\[ \varepsilon_{s,t}, \varepsilon_{f,t} = \text{residuals from VECM for the spot and futures markets respectively.} \]

\[ \Omega_{t-1} = \text{set of information available at time } t-1. \]

\[ H_t = \text{variance-covariance matrix} \]

\[ \nu = \text{degrees of freedom} \]

\[ \rho = \text{coefficient of correlation between spot and futures markets.} \]

\[ \sigma_{s,t}^2, \sigma_{f,t}^2 = \text{conditional variance for the spot and futures markets respectively.} \]
The above sets of equations are jointly estimated by maximizing the following log likelihood function:

$$ L(\Theta) = \sum_{t=1}^{T} \ln \{ l_t(\Theta) \} \quad \cdots \quad \cdots \quad \cdots \quad (3.19) $$

$$ l_t(\Theta) = \frac{T[(2 + \nu)/2]}{T(\nu/2)[\pi(\nu - 2)]} |H_t|^{-1/2} \left[ 1 \pm \frac{1}{\nu - 2} \varepsilon_t |H_t|^{-1} \varepsilon_t \right]^{-(2+\nu)/2} \quad \cdots \quad \cdots \quad \cdots \quad (3.20) $$

In the conditional variance equation above $\alpha_s$ and $\alpha_f$ describe market specific volatility clustering for the spot and futures markets respectively. $\gamma_f$ and $\gamma_s$ measure volatility spillover from futures to spot market, and spot to futures market respectively. For modeling covariances, $\rho$ denotes constant conditional correlation as in Bollerslev (1990), Chan, Chan and Karolyi (1991), Tse (1999) among others. Student-t distribution has been used for capturing excessive kurtosis and fat tails commonly exhibited by returns of financial assets.

2. Bivariate EGARCH (1,1)-

To complement the results from bivariate GARCH (1,1), one more formulation of multivariate GARCH models has been employed. The model is bivariate EGARCH (1,1) used by Koutmos and Tucker (1996), Koutmos (1996) and Tse (1999) among others.

$$ \varepsilon_t = \begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{bmatrix} |\Omega_{t-1} \sim \text{Student} - t(0, H_t, \nu), \quad \cdots \quad \cdots \quad \cdots \quad (3.21) $$

$$ H_t = \begin{bmatrix} \sigma_{s,t}^2 & \rho \sigma_{s,t} \sigma_{f,t} \\ \rho \sigma_{f,t} \sigma_{s,t} & \sigma_{f,t}^2 \end{bmatrix} \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (3.22) $$

$$ \ln(\sigma_{st}^2) = \omega_s + \alpha_s G_{s,t-1} + \gamma_f G_{f,t-1} + \beta_s \ln (\sigma_{s,t-1}^2) \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (3.23) $$
\[
\ln\left(\sigma_{t}^{2}\right) = \omega + \gamma s_{t-1} + \alpha f_{t-1} + \beta \ln\left(\sigma_{t-1}^{2}\right) \quad \ldots \quad \ldots \quad \ldots \quad (3.24)
\]

\[
G_{s,t-1} = \left(\left|\xi_{s,t-1}\right| - E\left|\xi_{s,t-1}\right|\right) + \theta s_{s,t-1} \quad \ldots \quad \ldots \quad \ldots \quad (3.25)
\]

[where \(\xi_{s,t-1} = e_{s,t-1}/\sigma_{s,t-1}\) ]

\[
G_{f,t-1} = \left(\left|\xi_{f,t-1}\right| - E\left|\xi_{f,t-1}\right|\right) + \theta f_{f,t-1} \quad \ldots \quad \ldots \quad \ldots \quad (3.26)
\]

[where \(\xi_{f,t-1} = e_{f,t-1}/\sigma_{f,t-1}\) ]

Where,

\(\xi_{s,t-1}\) and \(\xi_{f,t-1}\) are standardized residuals for spot and futures markets respectively.

\(\theta s\) and \(\theta f\) are asymmetric coefficients for the spot and futures markets respectively.

Other symbols are defined in bivariate GARCH formulation above.

The bivariate-EGARCH above is estimated by maximizing the following log-likelihood function:

\[
L(\Theta) = -0.5(NT) \ln(2\pi) - 0.5 \sum_{t=1}^{T} \left(\ln|\Sigma_t| + \xi_t^\prime \Sigma_t^{-1} \xi_t\right) \quad \ldots \quad \ldots \quad \ldots \quad (3.27)
\]

Where,

\(N\) = number of equations (two in the present case)

\(T\) = number of observations

\(\xi_t^\prime = [\epsilon_{st}, \epsilon_{ft}]\) 1 \times 2 vector of innovations at time \(t\)

\(\Sigma_t\) = time – varying conditional variance – covariance matrix of innovations
Volatility Spillover based on VAR-

To complement the results from GARCH family of models, the present study has also examined volatility spillover in the framework of a vector autoregressive (VAR) model. The following bivariate-VAR has been used—

\[
\epsilon_{s,t}^2 = \phi_s + \sum_{i=1}^{k} \alpha_i \epsilon_{s,t-i}^2 + \sum_{i=1}^{k} \beta_i \epsilon_{s,t-1}^2 + \eta_{s,t} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.28)
\]

\[
\epsilon_{f,t}^2 = \phi_f + \sum_{i=1}^{k} \gamma_i \epsilon_{s,t-1}^2 + \sum_{i=1}^{k} \delta_i \epsilon_{f,t-1}^2 + \eta_{f,t} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.29)
\]

Where,

\(\epsilon_{s,t}, \epsilon_{f,t}\) = residuals from VECM for spot and futures markets respectively

\(\eta_{s,t}, \eta_{f,t}\) = white noise error terms

In the above formulation if all \(\beta_i\) are jointly found to be zero, then it would imply that there is no volatility spillover from futures to spot market. Similarly, if all \(\gamma_i\) are all jointly zero, it would mean that there is no volatility spillover from spot to futures market.

3.5 CONCLUSION

This chapter describes objectives and hypotheses of the study. Then, data and methodology used in the thesis to achieve the stated objectives are described. For studying the returns relationship between spot and futures markets, the study proposes to use Johansen cointegration analysis and Granger causality based on vector autoregression. For studying volatility spillovers between the two markets, the study proposes to use bivariate GARCH (1,1) and EGARCH (1,1) with constant conditional
correlation. In addition, to complement the results from GARCH type models, volatility spillover is also to be studied in the framework of a VAR model.