5.1.1 White noise model

Suppose ‘f’ is observed from the white noise model

\[ Y(dx) = f(x)dx + \tau W_e(dx), x \in [0, 1] \]  

where \( W_e \) is a Weiner process, \( \tau \) is a formal noise level parameter, and ‘f’ is an unknown function which have jumps and/or cusps. The problem is to detect these jumps and/or cusps.

The white noise model (5.3) is closely related to the following non-parametric regression model

\[ y_i = f(x_i) + \epsilon_i, \]  

where \( \epsilon_i \) is some noise process with variance \( \sigma^2 \), \( x_i = \frac{i}{n} \). The idea is to estimate the “jump” or “cusp” in an unknown function ‘f’ from the observations \( y_i, i = 1, 2, 3, \ldots, n \).

In vector notation, the model (5.4) can be written as

\[ Y = F + E, \]

where \( Y = (y_1, y_2, \ldots, y_n) \), \( F = (f_1, f_2, \ldots, f_n) \) and \( E = (\epsilon_1, \epsilon_2, \ldots, \epsilon_n) \) the observed data, signal and noise respectively.

The wavelet transformation of the white noise \( W_e(dx) \) is defined to be \( WW_e(a, b) = \int \psi_{a,b} W_e(dx) \). The wavelet transform of \( Y \) is

\[ WY(a, b) = \int \psi_{a,b} Y(dx) = Wf(a, b) + \tau WW_e(a, b) \]

For a compactly supported wavelet the value of \( Wf(a, b) \) depends upon the value of ‘f’ in a neighbourhood of ‘b’ of size proportional to the scale ‘a’. At a small scale \( Wf(a, b) \) provides “localized” information such as local regularity on \( f(x) \).

The local regularity is often measured by Lipschitz. Note that \( WW_e(a, b) \) follows standard normal distribution and that the orders of \( Wf(a, b) \) are respectively