CHAPTER III
MULTIOBJECTIVE CAPACITATED FRACTIONAL TRANSPORTATION PROBLEM WITH MIXED CONSTRAINTS: AN INTEGER SOLUTION

3.1. INTRODUCTION

The Transportation problem (TP) is a special type of linear programming problem where the product/products is/are to be transported from several sources (also called origin, supply or capacity centers) to several sinks (also called destination, demand or requirement centers). Hitchcock (1941) developed the basic transportation problem. The TP in which the objective function is of fractional type is known as Fractional transportation problem (FTP). The FTP was originally proposed by Swarup (1966). The TP with fractional objective functions have been extensively used by several authors. Verma and Puri (1991) worked on paradox in linear fractional TP; Gupta et al. (1993) presented a paradox in linear fractional TPs with mixed constraints, etc. Recently, some authors who have discussed FTPs are Khurana and Arora (2006), Gupta and Arora (2012a, 2012b, 2012c). Joshi and Gupta (2011) investigated the TP with fractional objective function when the demand and supply quantities are varying.

Real life TPs are mostly multiobjective and in the case of multiple conflicting objectives, it is not necessary that the optimum solution for one objective is also optimum for the others. So, in order to deal with such solutions, a compromise criterion is used in which a solution is obtained which is optimum for all the objectives in some sense. Also, real life TPs have mixed constraints but no systematic method for finding an optimal solution for TPs with mixed constraints are revealed in literature. Recently, some authors consider this situation such as Adlakha (2006), Mondal (2012), Gupta and Bari (2014) etc.

The present chapter deals with multiobjective capacitated fractional transportation problem (MOCFTP) with mixed constraints. The model considered in this chapter consists of fractional objectives, that is, each objective function is a ratio of two linear functions. The compromise solution is obtained by using fuzzy programming and
minimum distance method of lexicographic goal programming. The solution procedure for solving a numerical example is also given to demonstrate the applications.

3.2 Formulation of the problem

Consider a fractional transportation problem with \( m \) origins having \( a_i \) (\( i = 1,2,\ldots,m \)) units of supply to be transported among \( n \) destinations with \( b_j \) (\( j = 1,2,\ldots,n \)) units of demand. Here, we consider three fractional objective functions, which are as follows:

- Units transporting cost \( c_{ij} \) due to the traveled route and unit transporting cost due to preferring route \( r_{ij} \), for transporting the product from \( i^{th} \) origin to \( j^{th} \) destination.
- Actual transportation time \( t_{ij}^a \) and a standard transportation time \( t_{ij}^s \), for transporting the product from \( i^{th} \) origin to \( j^{th} \) destination.
- Unit transporting damage cost \( d_{ij} \) (loss of quality and quantity of transportation) due to the traveled route and unit transporting damage cost due to preferring route \( r_{ij} \), for transporting the product from \( i^{th} \) origin to \( j^{th} \) destination.
- The problem is to determine the transportation schedule of transporting the available quantity of products, to satisfy the demand that minimizes the total transportation cost, time and damage charges. Let \( x_{ij} \) be the number of units transported from \( i^{th} \) origin to \( j^{th} \) destination.

Then, the mathematical model of the MOCFTP with mixed constraints can be expressed as follows: -

Minimize \( f_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij} / \sum_{i=1}^{m} \sum_{j=1}^{n} r_{ij}x_{ij} \)

Minimize \( f_2 = \max \left\{ t_{ij}^a x_{ij} > 0 \right\} \) for \( t_{ij}^s x_{ij} > 0 \)
Minimize $f_3 = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} x_{ij}}{\sum_{i=1}^{m} \sum_{j=1}^{n} r_{ij} x_{ij}}$ \hspace{1cm} (3.2.1)

subject to $\sum_{i=1}^{m} x_{ij} \{ l = \geq \} a_i,$

$\sum_{j=1}^{n} x_{ij} \{ l = \geq \} b_j,$

$l_{ij} \leq x_{ij} \leq s_{ij}; x_{ij} \geq 0$ and integer.

where, $l_{ij}$ be the minimum and $s_{ij}$ be the maximum amount of quantity transported from $i^{th}$ source to $j^{th}$ destination i.e., $x_{ij} \leq s_{ij}$ or the capacitated restriction on the route $i$ to $j$. And all the variables $x_{ij}$ must be integer.

3.3 Optimization Techniques

3.3.1 Fuzzy programming with different membership functions

The compromise solution of MOCFTP is obtained by using fuzzy programming (FzP). Before formulation of the fuzzy model, first we define a payoff matrix as follows:

$$
\text{Payoff Matrix} = \begin{pmatrix}
C & D & T \\
x_{ij}^{(1)} & C(x_{ij}^{(1)}) & D(x_{ij}^{(1)}) & T(x_{ij}^{(1)}) \\
x_{ij}^{(2)} & C(x_{ij}^{(2)}) & D(x_{ij}^{(2)}) & T(x_{ij}^{(2)}) \\
\vdots & \vdots & \vdots & \vdots \\
x_{ij}^{(k)} & C(x_{ij}^{(k)}) & D(x_{ij}^{(k)}) & T(x_{ij}^{(k)})
\end{pmatrix}
$$

where, $x_{ij}^{(k)}; k = 1,2,\ldots, K$ are the individual optimum solutions.

In the following subsection, the membership functions used in solving the problem are defined. Let the membership functions for the cost objective can be defined as follows:
3.3.1.1 Linear membership function

The objective function of cost is defined by a linear membership function, $\mu^L(C)$ as follows:

$$
\mu^L(C) = \begin{cases} 
1, & \text{if } C \leq C_l, \\
\frac{C_u - C}{C_u - C_l}, & \text{if } C_l < C < C_u, \\
0, & \text{if } C \geq C_u,
\end{cases}
$$

where, $C_l$ and $C_u$ are the lower and upper tolerance limits of the objective functions.

3.3.1.2 Exponential membership function

The objective function of cost is defined by an exponential membership function, $\mu^E(C)$ as follows:

$$
\mu^E(C) = \begin{cases} 
1, & \text{if } C \leq C_l, \\
\frac{\exp\left(-\alpha(C - C_l)\right) - \exp(-\alpha)}{1 - \exp(-\alpha)}, & \text{if } C_l < C < C_u, \\
0, & \text{if } C \geq C_u \text{ and } \alpha \to \infty.
\end{cases}
$$

here, $\alpha$ is a non-zero parameter.

3.3.1.3 Hyperbolic membership function

The objective function of cost is defined by the hyperbolic membership function $\mu^H(C)$ as follows:

$$
\mu^H(C) = \begin{cases} 
1, & \text{if } C \leq C_l, \\
\frac{1}{2} \tanh\left(\frac{C_u + C_i - C}{2} \alpha_k\right) + \frac{1}{2}, & \text{if } C_l < C < C_u, \\
0, & \text{if } C \geq C_u
\end{cases}
$$

here, $\alpha_k = \frac{6}{(C_u - C_l)}$.  

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This membership function has the following formal properties, (see Zimmerman (1985) which are as follows:

i. \( \mu^H(C) \) is strictly monotonously decreasing function with respect to \( C \)

\[ \mu^H(C) = \frac{1}{2} \Leftrightarrow C = \frac{1}{2}(C_u + C_i) \]

ii. \( \mu^H(C) \) is strictly convex for \( C \geq \frac{1}{2}(C_u + C_i) \) and strictly concave for

\[ C \leq \frac{1}{2}(C_u + C_i) . \]

iii. \( \mu^H(C) \) satisfies \( 0 < \mu^H(C) < 1 \) for \( C_i < \mu^H(C) < C_u \) and approaches asymptotically \( \mu^H(C) = 0 \) and \( \mu^H(C) = 1 \) as \( C \rightarrow \infty \) and \(-\infty\) respectively.

Similarly, the membership functions can also be defined for the objective functions of time and damage charges respectively.

The mathematical model of MOCFTP with mixed constraints using linear membership function can be given as follows:

Minimize \( \delta \)

subject to \( \frac{C_u - C}{C_u - C_i} \leq \delta ; \)

\( \frac{T_u - T}{T_u - T_i} \leq \delta ; \quad \frac{D_u - D}{D_u - D_i} \leq \delta ; \) \hspace{1cm} (3.3.1)

\( \sum_{i=1}^{m} x_{ij} \{ \leq \ l = l \geq \} a_i , \sum_{j=1}^{n} x_{ij} \{ \leq \ l = l \geq \} b_j , \)

\( l_{ij} \leq x_{ij} \leq s_{ij} ; \delta \geq 0 ; x_{ij} \geq 0 \) and integer.

Similarly, the mathematical model of MOCFTP with mixed constraints using exponential membership function can be given as follows:

Minimize \( \delta \)
subject to \[ \frac{\exp\left(-\alpha\left(C - C_i\right)\right) - \exp(-\alpha)}{1 - \exp(-\alpha)} \leq \delta; \]

subject to \[ \frac{\exp\left(-\alpha\left(T - T_i\right)\right) - \exp(-\alpha)}{1 - \exp(-\alpha)} \leq \delta; \]

subject to \[ \frac{\exp\left(-\alpha\left(D - D_i\right)\right) - \exp(-\alpha)}{1 - \exp(-\alpha)} \leq \delta; \]

\[ \sum_{i=1}^{m} x_{ij} \leq l \geq a_i, \sum_{j=1}^{n} x_{ij} \leq l \geq b_j, \]

\[ l_{ij} \leq x_{ij} \leq s_{ij}; \delta \geq 0; x_{ij} \geq 0 \text{ and integer}. \]

And, the mathematical model of MOCFTP with mixed constraints using hyperbolic membership function is given as follows:

Minimize \[ \delta \]

subject to \[ \frac{1}{2} \tanh\left(\left(C + C_i - C\right)\alpha_k\right) + \frac{1}{2} \leq \delta; \]

subject to \[ \frac{1}{2} \tanh\left(\left(T + T_i - C\right)\alpha_k\right) + \frac{1}{2} \leq \delta; \]

subject to \[ \frac{1}{2} \tanh\left(\left(D + D_i - C\right)\alpha_k\right) + \frac{1}{2} \leq \delta; \]

\[ \sum_{i=1}^{m} x_{ij} \leq l \geq a_i, \sum_{j=1}^{n} x_{ij} \leq l \geq b_j, \]

\[ l_{ij} \leq x_{ij} \leq s_{ij}; \delta \geq 0; x_{ij} \geq 0 \text{ and integer}. \]

where, \( \delta \) is the deviational variable.
3.3.2 Lexicographic goal programming with minimum distance

Lexicographic goal programming (LGP), has priority levels and each priority level contains a number of unwanted deviations which are to be minimized or in other words, unwanted deviations are placed into priority levels. LGP with minimum distance, is an improved form of the original LGP. For solving LGP with the minimum distance, firstly the priorities are given to objectives one after the other and a set of solution is obtained, then an ideal solution is identified. The Table (3.1) given below is for the calculation of ideal solutions.

Table 3.1: Calculations of ideal solutions.

<table>
<thead>
<tr>
<th>Priority Structure</th>
<th>$x_{11}$</th>
<th>$x_{22}$</th>
<th>…</th>
<th>$x_{pq}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P^{(1)}$</td>
<td>$x_{11}^{(1)}$</td>
<td>$x_{22}^{(1)}$</td>
<td>…</td>
<td>$x_{pq}^{(1)}$</td>
</tr>
<tr>
<td>$P^{(2)}$</td>
<td>$x_{11}^{(2)}$</td>
<td>$x_{22}^{(2)}$</td>
<td>…</td>
<td>$x_{pq}^{(2)}$</td>
</tr>
<tr>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
</tr>
<tr>
<td>$P^{(r)}$</td>
<td>$x_{11}^{(r)}$</td>
<td>$x_{22}^{(r)}$</td>
<td>…</td>
<td>$x_{pq}^{(r)}$</td>
</tr>
<tr>
<td>Ideal Solution</td>
<td>$x_{11}^*$</td>
<td>$x_{22}^*$</td>
<td>…</td>
<td>$x_{pq}^*$</td>
</tr>
</tbody>
</table>

From this table, the ideal solution can be identified as follows:

$$\text{Ideal Solution} = x_{ij}^* = \{ \min(x_{11}^{(1)}, \ldots, x_{ij}^{(r)}), \min(x_{22}^{(1)}, \ldots, x_{22}^{(r)}), \ldots, \min(x_{pq}^{(1)}, \ldots, x_{pq}^{(r)}) \}$$

$$= \{ x_{11}^*, x_{22}^*, \ldots, x_{pq}^* \}.$$

A general procedure with K objectives is as follows.

As explained above, we will obtain K! (Factorial) different solutions by solving the K! problems arising for K! different priority structures.

Let $x_{ij}^{(r)} = \{ x_{11}^{(r)}, x_{22}^{(r)}, \ldots, x_{pq}^{(r)} \}, 1 \leq r \leq K!$ be the K! number of solutions obtained by giving priorities to K objective functions. Let $(x_{11}^*, x_{22}^*, \ldots, x_{pq}^*)$ be the ideal solution. But in practice, an ideal solution can never be achieved. The solution, which is closest to the ideal solution, is acceptable as the best compromise solution, and the corresponding priority structure is identified as the most appropriate priority structure in the planning context. The $D_i$-distances of different solutions from the ideal
solution defined below are then calculated. The solution corresponding to the minimum $D_1$-distance gives the best compromise solution.

Now, $(D_1)' = \sum_{i=1}^{p} \sum_{j=1}^{q} |x_{ij}^* - x_{ij}^{(r)}|$ is defined as the $D_1$-distance from the ideal solution $(x_{11}^*, x_{22}^*, \ldots, x_{pq}^*)$, of the $r^{th}$ solution $\{x_{11}^{(r)}, x_{22}^{(r)}, \ldots, x_{pq}^{(r)}\}, 1 \leq r \leq P$.

Therefore,

$$(D_1)'_{opt} = \min_{1 \leq r \leq P} (D_1)' = \min_{1 \leq r \leq P} \left[ \sum_{i=1}^{p} \sum_{j=1}^{q} |x_{ij}^* - x_{ij}^{(r)}| \right]$$

$D_1$-distances are calculated from the ideal solution given below in table 3.2 as follows:-

**Table 3.2: $D_1$-distances from the ideal solution.**

<table>
<thead>
<tr>
<th>Priorities</th>
<th>$x_{11}$</th>
<th>$\cdots$</th>
<th>$x_{pq}$</th>
<th>$(D_1)'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P^{(1)}$</td>
<td>$</td>
<td>x_{11}^* - x_{11}^{(1)}</td>
<td>$</td>
<td>$\cdots$</td>
</tr>
<tr>
<td>$P^{(2)}$</td>
<td>$</td>
<td>x_{11}^* - x_{11}^{(2)}</td>
<td>$</td>
<td>$\cdots$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$</td>
<td>x_{11}^* - x_{11}^{(r)}</td>
<td>$</td>
<td>$\cdots$</td>
</tr>
<tr>
<td>$P^{(r)}$</td>
<td>$</td>
<td>x_{11}^* - x_{11}^{(r)}</td>
<td>$</td>
<td>$\cdots$</td>
</tr>
</tbody>
</table>

Let the minimum be attained for $r = t$. Then $\{x_{11}^{(r)}, x_{22}^{(r)}, \ldots, x_{pq}^{(r)}\}$ is the best compromise solution of the problem.

### 3.4 Numerical Illustration

In order to demonstrate the problem and the utility of the approaches discussed above, a numerical problem is considered. Here, we are assuming a TP of three origins and three destinations. The TP cost, time and damage charges (both quality and quantity damage) during the transportation are given in the Table (3.3), (3.4) and (3.5) respectively.
Table 3.3: Cost Charge Matrix

<table>
<thead>
<tr>
<th>a_i</th>
<th>b_1</th>
<th>b_2</th>
<th>b_3</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>a_1</td>
<td>5/3</td>
<td>7/4</td>
<td>15/13</td>
<td>≤12</td>
</tr>
<tr>
<td>a_2</td>
<td>8/12</td>
<td>17/14</td>
<td>12/7</td>
<td>=15</td>
</tr>
<tr>
<td>a_3</td>
<td>19/15</td>
<td>10/6</td>
<td>13/8</td>
<td>≥20</td>
</tr>
</tbody>
</table>

Demand    ≥ 9  = 13  ≤ 21

Table 3.4: Time Matrix

<table>
<thead>
<tr>
<th>a_i</th>
<th>b_1</th>
<th>b_2</th>
<th>b_3</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>a_1</td>
<td>17/9</td>
<td>5/2</td>
<td>10/3</td>
<td>≤12</td>
</tr>
<tr>
<td>a_2</td>
<td>1/2</td>
<td>11/4</td>
<td>6/5</td>
<td>=15</td>
</tr>
<tr>
<td>a_3</td>
<td>13/8</td>
<td>16/12</td>
<td>10/11</td>
<td>≥20</td>
</tr>
</tbody>
</table>

Demand    ≥ 9  = 13  ≤ 21

Table 3.5: Damage Charges Matrix

<table>
<thead>
<tr>
<th>a_i</th>
<th>b_1</th>
<th>b_2</th>
<th>b_3</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>a_1</td>
<td>13/8</td>
<td>15/9</td>
<td>8/11</td>
<td>≤12</td>
</tr>
<tr>
<td>a_2</td>
<td>15/11</td>
<td>14/6</td>
<td>19/7</td>
<td>=15</td>
</tr>
<tr>
<td>a_3</td>
<td>9/7</td>
<td>15/6</td>
<td>8/17</td>
<td>≥20</td>
</tr>
</tbody>
</table>

Demand    ≥ 9  = 13  ≤ 21

Case I: When the \( l_{ij} = 0 \)

Using the data given in Tables (3.3), (3.4) and (3.5), the MOCFTP with mixed constraints and with \( l_{ij} = 0 \) is given as follows:

Minimize \( C = \frac{5x_{11} + 7x_{12} + 15x_{13} + 8x_{21} + 17x_{22} + 12x_{32} + 19x_{31} + 10x_{32} + 13x_{33}}{3x_{11} + 4x_{12} + 13x_{13} + 12x_{21} + 14x_{22} + 7x_{23} + 15x_{31} + 6x_{32} + 8x_{33}} \)

Minimize \( D = \frac{13x_{11} + 15x_{12} + 8x_{13} + 15x_{21} + 14x_{22} + 19x_{23} + 9x_{31} + 15x_{32} + 8x_{33}}{8x_{11} + 9x_{12} + 11x_{13} + 15x_{21} + 6x_{22} + 7x_{23} + 7x_{31} + 6x_{32} + 17x_{33}} \)

Minimize \( T = \frac{17x_{11} + 5x_{12} + 10x_{13} + x_{21} + 11x_{22} + 6x_{23} + 13x_{31} + 16x_{32} + 10x_{33}}{9x_{11} + 2x_{12} + 3x_{13} + 2x_{21} + 4x_{22} + 5x_{23} + 8x_{31} + 12x_{32} + 11x_{33}} \)
subject to \[ \sum_{i=1}^{3} x_{1j} \leq 12; \quad \sum_{i=1}^{3} x_{2j} = 15; \quad \sum_{i=1}^{3} x_{3j} \geq 20; \]
\[ \sum_{i=1}^{3} x_{ij} \geq 9; \quad \sum_{i=1}^{3} x_{ij} = 13; \quad \sum_{i=1}^{3} x_{ij} \leq 21; \quad x_{ij} \geq 0 \text{ and integer.} \]

The capacitated constraints are given as follows:

\[ 0 \leq x_{11} \leq 6, \quad 0 \leq x_{12} \leq 7, \quad 0 \leq x_{13} \leq 13, \quad 0 \leq x_{21} \leq 6, \quad 0 \leq x_{22} \leq 2, \quad 0 \leq x_{23} \leq 13, \quad 0 \leq x_{31} \leq 4 \]
\[ 0 \leq x_{32} \leq 7, \quad 0 \leq x_{33} \leq 14. \]

3.4.1 Fuzzy programming approach with different membership functions (Case I)

The payoff matrix for the case \([l_{ij} = 0]\) obtained after solving the above problem separately for each problem using the optimizing software LINGO is as follows:

\[
\begin{bmatrix}
C & D & T \\
X_{ij}^{(1)} & 1.316832 & 1.16129 & 1.34472 \\
X_{ij}^{(2)} & 1.37988 & 1.068410 & 1.79661 \\
X_{ij}^{(3)} & 1.406433 & 1.170886 & 1.168285 \\
\end{bmatrix}
\]

\[ C_u^1 = 1.406433, \quad C_l^1 = 1.316832, \quad D_u^2 = 1.170886, \quad D_l^2 = 1.068410, \quad T_u^3 = 1.79661 \text{ and } T_l^3 = 1.168285. \]

Individual optimum solutions are obtained by solving the above problem separately for each objective function. They are summarised in table (3.6) which is given as follows.

**Table 3.6: Individual optimum solution for case I**

<table>
<thead>
<tr>
<th>Objective</th>
<th>Objective values</th>
<th>Allocations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(x_{11})</td>
</tr>
<tr>
<td>Cost Charge</td>
<td>1.316832</td>
<td>0</td>
</tr>
<tr>
<td>Damage Charge</td>
<td>1.068410</td>
<td>0</td>
</tr>
<tr>
<td>Time</td>
<td>1.168285</td>
<td>0</td>
</tr>
</tbody>
</table>
The MOCFTP model with the linear membership function can be given as follows:

Minimize $\delta$

subject to

$$\frac{5x_{11} + 7x_{12} + 15x_{13} + 8x_{21} + 17x_{22} + 12x_{23} + 19x_{31} + 10x_{32} + 13x_{33}}{3x_{11} + 4x_{12} + 13x_{13} + 12x_{21} + 14x_{22} + 7x_{23} + 15x_{31} + 6x_{32} + 8x_{33}} \leq 0.089601\delta;$$

$$\frac{13x_{11} + 15x_{12} + 8x_{13} + 11x_{21} + 14x_{22} + 19x_{23} + 9x_{31} + 15x_{32} + 8x_{33}}{8x_{11} + 9x_{12} + 11x_{13} + 15x_{21} + 6x_{22} + 7x_{23} + 7x_{31} + 6x_{32} + 17x_{33}} \leq 0.628325\delta;$$

$$\frac{17x_{11} + 5x_{12} + 10x_{13} + x_{21} + 11x_{22} + 6x_{23} + 13x_{31} + 6x_{32} + 10x_{33}}{9x_{11} + 2x_{12} + 3x_{13} + 2x_{21} + 4x_{22} + 5x_{23} + 8x_{31} + 12x_{32} + 11x_{33}} \leq 0.102476\delta;$$

$$\sum_{j=1}^{3} x_{ij} \leq 12; \sum_{j=1}^{3} x_{2j} = 15; \sum_{j=1}^{3} x_{3j} \geq 20; \sum_{i=1}^{3} x_{1i} \geq 9; \sum_{i=1}^{3} x_{2i} = 13; \sum_{i=1}^{3} x_{3i} \leq 21;$$

$$0 \leq x_{11} \leq 6, 0 \leq x_{12} \leq 7, 0 \leq x_{13} \leq 13, 0 \leq x_{21} \leq 6, 0 \leq x_{22} \leq 2, 0 \leq x_{23} \leq 13, 0 \leq x_{31} \leq 4$$

$$0 \leq x_{32} \leq 7, 0 \leq x_{33} \leq 14, x_{ij} \in \text{integer}.$$

The compromise solution of MOCFTP with linear membership function is as follows:-

$$x_{11}^* = 4, x_{12}^* = 4, x_{13}^* = 4, x_{21}^* = 5, x_{22}^* = 2, x_{23}^* = 8, x_{31}^* = 4, x_{32}^* = 7, x_{33}^* = 9, \text{and} \delta = 0.$$

The MOCFTP model with the exponential membership function with the parameter $\alpha = 1$, can be given as follows:-

Minimize $\delta$

subject to

$$e^{-0.89601} - e^{-1} \leq \delta;$$
\[ \frac{e^{-D-1.168285}}{0.628325} - e^{-1} \leq \delta; \quad \frac{e^{-T-1.068410}}{1.02476} - e^{-1} \leq \delta; \]

\[ \sum_{j=1}^{3} x_{ij} \leq 12; \quad \sum_{j=1}^{3} x_{2j} = 15; \quad \sum_{j=1}^{3} x_{3j} \geq 20; \quad \sum_{i=1}^{3} x_{ij} \geq 9; \quad \sum_{i=1}^{3} x_{2i} = 13; \quad \sum_{i=1}^{3} x_{3i} \leq 21; \]

\[ 0 \leq x_{11} \leq 6, \quad 0 \leq x_{12} \leq 7, \quad 0 \leq x_{13} \leq 13, \quad 0 \leq x_{21} \leq 6, \quad 0 \leq x_{22} \leq 2, \quad 0 \leq x_{23} \leq 13, \quad 0 \leq x_{31} \leq 4 \]

\[ 0 \leq x_{32} \leq 7, \quad 0 \leq x_{33} \leq 14, \quad x_{ij} \in \text{integer}. \]

The compromise solution of the MOCFTP with the exponential membership function is as follows:

\[ x_{11}^* = 0, \quad x_{12}^* = 4, \quad x_{13}^* = 1, \quad x_{21}^* = 6, \quad x_{22}^* = 2, \quad x_{23}^* = 7, \quad x_{31}^* = 4, \quad x_{32}^* = 7, \quad x_{33}^* = 9, \quad \delta = 0. \]

The MOCFTP model with the hyperbolic membership function is as follows:

Minimize \( \delta \)

subject to

\[ \frac{1}{2} \tanh \left( \frac{1}{1.3616325} \left[ \begin{array}{c} 17x_{11} + 5x_{12} + 10x_{13} + x_{21} + \\ 11x_{22} + 6x_{23} + 13x_{31} + 6x_{32} + 10x_{33} \\ 9x_{11} + 2x_{12} + 3x_{13} + 2x_{21} + \\ 4x_{22} + 5x_{23} + 8x_{31} + 12x_{32} + 11x_{33} \end{array} \right] \right) \right) + \frac{1}{2} \leq \delta; \]

\[ \frac{1}{2} \tanh \left( \frac{1}{1.4824475} \left[ \begin{array}{c} 13x_{11} + 15x_{12} + 8x_{13} + 11x_{21} + \\ 14x_{22} + 19x_{23} + 9x_{31} + 15x_{32} + 8x_{33} \\ 8x_{11} + 9x_{12} + 11x_{13} + 15x_{21} + \\ 6x_{22} + 7x_{23} + 7x_{31} + 6x_{32} + 17x_{33} \end{array} \right] \right) \right) + \frac{1}{2} \leq \delta; \]

\[ \frac{1}{2} \tanh \left( \frac{1}{1.119648} \left[ \begin{array}{c} 17x_{11} + 5x_{12} + 10x_{13} + x_{21} + \\ 11x_{22} + 6x_{23} + 13x_{31} + 6x_{32} + 10x_{33} \\ 9x_{11} + 2x_{12} + 3x_{13} + 2x_{21} + \\ 4x_{22} + 5x_{23} + 8x_{31} + 12x_{32} + 11x_{33} \end{array} \right] \right) \right) + \frac{1}{2} \leq \delta; \]

\[ \sum_{j=1}^{3} x_{1j} \leq 12; \quad \sum_{j=1}^{3} x_{2j} = 15; \quad \sum_{j=1}^{3} x_{3j} \geq 20; \quad \sum_{i=1}^{3} x_{ij} \geq 9; \quad \sum_{i=1}^{3} x_{2i} = 13; \quad \sum_{i=1}^{3} x_{3i} \leq 21; \]

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The compromise solution of the MOCFTP with the hyperbolic membership function is as follows:

\[ x_{i1}^* = 3, x_{i2}^* = 4, x_{i3}^* = 1, x_{21}^* = 6, x_{22}^* = 2, x_{23}^* = 7, x_{31}^* = 4, x_{32}^* = 7, x_{33}^* = 9, \delta = 0. \]

**Case II:** When the \( l_{ij} \geq 0 \).

Using the data given in Tables (3.3), (3.4) and (3.5), the mathematical model of MOCFTP with mixed constraints and with \( l_{ij} \geq 0 \) can be given as:

Minimize \( C = \frac{5x_{11} + 7x_{12} + 15x_{13} + 8x_{21} + 17x_{22} + 12x_{23} + 19x_{31} + 10x_{32} + 13x_{33}}{3x_{11} + 4x_{12} + 13x_{13} + 12x_{21} + 14x_{22} + 7x_{23} + 15x_{31} + 6x_{32} + 8x_{33}} \)

Minimize \( D = \frac{13x_{11} + 15x_{12} + 8x_{13} + 15x_{21} + 14x_{22} + 19x_{23} + 9x_{31} + 15x_{32} + 8x_{33}}{8x_{11} + 9x_{12} + 11x_{13} + 15x_{21} + 6x_{22} + 7x_{23} + 7x_{31} + 6x_{32} + 17x_{33}} \)

Minimize \( T = \frac{17x_{11} + 5x_{12} + 10x_{13} + x_{21} + 11x_{22} + 6x_{23} + 13x_{31} + 16x_{32} + 10x_{33}}{9x_{11} + 2x_{12} + 3x_{13} + 2x_{21} + 4x_{22} + 5x_{23} + 8x_{31} + 12x_{32} + 11x_{33}} \)

subject to \( \sum_{j=1}^{3} x_{ij} \leq 12; \sum_{j=1}^{3} x_{2j} = 15; \sum_{j=1}^{3} x_{3j} \geq 20; \)

\( \sum_{j=1}^{3} x_{1j} \geq 9; \sum_{j=1}^{3} x_{2j} = 13; \sum_{j=1}^{3} x_{3j} \leq 21; x_{ij} \geq 0 \) and integer.

The capacitated constraints are given below:

\( 1 \leq x_{11} \leq 6, 2 \leq x_{12} \leq 7, 4 \leq x_{13} \leq 13, 2 \leq x_{21} \leq 6, 0 \leq x_{22} \leq 2, 5 \leq x_{23} \leq 13, 1 \leq x_{31} \leq 4, \)
\( 2 \leq x_{32} \leq 7, \leq x_{33} \leq 14. \)

### 3.4.2 Fuzzy programming approach with different membership functions (Case II)

The payoff matrix for the case \( [l_{ij} \geq 0] \) obtained by solving the above problem separately for each problem using the optimizing software LINGO is as follows:
Chapter 3

\[
\begin{bmatrix}
C & D & T \\
x_i(1) & 1.31941 & 1.169133 & 1.360927 \\
x_i(2) & 1.33 & 1.147303 & 1.333333 \\
x_i(3) & 1.333333 & 1.152542 & 1.317881 \\
\end{bmatrix}
\]

Payoff Matrix =

\[
\begin{bmatrix}
C^1_u = 1.33333, & C^1_l = 1.31942 & D^2_u = 1.169133 & D^2_l = 1.147303 & T^3_u = 1.360927 & T^3_l = 1.317881.
\end{bmatrix}
\]

Individual optimum solutions are obtained by solving the above problem separately for each objective function. They are summarised in table 3.7 given below.

**Table 3.7: Individual optimum solution for case II**

<table>
<thead>
<tr>
<th>Objective</th>
<th>Objective values</th>
<th>Allocations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(x_1)</td>
</tr>
<tr>
<td>Cost Charge</td>
<td>1.31941</td>
<td>1</td>
</tr>
<tr>
<td>Damage Charge</td>
<td>1.147303</td>
<td>1</td>
</tr>
<tr>
<td>Time</td>
<td>1.317881</td>
<td>1</td>
</tr>
</tbody>
</table>

The MOCFTP model with the linear membership function can be obtained as follows:

Minimize \( \delta \)

subject to

\[
\begin{align*}
\left(1.33333 - \frac{5x_{11} + 7x_{12} + 15x_{13} + 8x_{21} + 17x_{22} + 12x_{23} + 19x_{31} + 10x_{32} + 13x_{33}}{3x_{11} + 4x_{12} + 13x_{13} + 12x_{21} + 14x_{22} + 7x_{23} + 15x_{31} + 6x_{32} + 8x_{33}}\right) & \leq 0.01392\delta; \\
\left(1.360927 - \frac{13x_{11} + 15x_{12} + 8x_{13} + 11x_{21} + 14x_{22} + 19x_{23} + 9x_{31} + 15x_{32} + 8x_{33}}{8x_{11} + 9x_{12} + 11x_{13} + 15x_{21} + 6x_{22} + 7x_{23} + 7x_{31} + 6x_{32} + 17x_{33}}\right) & \leq 0.043046\delta;
\end{align*}
\]
(1.169133 - \begin{vmatrix}
17x_{11} + 5x_{12} + 10x_{13} + x_{21} + 11x_{22} + 6x_{23} + 13x_{31} + 6x_{32} + 10x_{33} \\
9x_{11} + 2x_{12} + 3x_{13} + 2x_{21} + 4x_{22} + 5x_{23} + 8x_{31} + 12x_{32} + 11x_{33}
\end{vmatrix}

\leq .02183\delta;

\sum_{j=1}^{3} x_{1j} \leq 12; \sum_{j=1}^{3} x_{2j} = 15; \sum_{j=1}^{3} x_{3j} \geq 20; \sum_{i=1}^{3} x_{1i} \geq 9; \sum_{i=1}^{3} x_{2i} = 13; \sum_{i=1}^{3} x_{3i} \leq 21; x_{ij} \geq 0 \text{ and integer.}

1 \leq x_{11} \leq 6, 2 \leq x_{12} \leq 7, 4 \leq x_{13} \leq 13, 2 \leq x_{21} \leq 6, 0 \leq x_{22} \leq 2, 5 \leq x_{23} \leq 13, 1 \leq x_{31} \leq 4

2 \leq x_{32} \leq 7, 5 \leq x_{33} \leq 14.

The compromise solution of MOCFTP with linear membership function is obtained as follows:

\begin{align*}
x_{11}^* &= 4, x_{12}^* = 4, x_{13}^* = 4, x_{21}^* = 6, x_{22}^* = 2, x_{23}^* = 7, x_{31}^* = 3, x_{32}^* = 7, x_{33}^* = 10.
\end{align*}

The MOCFTP model with the exponential membership function with the parameter \( \alpha = 1 \), can be given as follows:

Minimize \( \delta \)

subject to

\begin{align*}
e^{-\frac{(C-1.31941)}{0.01392}} - e^{-1} &\leq \delta; \\
e^{-\frac{(D-1.169133)}{0.021803}} - e^{-1} &\leq \delta; \quad e^{-\frac{(T-1.360927)}{0.436046}} - e^{-1} &\leq \delta;
\end{align*}

\sum_{j=1}^{3} x_{1j} \leq 12; \sum_{j=1}^{3} x_{2j} = 15; \sum_{j=1}^{3} x_{3j} \geq 20; \sum_{i=1}^{3} x_{1i} \geq 9; \sum_{i=1}^{3} x_{2i} = 13; \sum_{i=1}^{3} x_{3i} \leq 21;

1 \leq x_{11} \leq 6, 2 \leq x_{12} \leq 7, 4 \leq x_{13} \leq 13, 2 \leq x_{21} \leq 6, 0 \leq x_{22} \leq 2, 5 \leq x_{23} \leq 13, 1 \leq x_{31} \leq 4

2 \leq x_{32} \leq 7, 5 \leq x_{33} \leq 14, x_{ij} \geq 0 \text{ and integer.}

The compromise solution of MOCFTP with the exponential membership function is obtained as follows:

\begin{align*}
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\end{align*}
\[ x_{11}^* = 4, x_{12}^* = 4, x_{13}^* = 4, x_{21}^* = 6, x_{22}^* = 2, x_{23}^* = 7, x_{31}^* = 4, x_{32}^* = 7, x_{33}^* = 9. \]

The MOCFTP model with the hyperbolic membership function is as follows:

Minimize \( \delta \)

subject to

\[
\frac{1}{2} \tanh \left( 1.32637 - \left( \begin{array}{c}
17x_{11} + 5x_{12} + 10x_{13} + x_{21} + 11x_{22} + 6x_{23} \\
+ 13x_{31} + 6x_{32} + 10x_{33} \\
9x_{11} + 2x_{12} + 3x_{13} + 2x_{21} + 4x_{22} + 5x_{23} \\
+ 8x_{31} + 12x_{32} + 11x_{33}
\end{array} \right) \right) \leq \frac{1}{2} \delta;
\]

\[
\frac{1}{2} \tanh \left( 1.15848 - \left( \begin{array}{c}
13x_{11} + 15x_{12} + 8x_{13} + 11x_{21} + 14x_{22} + 19x_{23} \\
+ 9x_{31} + 15x_{32} + 8x_{33} \\
8x_{11} + 9x_{12} + 11x_{13} + 15x_{21} + 6x_{22} + 7x_{23} \\
+ 7x_{31} + 6x_{32} + 17x_{33}
\end{array} \right) \right) \leq \frac{1}{2} \delta;
\]

\[
\frac{1}{2} \tanh \left( 1.339404 - \left( \begin{array}{c}
17x_{11} + 5x_{12} + 10x_{13} + x_{21} + 11x_{22} + 6x_{23} \\
+ 13x_{31} + 6x_{32} + 10x_{33} \\
9x_{11} + 2x_{12} + 3x_{13} + 2x_{21} + 4x_{22} + 5x_{23} \\
+ 8x_{31} + 12x_{32} + 11x_{33}
\end{array} \right) \right) \leq \frac{1}{2} \delta;
\]

\[
\sum_{j=1}^{3} x_{ij} \leq 12; \sum_{j=1}^{3} x_{ij} = 15; \sum_{j=1}^{3} x_{ij} \geq 20; \sum_{i=1}^{3} x_{ij} = 9; \sum_{i=1}^{3} x_{ij} = 13; \sum_{i=1}^{3} x_{ij} = 21; x_{ij} \geq 0 \text{ and integer.}
\]

\[
1 \leq x_{11} \leq 6, 2 \leq x_{12} \leq 7, 4 \leq x_{13} \leq 13, 2 \leq x_{21} \leq 6, 0 \leq x_{22} \leq 2, 5 \leq x_{23} \leq 13, 1 \leq x_{31} \leq 4
\]

\[
2 \leq x_{32} \leq 7, 5 \leq x_{33} \leq 14.
\]

The compromise solution of MOCFTP with the hyperbolic membership function is obtained as follows:

\[
x_{11}^* = 3, x_{12}^* = 4, x_{13}^* = 4, x_{21}^* = 6, x_{22}^* = 2, x_{23}^* = 7, x_{31}^* = 3, x_{32}^* = 7, x_{33}^* = 10.
\]

3.4.3 Minimum \( D_1 \) – distances method of lexicographic goal programming

Since, we have three objectives of minimizing transportation cost, time and damage charges in our problem, so we have to solve \( 3! = 6 \) problems according to the priority. The solutions obtained by giving priority to each of the objectives one by one is given below in the Table (3.8) and Table (3.9) for both the case I and II respectively.
### Table 3.8: Ideal solution for case I

<table>
<thead>
<tr>
<th>Priority structure</th>
<th>$x_{11}$</th>
<th>$x_{12}$</th>
<th>$x_{13}$</th>
<th>$x_{21}$</th>
<th>$x_{22}$</th>
<th>$x_{23}$</th>
<th>$x_{31}$</th>
<th>$x_{32}$</th>
<th>$x_{33}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P[C,D,T]$</td>
<td>0</td>
<td>4</td>
<td>1.8892</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>3</td>
<td>7</td>
<td>11.4361</td>
</tr>
<tr>
<td>$P[C,T,D]$</td>
<td>0</td>
<td>4</td>
<td>1.8892</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>3</td>
<td>7</td>
<td>11.4361</td>
</tr>
<tr>
<td>$P[D,C,T]$</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>3.1340</td>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>$P[D,T,C]$</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>3</td>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>$P[T,C,D]$</td>
<td>0</td>
<td>5.2547</td>
<td>0</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>3</td>
<td>5.7453</td>
<td>14</td>
</tr>
<tr>
<td>$P[T,D,C]$</td>
<td>0</td>
<td>5.2547</td>
<td>0</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>3</td>
<td>5.7453</td>
<td>14</td>
</tr>
<tr>
<td>IDEAL</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>3</td>
<td>4</td>
<td>11.4361</td>
</tr>
</tbody>
</table>

### Table 3.9: Ideal solution for case II.

<table>
<thead>
<tr>
<th>Priority structure</th>
<th>$x_{11}$</th>
<th>$x_{12}$</th>
<th>$x_{13}$</th>
<th>$x_{21}$</th>
<th>$x_{22}$</th>
<th>$x_{23}$</th>
<th>$x_{31}$</th>
<th>$x_{32}$</th>
<th>$x_{33}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P[C,T,D]$</td>
<td>1</td>
<td>4</td>
<td>4.277</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>3.363</td>
<td>7</td>
<td>9.637</td>
</tr>
<tr>
<td>$P[D,T,C]$</td>
<td>1</td>
<td>4.341</td>
<td>4</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>3.341</td>
<td>6.659</td>
<td>10</td>
</tr>
<tr>
<td>$P[T,C,D]$</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>3</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>$P[T,D,C]$</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>3</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>IDEAL</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>3</td>
<td>6.444</td>
<td>9.637</td>
</tr>
</tbody>
</table>

Using the ideal solutions, the $D_i$-distances are calculated for both the cases (I and II). They are shown in Table (3.10) and Table (3.11) respectively.
Table 3.10: The $D_1$-distance from the ideal solutions for case I.

<table>
<thead>
<tr>
<th>Priority structure</th>
<th>$x_{11}$</th>
<th>$x_{12}$</th>
<th>$x_{13}$</th>
<th>$x_{21}$</th>
<th>$x_{22}$</th>
<th>$x_{23}$</th>
<th>$x_{31}$</th>
<th>$x_{32}$</th>
<th>$x_{33}$</th>
<th>$(D_1)'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P[C,D,T]</td>
<td>0</td>
<td>0</td>
<td>1.8892</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>4.882</td>
<td></td>
</tr>
<tr>
<td>P[C,T,D]</td>
<td>0</td>
<td>0</td>
<td>1.8892</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>4.8892</td>
<td></td>
</tr>
<tr>
<td>P[D,C,T]</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.134</td>
<td>0</td>
<td>2.5639</td>
</tr>
<tr>
<td>P[D,T,C]</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2.5639</td>
<td>5.6979</td>
</tr>
<tr>
<td>P[T,C,D]</td>
<td>0</td>
<td>1.2547</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.7453</td>
<td>2.5639</td>
<td>5.6979</td>
</tr>
<tr>
<td>P[T,D,C]</td>
<td>0</td>
<td>1.2547</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.7453</td>
<td>2.5639</td>
<td>5.6979</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.11: The $D_1$-distance from the ideal solutions for case II.

<table>
<thead>
<tr>
<th>Priority structure</th>
<th>$x_{11}$</th>
<th>$x_{12}$</th>
<th>$x_{13}$</th>
<th>$x_{21}$</th>
<th>$x_{22}$</th>
<th>$x_{23}$</th>
<th>$x_{31}$</th>
<th>$x_{32}$</th>
<th>$x_{33}$</th>
<th>$(D_1)'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P[C,D,T]</td>
<td>0</td>
<td>0.002</td>
<td>0.323</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.325</td>
<td>0.554</td>
<td>0.04</td>
<td>1.244</td>
</tr>
<tr>
<td>P[C,T,D]</td>
<td>0</td>
<td>0</td>
<td>0.277</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.363</td>
<td>0.556</td>
<td>0</td>
<td>1.196</td>
</tr>
<tr>
<td>P[D,C,T]</td>
<td>0</td>
<td>0.556</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.556</td>
<td>0.363</td>
<td>1.475</td>
</tr>
<tr>
<td>P[D,T,C]</td>
<td>0</td>
<td>0.341</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.341</td>
<td>0.215</td>
<td>0.363</td>
<td>1.26</td>
</tr>
<tr>
<td>P[T,C,D]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.556</td>
<td>0.363</td>
<td>0.919</td>
</tr>
<tr>
<td>P[T,D,C]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.363</td>
<td>0.363</td>
<td></td>
</tr>
</tbody>
</table>
3.5 Discussion

3.5.1 Case I

A table of result’s summary is given in table (3.12), it can be seen that FP with hyperbolic membership function gives the minimum cost and damage charges, but minimum time is obtained by LGP with minimum distance approach. It can also be seen that in LGP with minimum distance approach, both the priority structures, that is P[C,D,T] and P[C,T,D] derives the same optimum solutions.

3.5.2 Case II

When \( l_{ij} \geq 0 \), it is observed that the LGP with priority structure, that is, P[T,C,D] and P[T,D,C] gives the best result in comparison of all the other used methods. However, FP with different membership functions, the hyperbolic membership function gives the minimum values of cost, damage charges and time.

3.6 Conclusion and Summary

This chapter derives the compromise integer solution of MOCFTP with mixed constraints using FP approach, in which three different forms of membership functions viz. linear, exponential and hyperbolic are used along with LGP with minimum distances approach. Two cases are studied. The results are summarized in the Table (3.12) and (3.13)

| Table (3.12): Compromise optimum solution for case I |
|---|---|---|
| **Methods** | **Objective Values** |
| | Cost | Damage Charges | Time |
| **FP with linear membership function** | 1.359296 | 1.238494 | 1.389058 |
| **FP with exponential membership function** | 1.349030 | 1.229213 | 1.314935 |
| **FP with exponential membership function** | 1.359296 | 1.238494 | 1.389058 |
| **LGP with D1-distance** | 1.353103 | 1.129854 | 1.237344 |
Table (3.13): Compromise optimum solution for case II

<table>
<thead>
<tr>
<th>Methods</th>
<th>Objective Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cost</td>
</tr>
<tr>
<td>FP with linear membership function</td>
<td>1.359296</td>
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<tr>
<td>FP with exponential membership function</td>
<td>1.332506</td>
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<tr>
<td>FP with exponential membership function</td>
<td>1.359296</td>
</tr>
<tr>
<td>LGP with $D_1$-distance</td>
<td>1.3333</td>
</tr>
</tbody>
</table>