This thesis entitled “Applications of Integer Optimization” is submitted to the Aligarh Muslim University, Aligarh, INDIA, for the degree of Doctor of Philosophy in Operations Research. It embodies the research work carried out by me in the Department of Statistics and Operations Research, Aligarh Muslim University, Aligarh.

Optimization is an act or methodology for obtaining the best result of any situation or a problem as effective as possible, keeping in view all the restrictions associated with that problem or situation. The object is to reach the best possible goal in minimum number of steps. Mathematical programming (MP) helps to represent the complicated problems into systematic manner. An MP problem includes an objective function which has to be optimised (maximised/minimised). This objective function is dependent on several decision variables whose optimal values are to be obtained. Some restrictions are also placed on the objective function which has to be satisfied before optimizing it. After the systematic representation of the problem, the next step is to choose a mathematical model to solve the optimization problem effectively. These methods are usually called optimization techniques or algorithm.

The optimization problem in which there is a restriction on certain or all decision variables to be integer is referred to as integer programming problem (IPP). In real life problems, there arise certain situations where it is required that the solution must be integer. For dealing with such situations, integer optimization is applied on these problems. Some areas where IPP can be used include production planning, scheduling, telecommunication networks, cellular networks etc. In this thesis, integer optimization is applied on various programming problems like multiobjective linear plus linear fractional programming problems and different types of transportation problems.

With the development of the world, the optimization problems have also become complicated. The problems are not single objective but in order to face the increased competition and to survive and excel in it, the objectives involving in a problem has also increased. Such type of optimization problem with multiple and conflicting objectives give rise to multiobjective optimization problem (MOOP). At the same time, these problems are comparatively difficult to solve but availability of powerful
computational tools and techniques made it easy to tackle such problems in a very short time.

Multiple objectives in transportation problems (TP) include not only the cost objective which is to be minimized (as used in classical TPs), but also objectives like minimizing the distance or minimizing the time of transportation or minimizing the deterioration of the products etc. Moreover, sometimes there are situations when the data which is available to us may be incomplete, or imprecise or vague. To deal with this uncertainty in data, the use of fuzzy set theory can become very useful.

In this thesis integer optimization is applied on some type of transportation problems along with multiobjective linear plus linear fractional programming problem.

The detail of the work contained in this thesis is summarised as follows:

Chapter I provides an introduction to optimization along with brief historical sketch of mathematical programming (linear and nonlinear), integer programming and its various methods to solve IPP. Multiobjective Programming (MOPP) along with some methods to solve MOPP like Weighted Sum Method, Goal Programming (GP) are also discussed. Programming under uncertainty is also discussed which includes Stochastic Programming (SP), Fuzzy Programming (FzP) and Interval Programming Problem (InPP). Fractional Programming (FP) is also defined. In the end the applications of Integer Optimization in various types of Transportation Problem (TP) is discussed.

In the second chapter, the Multi-Objective Linear Plus Linear Fractional Programming Problem (MOLPLFPP) is considered. The purpose is to obtain the compromise integer solution of MOLPLFPP. Firstly, the problem is formulated. The two optimization methods i.e., value function and Chebyshev goal programming methods are used to solve the problem. Then, the stepwise algorithm to solve the problem is given. Finally, numerical examples are presented with the solution procedure to demonstrate the procedure. The compromise solution (with and without integer restrictions) is obtained.

In the third chapter, a multiobjective capacitated fractional transportation problem (MOCFTP) with mixed constraints is considered to obtain compromise integer solution. The two optimization methods used are the fuzzy programming with different membership functions and the lexicographic goal programming with
minimum distance methods. The membership functions used are linear, exponential, and hyperbolic membership functions. As the problem considered is the capacitated transportation problem. Therefore, \( l_{ij} \leq x_{ij} \leq s_{ij} \) is the capacitated restriction on \( x_{ij} \) (the number of units to be transported) where, \( l_{ij} \) be the minimum and \( s_{ij} \) be the maximum amount of quantity transported. Two cases are considered in order to illustrate the efficiency of the methods used, the first case is \( l_{ij} = 0 \) when and second case is \( l_{ij} \geq 0 \).

For demonstration purpose, a numerical example is solved at the end.

In the fourth chapter, a Fully Fuzzy Multiobjective Fractional Transportation problem (FFMOFTP) is considered. Each of the coefficients in the numerator and denominator of all the objective functions, supplies and demands are considered as triangular and trapezoidal fuzzy numbers respectively. These fuzzy numbers are converted into the crisp form by using the Yager’s ranking function and the Maleki ranking function each time. The non-linear objective functions obtained are linearized by using Taylor series. And the linearized problem is solved by fuzzy goal programming approach. The problem is illustrated by taking two numerical problems of each type- one with data of triangular fuzzy numbers and other with trapezoidal fuzzy numbers. The use of the two ranking functions also helps in finding their respective efficiencies.

In the fifth chapter, we have suggested two techniques for solving Multiobjective multi-item solid transportation problem (MOMISTP) in fuzzy environment. The fuzzy problem is converted into crisp problem by using \( \alpha \)-cut. Then, the compromise integer solution of the crisp problem is derived by using Fuzzy Programming Technique and Interactive Fuzzy Goal Programming (IFGP) technique. An illustrative example is also solved to demonstrate the techniques.

In the sixth chapter, the Fully Fuzzy Multiobjective Solid Transportation (FFMOSTP) is considered. The problem is initially converted into its deterministic equivalent, and for it, the basic concept of fuzzy set theory along with the approach proposed by Kumar et al. (2011) is applied to each individual objective functions and the constraints. The classical weighted sum method is applied to the deterministic multi-objective problem to reach the compromise solution (with or without integer restriction). A numerical problem is solved to illustrate the proposed method of solving FFMOSTP.