Pair Production in Nonabelian Gauge Fields-A Resonance Phenomenon - Model 2

Chapter 4

Pair Production in Nonabelian
Gauge Fields-A Resonance
Phenomenon -Model 2

In this chapter we calculate the \((q\bar{q})\) pair production probability in the colour flux tube model by taking the effect of non-Abelian interactions in the theory. By solving \(SU(2)\) Yang Mills equation we obtained an exact expression for electric field which is found to be time dependent and oscillating with a characteristic frequency \(\omega_0\) which depends on the amplitude of the field strength. By using WKB approximation in complex time we calculated the pair creation probability and it is seen that when ever the strength of the field \((gE)\) is comparable to the quark-aniquark energy \((p^2+m^2)\) the corresponding pair creation probability is maximum, and for the static field \(\omega_0 \rightarrow 0\), we recovered the famous Schwinger result.
4.1 Introduction

What happens in Ultra Relativistic Heavy Ion Collisions (URHIC)? The question has attracted a lot of attention supported by extensive research from the early days of strong interactions. In high energy Physics, the quark antiquark creation probability in hadron-hadron or nucleus-nucleus collisions had received great attention. It is believed that a large amount of energy is deposited in a small space-time region in all such collisions, stored in the form of color electric field energy, it is thought to be responsible for the production of quark antiquark (q̅q) pairs which eventually results in quark-gluon plasma (QGP). Here we discuss, once more, the gluons induced pair creation, in the presence of strong color electric field, using a new non-abelian solution. This approach we find, produce surprising results.

What are the advantages of this approach? The basic one is that the solution is non-abelian actually and shines over several others from authors [1-3] which are only models. The second advantage is that we do not rely or resort to any approximation, simply as we depend, on pure calculations. Final advantage is that it leads to a result consistent, with real physics, and when compared with former results from authors [2-5], our results are better.

4.2 Visualization of the Problem

When two high energy nucleons collide, they almost pass through each other, exciting themselves. But in addition in the space they pass through they leave behind a flux tube of deposited energy with great rapidity. This energy get
rapidly transformed into hadrons. The depositions of energy at higher rate is about $\frac{1}{3} GeV\,fm^{-3}$, as can be ascertained from proton-proton collision data. This increase can occur for energy deposition even if collision energy increase further, but it elongates the flux tube further.

However, a given region of flux tube can receive a much larger deposition of energy[6], when a multiple superposition of two heavy nuclei collides occur, to the tune of at least $\varepsilon \sim NA^{1/3} GeV\,fm^{-3}$, as can be deduced from simple geometrical arguments. This means that an energetic collision of two $^{238}U$ nuclei an average deposition of about $6 GeV\,fm^{-3}$, well above, what is required for plasma formation[7].

Pair production in the flux tube model[4,5,8-14], had been considered by many authors. In flux tubes, the strong electric color field that is set up, makes the vacuum unstable to pair production via Schwinger Mechanism[1]. The color field in the flux tube is transformed into the energy of pairs. In the collision, a large number of pairs and in addition gluons get realized, leading ultimately to the formation of a quark-gluon plasma(QGP). The self interaction of the gluons in the flux tube is likely to polarize the medium between two receding nuclei. This leads to characteristic normal mode of oscillations.

4.3 Solution of SU(2) Yang-Mills equation

According to our present level of understanding all fundamental interactions except gravity are described by non-Abelian gauge theories. The gauge bosons of various non-Abelian symmetries mediate different interactions. All
known fundamental fermions fields are divided into two broad classes. One class, called Leptons, such as the electrons, muon and neutrinos, do not have any strong interactions. In gauge theory parlance we can say that these are singlets under the strong gauge interaction gauge group. The other class, called quarks, can be described by $SU(3)$ symmetry and therefore have strong interactions. Under a global $SU(3)$ symmetry the three colors of quarks transform like a triplet. If try to gauge this color symmetry, we obtained a theory called Quantum chromodynamics (QCD). This will contain eight gauge fields $G^{a}_{\mu\nu}$, which are called gluons. To understand the features due to non-abelian effect one has to solve the Yang Mills equation. For simplicity here we consider the time dependent vacuum solution of $SU(2)$ Yang-Mills equation, which satisfies

$$\partial_{\mu}G_{a}^{\mu\nu} + g\varepsilon_{abc}A_{\mu b}G_{c}^{\mu\nu} = 0$$

Where the field tensor

$$G_{a}^{\mu\nu} = \partial^{\mu}A_{a}^{\nu} - \partial^{\nu}A_{a}^{\mu} + g\varepsilon_{abc}A_{c}^{\mu}A_{b}^{\nu}$$

$a, b, c$ are color indices which take values 1,2,3 and Lorentz indices $\mu, \nu = 0,1,2,3$ with metric $(1, -1, -1, -1)$. $\varepsilon_{abc}$ is an anti symmetric LeviCivita tensor. Taking the temporal gauge $A^{0} = 0$, space homogeneity ($\nabla \Phi = 0$), the Yang-Mills equation becomes

$$A_{a} + g^{2}A_{a}(A^{2} - A_{a}^{2}) = 0$$

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Where \( A^2 = A_1^2 + A_2^2 + A_3^2 \) We assume that all three colors vary in time in the same way:

\[
A_1(t) = A_2(t) = A_3(t) = A(t)
\]

Then the Yang-mills equation reduces to

\[
A(t) + 2g^2 A^3(t) = 0
\]

It is a non-linear equation in one degree of freedom and admitting an analytical solution as the Jacobian elliptic cosine function. When the modulus of the cos function is taken to be one, solution becomes hyperbolic secant function. The color potential is evaluated to be

\[
A(t) = E_0 / \omega_0 \text{sech} \omega_0 t
\]

where

\[
\omega_0 = (8g^2/3)^{1/4} \sqrt{E_0}/2
\]

Therefore

\[
E = -\frac{\partial A}{\partial t} = E_0 \text{sech} \omega_0 t \tan h \omega_0 t
\]

It shows that the color field in the flux tube is time dependant (color particles are coupled to each other via the gauge fields) and the characteristic frequency of oscillation depends on the amplitude of oscillation. It is actually one of the hallmarks of non-linear oscillators opposed to linear ones is that their frequency verses with the amplitude. Here we wish to calculate the quark- anti quark pair production amplitude in quark- gluon plasma. For this, we follow the method given by S. Biswas and S. Guha [9]. The basic
ingredient in their calculation is the evaluation of action integral \( S([t_1, t_2]) \). This will be evaluated by solving the color coupled Klein Gordan equation in the external color potential. For the color \( SU(2) \) group the equations to be solved are (\( \tau_\alpha \), with \( \alpha = 1, 2, 3 \) are paulispin matrices)

\[
[\partial^2_0 - \nabla^2 + 2ig\tau_\alpha A_\alpha \partial_0 + g^2 A_\alpha^2 + m^2] \begin{pmatrix} \phi_+ \\ \phi_- \end{pmatrix} = 0 \tag{4.1}
\]

with

\[
\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

It is due to non-abelian effect in the equation 2, it may be seen that \( \phi_+ \) and \( \phi_- \) are coupled. So, to solve the above equation, it has to be decoupled. It may be shown that the above equation can be decoupled by a unitary transformation in color space defined by

\[
U^+ = \begin{pmatrix} \sqrt{E^2 - E_3^2} & E_1 - iE_2 \\ E - E_3 \sqrt{E^2 - E_3^2} & E + iE_3 \sqrt{E^2 - E_3^2} \end{pmatrix}, \text{ where } E^2 = E_1^2 + E_2^2 + E_3^2. \text{ The (column vector) wave function in turn transforms into}
\]

\[
U^+ \begin{pmatrix} \phi_+ \\ \phi_- \end{pmatrix} = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix} \quad \text{The decoupled equations are evaluated to be}
\]

\[
[\partial^2_0 + \omega(t)] \psi_+ = 0 \text{ and } [\partial^2_0 - \omega(t)] \psi_- = 0 \tag{4.2}
\]

With \( \omega(t) = [\omega^2 + (p_3 + \sqrt{\frac{\sqrt{3}gE}{\omega_0}} tanh \omega_0 t)^2]^{1/2} \), where \( \omega^2 = p_1^2 + p_2^2 + m^2 = p_1^2 + m^2 \)

This is nothing but our Klein-Gordan (K.G) equation for one dimension.

Now, we try to solve the decoupled equations using the W.K.B method.
The validity of the approximation can be easily checked.\cite{2}

\[ \frac{d}{dt}\left(\frac{1}{\omega(t)}\right) \ll \frac{\omega(t)}{\omega'(t)} \ll 1 \]

So as to solve our K.G. equation, we follow the method, W.K.B approximation in complex time, given by S.Biswas and J.Guha\cite{9}.

### 4.4 Evaluation of reflection coefficient using W.K.B. approximation in complex time.

In standard W.K.B approximation with a real time \( t \), wave function is written as

\[ \psi = \frac{c_1}{[\omega(t)]^{1/2}} e^{i\omega(t)dt} + \frac{c_2}{[\omega(t)]^{1/2}} e^{-i\omega(t)dt} \]

In complex time \( t \), we define

\[ \int_{t_1}^{t} \omega(t)dt = s(t, t_1) = \int_{t_1}^{t} [\omega^2 + V(t)]^{1/2} dt \]

Where \( V(t) = (p_3 - \frac{gE}{\omega_0} \text{sech}\omega_0 t)^2 \) The turning points are determined from \( \omega(t) = 0 \) i.e, \( [\omega^2 + V(t)]^{1/2} = 0 \) The boundary conditions are chosen such that,

\[ \psi \sim e^{is(t,t_1)} \sim e^{\int_{t_1}^{t} \omega(t)dt} \]

when \( t \to -\infty \)

\[ \psi \sim e^{is(t,t_1)} + be^{-is(t,t_1)} \]

i.e, \( \psi \sim e^{i\int \omega(t)dt} + be^{-i\int \omega(t)dt} \) When \( t \to \infty \)
Here $b$ is called the reflection coefficient. Consider the pair production as a consequence of reflection in time; the reflection coefficient will be identified as pair production amplitude [9].

The reflection coefficient is calculated to be.

$$ b = -\frac{ie^{2i\alpha(t_1,t_0)}}{1 + e^{2i\alpha(t_1,t_2)}} $$

(4.3)

From this, the reflection amplitude $b^2$ can be calculated which is our pair creation amplitude according to the complex time W.K.B approximation.

### 4.5 Calculation of action integral and the pair creation amplitude

$$ s(t_1,t_2) = \int_{t_2}^{t_1} \left[ \omega^2 + \left( p_3 + \frac{\sqrt{2}gE}{\omega_0} \tanh \omega_0 t \right)^2 \right]^{1/2} dt \tag{4.4} $$

where $t_1 = \frac{1}{\omega_0} \tanh^{-1} \frac{\omega_0 p_3}{\sqrt{2}gE} (i\omega - p_3)$ and $t_2 = -\frac{1}{\omega_0} \tanh^{-1} \frac{\omega_0 p_3}{\sqrt{2}gE} (i\omega + p_3)$

To solve equation 4.4, put $(p_3 + \frac{\sqrt{2}gE}{\omega_0} \tanh \omega_0 t) = i\omega \sin \theta$

$$ \therefore s(t_1,t_2) = \frac{ie^2}{\sqrt{2}gE} \int_{-\pi/2}^{\pi/2} \frac{\cos^2 \theta d\theta}{\left( p_3 - i\omega \sin \theta \right) \sqrt{1 - \frac{(i\omega \sin \theta - p_3)^2}{2gE^2}}} $$

This can be easily integrated to give

$$ s(t_1,t_2) = \frac{i\pi}{2} + \frac{i\pi}{2\sqrt{2}gE} \left[ \sqrt{\omega^2 + \left( \frac{\omega_0}{2} + p_3 \right)^2} + \sqrt{\omega^2 + \left( \frac{\omega_0}{2} - p_3 \right)^2} \right] \tag{4.5} $$
Similarly \(s(t_1, t_0)\) can be evaluated to be

\[
s(t_1, t_0) = \frac{i\pi}{4} + \frac{i\pi}{4\sqrt{2}qE} \left[ \sqrt{\omega^2 + \left(\frac{\omega_0}{2} + p_3\right)^2} + \sqrt{\omega^2 + \left(\frac{\omega_0}{2} - p_3\right)^2} \right] \tag{4.6}
\]

Putting the values of \(s(t_1, t_2)\) and \(s(t_1, t_0)\) in equation 4.3, we get

\[
b = \frac{e^{-\pi} \times e^{\frac{\pi}{\sqrt{2}qE}} \left[ \sqrt{\omega^2 + \left(\frac{\omega_0}{2} + p_3\right)^2} + \sqrt{\omega^2 + \left(\frac{\omega_0}{2} - p_3\right)^2} \right]}{1 + e^{-\pi} \times e^{\frac{\pi}{\sqrt{2}qE}} \left[ \sqrt{\omega^2 + \left(\frac{\omega_0}{2} + p_3\right)^2} + \sqrt{\omega^2 + \left(\frac{\omega_0}{2} - p_3\right)^2} \right]^2} \tag{4.7}
\]

\[
\therefore \text{The pair creation probability}
\]

\[
b^2 = \frac{e^{-\pi} \times e^{\frac{\pi}{\sqrt{2}qE}} \left[ \sqrt{\omega^2 + \left(\frac{\omega_0}{2} + p_3\right)^2} + \sqrt{\omega^2 + \left(\frac{\omega_0}{2} - p_3\right)^2} \right]}{[1 + e^{-\pi} \times e^{\frac{\pi}{\sqrt{2}qE}} \left[ \sqrt{\omega^2 + \left(\frac{\omega_0}{2} + p_3\right)^2} + \sqrt{\omega^2 + \left(\frac{\omega_0}{2} - p_3\right)^2} \right]^2]}
\tag{4.8}
\]

To estimate the pair creation probability we set \(p_3 = 0\), the range of integration being of order of magnitude \(\frac{2gE}{\omega_0}\) as suggested by classical equation of motion.

\[
\therefore
\]

\[
b^2 = \frac{e^{-\pi} \times e^{\frac{\pi}{\sqrt{2}qE}} \left[ \sqrt{p^2 + m^2 + \frac{\omega_0^2}{4}} \right]}{[1 + e^{-\pi} \times e^{\frac{\pi}{\sqrt{2}qE}} \left[ \sqrt{p^2 + m^2 + \frac{\omega_0^2}{4}} \right]^2]}
\tag{4.9}
\]

From equation 4.9, it can be seen that, when \(\frac{db^2}{d(gE)} = 0\), we get

\[
gE = p^2 + m^2 \text{ and } \frac{d^2(b)^2}{d(gE)^2} \text{ is found to be negative. It shows that the pair creation becomes maximum when } gE = p^2 + m^2 \text{. i.e. when ever the quark}
\]

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anti quark energy is comparable with the field strength the pair creation probability dominates. Thus we can say that it is a resonance phenomenon. Further equation 4.9 shows that the pair creation probability depends crucially upon the collective oscillation frequency $\omega_0$ a result which is expected. Put $\omega_0 = 0$ in equation 4.9, we get

\[
\therefore \quad b^2 = \frac{e^{-\pi} \times e^{-\frac{4\pi}{\sqrt{2}a}\sqrt{p^2+m^2}}}{[1 + e^{-\pi} \times e^{-\frac{4\pi}{\sqrt{2}a}\sqrt{p^2+m^2}}]^2}
\]

4.6 Result

To conclude, we described an important non-abelian effect on the pair creation probability in collisions in high energy physics. Once again it is established that the colour field should be time dependent due to non-abelian interactions and should oscillate with a collective frequency $\omega_0$ which should depend on field strength. Our model has several advantages over other models[2-5]. We assumed a non-abelian colour field model which is an exact solution of Yang-Mills equation, the second is that our calculation is completely analytic and no assumption is made to simplify it, thirdly we need not go for any cumbersome calculations as the authors adopted [1-4]. Evaluation of the action integral $s(t_1, t_2)$, $s(t_1, t_0)$ is simple even though lengthy. Our results do not violate any of the former results in this field. Finally, we obtained a very consistent result. If our model is good and the pair creation probability is truly given by our equation 4.9, it should be possible to test it in heavy ion collision experiments. We hope that the same resonance effect will hold well even if we calculate the pair production probability using the
4.7 References