CHAPTER VI

A FULLY FUZZY MULTI-OBJECTIVE SOLID TRANSPORTATION PROBLEM: AN INTEGER SOLUTION

6.1 Introduction

The Solid Transportation Problem (STP) with multiple objective functions has already been discussed. However, sometimes the data available in real life problems can be insufficient, imprecise or vague. To overcome these difficulties, Bellman and Zadeh (1970) proposed the concept of decision making in fuzzy environment. In chapter 4, it has shown that this concept proved very useful in dealing with real life uncertainties. For example in transportation problems sometimes the availabilities, demands and conveyance capacities are not known exactly, so they can be assumed as fuzzy or arbitrary or both.

Weighted sum method is considered as an efficient and easy tool for solving multi-objective optimization problem. Initial work on the weighted sum method can be found in Zadeh (1963). Koski (1988) applied the weighted sum method to structural optimization.

When all the parameters and variables in an MOSTP are represented by fuzzy numbers, then it is known as fully fuzzy MOSTP (FFMOSTP). Kumar et al. (2011) proposed a new method for solving fully fuzzy linear programming problems (FFLPP). Allahviranloo et al. (2008) considered the ranking function to solve FFLPP. Lotfi et al. (2009) presented lexicography method and fuzzy approximate solution to solve FFLPP. Giri et al. (2015) presented fully fuzzy fixed charge MOMISTP and proposed different approaches to solve both balanced and unbalanced forms.

Involvement of fuzzy logic with linear programming problems often complicates the problems mathematically. The solution approaches so far reviewed in literature are very difficult to implement in the real life logistic scenarios. To overcome this complexity, this chapter presents a very simple method for solving FFMOSTP. The
proposed approach is very easy to implement on small as well as large scale solid transportation problems with very easy mathematical calculations.

In this chapter, the FFMOSTP is initially converted into its deterministic equivalent, and for it, the approach proposed by Kumar et al. (2011) is applied on each individual objective functions and the constraints. The classical weighted sum method is applied on the deterministic multi-objective problem to reach at the compromise solution (integer and non-integer). Then, in section 6.2, we have reviewed some very important definitions and the arithmetic operations involved in fuzzy logic. In section 6.3, the multi-objective model for solid transportation problem with fuzzy parameters is presented. The weighted sum approach along with the stepwise algorithm of solution procedure are discussed in section 6.4 and 6.5 respectively. In section 6.6, a numerical illustration is given to clearly understand the applicability of the presented approach. Finally, some conclusions and future scope of the method are drawn in the section 6.7.

6.2 Preliminaries

Some basic definitions have already discussed in introduction. Here, we are only discussing the necessary definitions and operations used in this chapter.

**Definition 1**: A triangular fuzzy number \((a, b, c)\) is said to be non-negative fuzzy number if and only if \(a \geq 0\)

**Definition 2**: Two triangular fuzzy numbers \(\tilde{A} = (a_1, b_1, c_1)\) and \(\tilde{B} = (a_2, b_2, c_2)\) are said to be equal if and only if \(a_1 = a_2, b_1 = b_2, c_1 = c_2\).

**Definition 3 (Ranking function)**: A ranking function \(\mathcal{R} : F(R) \rightarrow R\) is a function which maps each fuzzy number into real line. Where \(F(R)\) is a set of fuzzy numbers defined on set of real numbers i.e. \(R\).

The ranking function for fuzzy number \(\tilde{A}\) is defined as follows:

\[
\mathcal{R}(\tilde{A}) = \frac{a + 2b + c}{4} \tag{6.2.1}
\]

**Arithmetic operations**

Let \(\tilde{A} = (a_1, b_1, c_1)\) and \(\tilde{B} = (a_2, b_2, c_2)\) be two triangular fuzzy numbers then some basic arithmetic operations that can be applied on these fuzzy numbers are as follows:
\[ \tilde{A} \ominus \tilde{B} = (a_1, b_1, c_1) \ominus (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2) \]  
(6.2.2)

\[ \tilde{A} \odot \tilde{B} = (a_1, b_1, c_1) \odot (a_2, b_2, c_2) = (a_1 - a_2, b_1 - b_2, c_1 - c_2) \]  
(6.2.3)

\[ \tilde{A} \oplus \tilde{B} = (a_1, b_1, c_1) \oplus (a_2, b_2, c_2) = \min(a_1a_2, a_1c_2, c_1a_2, c_1c_2), b, b, \max(a_1a_2, a_1c_2, c_1a_2, c_1c_2) \]  
(6.2.4)

\[ \lfloor \tilde{A} \rfloor = \left\{ \begin{array}{ll}
(la_1, lb_1, lc_1), & l \geq 0 \\
(la_1, lb_1, lc_1), & l \leq 0
\end{array} \right. \]  
(6.2.5)

### 6.3 Fully Fuzzy MOSTP Decision Planning Model

STP is considered as a problem of planning an optimal transport policy for transporting the items from available sources to the destinations through more than one available conveyance options. The STP may involve more than one objective function as per the decision maker’s policy. The possible objective functions may be the minimization of some transportation parameters such as the total transportation cost, total transportation time, total deterioration. In many cases, it has been found that the available information about the availability of products, demand, conveyance capacity and objective function coefficients are not known (or difficult to predict) precisely. There exists some sort of uncertainty and vagueness in the available information. We have been considered this case of uncertainty and vagueness in the available information and formulated the transportation model.

The following assumptions have been taken into account while formulating the FFMOSTP model.

- All the parameters involved in the transportation problem such as objective function coefficients, decision variables, availabilities, demands, and transportation capacities are fuzzy variables.
- The total availabilities, total demands and total conveyance capacities are equal (Balanced condition for STP).

#### 6.3.1 Notations

The following notations are used in the formulation of the FFMOSTP model.
Nomenclature

Index set

\( i \) Index for sources, for all \( i = 1,2,\ldots, p \).

\( j \) Index for destinations, for all \( j = 1,2,\ldots, q \).

\( k \) Index for conveyance, for all \( k = 1,2,\ldots, r \).

\( T \) Index for objective, for all \( T = 1,2,\ldots, t \).

Decision variable

\( \tilde{x}_{ijk} \) Fuzzy amount of product transported from source \( i \) to destination \( j \) by conveyance \( k \)

Parameters

\( \tilde{C}_{ijk}^{(T)} \) Fuzzy objective function coefficient for Coefficient \( T^{th} \) objective

\( \tilde{a}_i \) Fuzzy amount of the product available at source \( i \)

\( \tilde{b}_j \) Fuzzy demand of the product at destination \( j \)

\( \tilde{e}_k \) Fuzzy transportation capacity of the conveyance \( k \)

6.3.2 Model Formulation

Minimize \( Z_T = \sum_{i=1}^{p} \sum_{j=1}^{q} \sum_{k=1}^{r} \tilde{C}_{ijk}^{(T)} \cdot \tilde{x}_{ijk} \quad \forall \ T = 1,2,\ldots,t \).

subject to

\[ \sum_{j=1}^{q} \sum_{k=1}^{r} \tilde{x}_{ijk} = \tilde{a}_i \quad \forall \ i = 1,2,\ldots, p; \]

\[ \sum_{i=1}^{p} \sum_{k=1}^{r} \tilde{x}_{ijk} = \tilde{b}_j \quad \forall \ j = 1,2,\ldots, q; \quad (6.3.1) \]

\[ \sum_{i=1}^{p} \sum_{j=1}^{q} \tilde{x}_{ijk} = \tilde{e}_k \quad \forall \ k = 1,2,\ldots, r; \]
\[ \tilde{x}_{ijk} \geq 0 \quad \forall \ i = 1,2,..., p, \ j = 1,2,..., q, \ k = 1,2,..., r. \]

The parameters- \( \tilde{C}^{(T)}_{ijk}, \tilde{x}_{ijk}, \tilde{a}_i, \tilde{b}_j \) and \( \tilde{e}_k \) are triangular fuzzy numbers and are represented as follows:

\[
\left( C^{(T)}_{ijk}, C^{(Tm)}_{ijk}, C^{(Ta)}_{ijk}, x^m_{ijk}, x^n_{ijk}, a^u_i, a^m_i, a^l_i, b^u_j, b^m_j, b^l_j \right) \quad \text{and} \quad \left( e^u_k, e^m_k, e^l_k \right) \tag{i}
\]

The (6.3.3) model can be rewritten in view of (i) as follow: -

\[
\text{Minimize } Z_T = \sum_{i=1}^{p} \sum_{j=1}^{q} \sum_{k=1}^{r} \left( C^{(T)}_{ijk}, C^{(Tm)}_{ijk}, C^{(Ta)}_{ijk} \right) \otimes \left( x^m_{ijk}, x^n_{ijk} \right) \quad \forall \ T = 1,2,..., t.
\]

subject to

\[
\sum_{j=1}^{q} \sum_{k=1}^{r} \left( x^m_{ijk}, x^n_{ijk} \right) = \left( a^m_i, a^l_i, a^u_i \right) \quad \forall \ i = 1,2,..., p;
\]

\[
\sum_{i=1}^{p} \sum_{k=1}^{r} \left( x^m_{ijk}, x^n_{ijk} \right) = \left( b^m_j, b^l_j, b^u_j \right) \quad \forall \ j = 1,2,..., q; \tag{6.3.2}
\]

\[
\sum_{i=1}^{p} \sum_{j=1}^{q} \left( x^m_{ijk}, x^n_{ijk} \right) = \left( e^m_k, e^l_k, e^u_k \right) \quad \forall \ k = 1,2,..., r;
\]

\[
\left( x^m_{ijk}, x^n_{ijk} \right) \geq 0 \quad \forall \ i = 1,2,..., p, \ j = 1,2,..., q, \ k = 1,2,..., r.
\]

The model (6.3.2) can also be solved in the same manner with integer restrictions. In the model with integer restrictions, each element of \( \tilde{x}_{ijk} \), the fuzzy amount of product transported from source \( i \) to destination \( j \) by conveyance \( k \) are assumed to be integer.

The model (6.3.2) is converted into crisp equivalent by using the basic concepts of fuzzy set theory defined in the preliminaries section.

Therefore, the equivalent crisp model (6.3.3) with integer restriction can be given as follows: -

\[
\text{Minimize } Z'_T = \left( \sum_{i=1}^{p} \sum_{j=1}^{q} \sum_{k=1}^{r} \frac{1}{4} \left( C^{(T)}_{ijk} x^m_{ijk} + 2 C^{(Tm)}_{ijk} x^n_{ijk} + C^{(Ta)}_{ijk} x^u_{ijk} \right) \right) \quad \forall \ T = 1,2,..., t.
\]

subject to
\[ \sum_{j=1}^{r} \sum_{k=1}^{l} x_{ijk}^{(l)} = a_{i}^{(l)} \quad \forall \ i = 1, 2, \ldots, p; \]
\[ \sum_{j=1}^{r} \sum_{k=1}^{l} x_{ijk}^{(u)} = a_{i}^{(u)} \quad \forall \ i = 1, 2, \ldots, p; \]
\[ \sum_{i=1}^{p} \sum_{k=1}^{r} x_{ijk}^{(m)} = b_{j}^{(m)} \quad \forall \ j = 1, 2, \ldots, q; \]
\[ \sum_{i=1}^{p} \sum_{k=1}^{r} x_{ijk}^{(a)} = b_{j}^{(a)} \quad \forall \ j = 1, 2, \ldots, q; \]
\[ \sum_{i=1}^{p} \sum_{j=1}^{q} x_{ijk}^{(f)} = e_{k}^{(f)} \quad \forall \ k = 1, 2, \ldots, r; \]
\[ \sum_{j=1}^{q} \sum_{k=1}^{r} x_{ijk}^{(a)} = e_{k}^{(a)} \quad \forall \ k = 1, 2, \ldots, r; \]
\[ x_{ijk}^{(l)} \geq 0, \quad \forall \ i = 1, 2, \ldots, p, \ j = 1, 2, \ldots, q, \ k = 1, 2, \ldots, r; \]
\[ x_{ijk}^{(m)} - x_{ijk}^{(l)} \geq 0, \quad \forall \ i = 1, 2, \ldots, p, \ j = 1, 2, \ldots, q, \ k = 1, 2, \ldots, r; \]
\[ x_{ijk}^{(f)} \geq 0, \quad \forall \ i = 1, 2, \ldots, p, \ j = 1, 2, \ldots, q, \ k = 1, 2, \ldots, r; \]
\[ (x_{ijk}^{f}, x_{ijk}^{m}, x_{ijk}^{a}) \geq 0 \text{ and integer} \quad \forall \ i = 1, 2, \ldots, p, \ j = 1, 2, \ldots, q, \ k = 1, 2, \ldots, r. \]

The model (6.3.3) is the resultant model which is deterministic in nature and therefore, weighted sum method is used for obtaining the compromise optimal solution.

### 6.4 The Weighted Sum Approach

This is the simplest method used to solve the multi-objective optimization problems in the literature. This method works on the concept of scalarising the set of objectives into a single objective by imposing some weights on each objective. The decision maker solely decides the weight to be assigned to each of the objective function according to its importance. The objective function which is more important for decision maker may be assigned higher weight as compare to the other objective function and so on as per decision maker’s priority. The sum of all the weights assigned to each objective function should be unity. Therefore, the FFMOSTP model (6.3.3) can be written as the single objective weighted sum model as follows: -
Minimize \[ Z'_T = \sum_{T=1}^{\ell} \sum_{i=1}^{\pi} \sum_{j=1}^{\sigma} \sum_{k=1}^{\varrho} \left( C_{ijk}^{(T)} \lambda_{ijk}^T + 2 \left( C_{ijk}^{(T)} \lambda_{ijk}^m \right) + C_{ijk}^{(T_a)} \lambda_{ijk}^u \right) \]

subject to

\[ \sum_{j=1}^{\sigma} \sum_{k=1}^{\varrho} x_{ijk}^{(l)} = a_i^{(l)} \quad \forall \ i = 1,2,\ldots, p; \sum_{j=1}^{\sigma} \sum_{k=1}^{\varrho} x_{ijk}^{(m)} = a_i^{(m)} \quad \forall \ i = 1,2,\ldots, p; \]

\[ \sum_{i=1}^{\pi} \sum_{j=1}^{\sigma} x_{ijk}^{(a)} = a_i^{(a)} \quad \forall \ i = 1,2,\ldots, p; \sum_{i=1}^{\pi} \sum_{k=1}^{\varrho} x_{ijk}^{(l)} = b_j^{(l)} \quad \forall \ j = 1,2,\ldots, q; \]

\[ \sum_{i=1}^{\pi} \sum_{k=1}^{\varrho} x_{ijk}^{(m)} = b_j^{(m)} \quad \forall \ j = 1,2,\ldots, q; \sum_{i=1}^{\pi} \sum_{j=1}^{\sigma} x_{ijk}^{(a)} = b_j^{(a)} \quad \forall \ j = 1,2,\ldots, q; \]

\[ \sum_{i=1}^{\pi} \sum_{j=1}^{\sigma} x_{ijk}^{(l)} = e_k^{(l)} \quad \forall \ k = 1,2,\ldots, r; \sum_{i=1}^{\pi} \sum_{j=1}^{\sigma} x_{ijk}^{(m)} = e_k^{(m)} \quad \forall \ k = 1,2,\ldots, r; \]

\[ \sum_{i=1}^{\pi} \sum_{j=1}^{\sigma} x_{ijk}^{(a)} = e_k^{(a)} \quad \forall \ k = 1,2,\ldots, r; \]

\[ x_{ijk}^{(l)} \geq 0, \quad \forall \ i = 1,2,\ldots, p; \quad x_{ijk}^{(m)} = x_{ijk}^{(l)} \geq 0, \quad \forall \ i = 1,2,\ldots, p; \quad x_{ijk}^{(a)} = x_{ijk}^{(m)} \geq 0, \quad \forall \ i = 1,2,\ldots, p; \]

\[ x_{ijk}^{(l)} - x_{ijk}^{(m)} \geq 0, \quad \forall \ i = 1,2,\ldots, p; \quad x_{ijk}^{(a)} - x_{ijk}^{(m)} \geq 0, \quad \forall \ i = 1,2,\ldots, p; \]

\[ \left( x_{ijk}^{(l)}, x_{ijk}^{(m)}, x_{ijk}^{(a)} \right) \geq 0 \text{ and integer} \quad \forall \ i = 1,2,\ldots, p; \quad j = 1,2,\ldots, q; \quad k = 1,2,\ldots, r. \]

\[ \sum_{T=1}^{\ell} \lambda_T = 1; \lambda_T \geq 0. \]

where, \( \lambda_T \) are the assigned weights set by the decision maker.

### 6.5 Stepwise Procedure for Solving FFMOSTP

The stepwise procedure is given to obtain the compromise optimal solution to the FFMOSTP is as follows:

**Step 1.** Formulate the STP with fuzzy parameters (triangular fuzzy numbers) as a multi-objective optimization problem as per the model (6.3.2).

**Step 2.** If the STP is in unbalanced form then balance the model by adding...
fuzzy slack or fuzzy surplus variables and if it is already balanced then go to
Step 3.

**Step 3.** Apply the ranking function to defuzzify all the objective functions
and equality condition on constraints to obtain the crisp model.

**Step 4.** Apply the weighted sum method to scalarize all the objectives into a
single objective by assigning the weights to each objective function.

The problem is solved and the solution is compromise optimal solution.

### 6.6 Numerical Illustration

In this section, we have presented a numerical example for a FFMOSTP with two
objective functions. The objective functions can be of minimization of cost, time,
deterioration and number of rejected items etc.

#### 6.6.1 Problem description and data

Let the given problem has two conveyance options, namely, $k_1$ and $k_2$ with
transportation capacities, $e_1$ and $e_2$. There are $i$ ($i = 1, 2, 3$) source points with product
availability $a_i$, from where the products is to be transported to $j$ ($j = 1, 2, 3$)
destination points having the demand $b_j$. All the variables involved in the problem
are assumed to be triangular fuzzy numbers. The total of demands, supplies and
conveyance capacities are assumed to be equal, which is referred to be as balanced
STP. The data of the problem are given in tables (6.1), (6.2) and (6.3).

#### Table 6.1 Data set of objective function coefficients for conveyance 1.

<table>
<thead>
<tr>
<th>$i$</th>
<th>$j$</th>
<th>$C_{ij}^{(1)}$</th>
<th>$C_{ij}^{(2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$i$</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>$(5,8,9)$</td>
<td>$(4,6,9)$</td>
<td>$(10,12,14)$</td>
</tr>
<tr>
<td>2</td>
<td>$(8,10,13)$</td>
<td>$(6,7,8)$</td>
<td>$(11,13,15)$</td>
</tr>
<tr>
<td>3</td>
<td>$(10,12,13)$</td>
<td>$(16,17,18)$</td>
<td>$(7,9,11)$</td>
</tr>
</tbody>
</table>
Table 6.2 Data set of the objective function coefficients for conveyance 2.

<table>
<thead>
<tr>
<th>i/j</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(9,11,13)</td>
<td>(6,8,10)</td>
<td>(7,9,12)</td>
<td>(6,7,8)</td>
<td>(4,6,7)</td>
<td>(5,7,9)</td>
</tr>
<tr>
<td>2</td>
<td>(10,11,13)</td>
<td>(6,8,10)</td>
<td>(14,16,18)</td>
<td>(4,6,8)</td>
<td>(7,9,11)</td>
<td>(9,10,11)</td>
</tr>
<tr>
<td>3</td>
<td>(8,10,12)</td>
<td>(5,6,7)</td>
<td>(11,13,15)</td>
<td>(10,12,14)</td>
<td>(8,9,10)</td>
<td>(4,5,6)</td>
</tr>
</tbody>
</table>

Table 6.3 Data set of availability, demand and conveyance capacities.

<table>
<thead>
<tr>
<th></th>
<th>Availability (a_i)</th>
<th>Demand (b_j)</th>
<th>Conveyance capacity (e_k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a_1=(21,24,26)</td>
<td>b_1=(22,26,28)</td>
<td>e_1=(35,41,45)</td>
</tr>
<tr>
<td>2</td>
<td>a_2=(28,32,35)</td>
<td>b_2=(25,27,30)</td>
<td>e_2=(39,42,45)</td>
</tr>
<tr>
<td>3</td>
<td>a_3=(25,27,29)</td>
<td>b_3=(27,30,32)</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>(74,83,90)</td>
<td>(74,83,90)</td>
<td>(74,83,90)</td>
</tr>
</tbody>
</table>

By using the values given in the tables (6.1), (6.2) and (6.3). And applying the suggested approach as given in stepwise procedure section. The FFMOSEP model for the given problem will be as follows:

Minimize \( Z' = \lambda \left( \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} \frac{1}{4} \left( C_{ijk}^{(1)} x_{ijk}^{(1)} + 2 \left( C_{ijk}^{(2)} x_{ijk}^{(2)} \right) \right) \right) \\
\quad \quad + (1-\lambda) \left( \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} \frac{1}{4} \left( C_{ijk}^{(3)} x_{ijk}^{(3)} + 2 \left( C_{ijk}^{(4)} x_{ijk}^{(4)} \right) \right) \right) \\
subject to

\( \sum_{j=1}^{3} \sum_{k=1}^{3} x_{ijk}^{(1)} = a_i^{(1)} \quad \forall \quad i = 1,2,3 \)

\( \sum_{j=1}^{3} \sum_{k=1}^{3} x_{ijk}^{(2)} = a_i^{(2)} \quad \forall \quad i = 1,2,3 \)

\( \sum_{j=1}^{3} \sum_{k=1}^{3} x_{ijk}^{(3)} = a_i^{(3)} \quad \forall \quad i = 1,2,3 \)
\[ \sum_{i=1}^{3} \sum_{k=1}^{2} x_{ijk}^{(l)} = b_j^{(l)} \quad \forall \quad j = 1,2,3 \]  
(6.6.1)

\[ \sum_{i=1}^{3} \sum_{k=1}^{2} x_{ijk}^{(m)} = b_j^{(m)} \quad \forall \quad j = 1,2,3 \]

\[ \sum_{i=1}^{3} \sum_{k=1}^{2} x_{ijk}^{(u)} = b_j^{(u)} \quad \forall \quad j = 1,2,3 \]

\[ \sum_{i=1}^{3} \sum_{j=1}^{2} x_{ijk} = e_k^{(l)} \quad \forall \quad k = 1,2 \]

\[ \sum_{i=1}^{3} \sum_{j=1}^{2} x_{ijk}^{(m)} = e_k^{(m)} \quad \forall \quad k = 1,2 \]

\[ \sum_{i=1}^{3} \sum_{j=1}^{2} x_{ijk}^{(u)} = e_k^{(u)} \quad \forall \quad k = 1,2 \]

\[ x_{ijk}^{(l)} \geq 0, \quad \forall \quad i = 1,2,3, \quad j = 1,2,3, \quad k = 1,2 \]

\[ x_{ijk}^{(m)} - x_{ijk}^{(l)} \geq 0, \quad \forall \quad i = 1,2,3, \quad j = 1,2,3, \quad k = 1,2 \]

\[ x_{ijk}^{(u)} - x_{ijk}^{(m)} \geq 0, \quad \forall \quad i = 1,2,3, \quad j = 1,2,3, \quad k = 1,2 \]

\[ \left( x_{ijk}^{j}, x_{ijk}^{m}, x_{ijk}^{u} \right) \geq 0 \quad \text{and integer} \quad \forall \quad i = 1,2,3, \quad j = 1,2,3, \quad k = 1,2. \]

Here, \( \lambda \) and \((1 - \lambda)\) are the weights for the individual objective functions as defined in the section 6.4.

### 6.7 Results and Conclusion

The proposed method gives fuzzy numbers as the solutions that can be defuzzified to obtain crisp solution. The ranking technique is applied to defuzzify the obtained fuzzy solutions. The values of the weights \( \lambda \) and \((1 - \lambda)\) is decided by the decision maker.

Integer values of the each element of \( x_{ijk} \), the fuzzy amount of product transported from source \( i \) to destination \( j \) by conveyance \( k \) can also be obtained, depending on the choice of decision maker. The fuzzy compromise solutions for both the cases i.e. one with integer restriction and other without integer restriction for different weight
sets are given in tables (6.4) and (6.5).

Table (6.4) Compromise optimal solutions for different weight sets for the model without integer restrictions

<table>
<thead>
<tr>
<th>((\lambda, 1-\lambda))</th>
<th>(Z'')</th>
<th>(\tilde{Z}_1^*)</th>
<th>(Z_1^*)</th>
<th>(\tilde{Z}_2^*)</th>
<th>(Z_2^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.1)</td>
<td>448</td>
<td>(593,803,1058)</td>
<td>814.25</td>
<td>(267,450,625)</td>
<td>448</td>
</tr>
<tr>
<td>(0.1,0.9)</td>
<td>484.625</td>
<td>(593,803,1058)</td>
<td>814.25</td>
<td>(267,450,625)</td>
<td>448</td>
</tr>
<tr>
<td>(0.2,0.8)</td>
<td>520.050</td>
<td>(581,789,1042)</td>
<td>814.25</td>
<td>(265,452,631)</td>
<td>450</td>
</tr>
<tr>
<td>(0.3,0.7)</td>
<td>554.363</td>
<td>(581,780,1033)</td>
<td>800.25</td>
<td>(265,453,636)</td>
<td>451.87</td>
</tr>
<tr>
<td>(0.4,0.6)</td>
<td>584.025</td>
<td>(551,750,988)</td>
<td>793.50</td>
<td>(250,468,681)</td>
<td>466.87</td>
</tr>
<tr>
<td>(0.5,0.5)</td>
<td>613.13</td>
<td>(551,750,985)</td>
<td>759.75</td>
<td>(250,468,684)</td>
<td>467.63</td>
</tr>
<tr>
<td>(0.6,0.4)</td>
<td>636.717</td>
<td>(509,3,704,916.34)</td>
<td>708.76</td>
<td>(341,521,731.7)</td>
<td>528.67</td>
</tr>
<tr>
<td>(0.7,0.3)</td>
<td>654.725</td>
<td>(509,3,704,916.34)</td>
<td>708.76</td>
<td>(341,521,731.7)</td>
<td>528.67</td>
</tr>
<tr>
<td>(0.8,0.2)</td>
<td>672.773</td>
<td>(509,3,704,916.34)</td>
<td>708.76</td>
<td>(341,521,731.7)</td>
<td>528.67</td>
</tr>
<tr>
<td>(0.9,0.1)</td>
<td>690.742</td>
<td>(509,3,704,916.34)</td>
<td>708.76</td>
<td>(341,521,731.7)</td>
<td>528.67</td>
</tr>
<tr>
<td>(1,0)</td>
<td>708.750</td>
<td>(509,3,704,916.34)</td>
<td>708.76</td>
<td>(341,521,731.7)</td>
<td>528.67</td>
</tr>
</tbody>
</table>

Table (6.5) Compromise optimal solutions for different weight sets for the model with integer restrictions

<table>
<thead>
<tr>
<th>((\lambda, 1-\lambda))</th>
<th>(Z'')</th>
<th>(\tilde{Z}_1^*)</th>
<th>(Z_1^*)</th>
<th>(\tilde{Z}_2^*)</th>
<th>(Z_2^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.1)</td>
<td>448</td>
<td>(593,803,1058)</td>
<td>814.25</td>
<td>(267,450,625)</td>
<td>448</td>
</tr>
<tr>
<td>(0.1,0.9)</td>
<td>484.625</td>
<td>(593,803,1058)</td>
<td>814.25</td>
<td>(267,450,625)</td>
<td>448</td>
</tr>
<tr>
<td>(0.2,0.8)</td>
<td>520.050</td>
<td>(581,789,1042)</td>
<td>814.25</td>
<td>(265,452,631)</td>
<td>450</td>
</tr>
<tr>
<td>(0.3,0.7)</td>
<td>554.363</td>
<td>(581,780,1033)</td>
<td>800.25</td>
<td>(265,453,636)</td>
<td>451.87</td>
</tr>
<tr>
<td>(0.4,0.6)</td>
<td>584.025</td>
<td>(551,750,988)</td>
<td>793.50</td>
<td>(250,468,681)</td>
<td>466.87</td>
</tr>
<tr>
<td>(0.5,0.5)</td>
<td>613.13</td>
<td>(551,750,985)</td>
<td>759.75</td>
<td>(250,468,684)</td>
<td>467.63</td>
</tr>
<tr>
<td>(0.6,0.4)</td>
<td>636.717</td>
<td>(509,3,704,916.34)</td>
<td>708.76</td>
<td>(341,521,731.7)</td>
<td>528.67</td>
</tr>
<tr>
<td>(0.7,0.3)</td>
<td>654.725</td>
<td>(509,3,704,916.34)</td>
<td>708.76</td>
<td>(341,521,731.7)</td>
<td>528.67</td>
</tr>
<tr>
<td>(0.8,0.2)</td>
<td>672.773</td>
<td>(509,3,704,916.34)</td>
<td>708.76</td>
<td>(341,521,731.7)</td>
<td>528.67</td>
</tr>
<tr>
<td>(0.9,0.1)</td>
<td>690.742</td>
<td>(509,3,704,916.34)</td>
<td>708.76</td>
<td>(341,521,731.7)</td>
<td>528.67</td>
</tr>
<tr>
<td>(1,0)</td>
<td>708.750</td>
<td>(509,3,704,916.34)</td>
<td>708.76</td>
<td>(341,521,731.7)</td>
<td>528.67</td>
</tr>
</tbody>
</table>
The compromise optimal planning scheme for the transportation of goods from the sources to destinations for the models (without and with integer restrictions) is given in table (6.4) and (6.5) respectively. The values of the decision variables are calculated for different weight sets \((\lambda, 1-\lambda)\). Decision maker can give weights according to his choice and preferences.

Table 6.6 Compromise optimal scheme for the transportation of goods without integer restrictions

<table>
<thead>
<tr>
<th>Weights ((\lambda, 1-\lambda))</th>
<th>Fuzzy compromise optimal transportation policy (x_{ijk}^{*})</th>
<th>Crisp equivalent compromise optimal values (x_{ijk}^{*})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.1)</td>
<td>(x_{111} = (0,3,4), x_{121} = (21,21,22), x_{221} = (4,6,8), x_{231} = (2,3,3), x_{331} = (8,8,8), x_{212} = (22,23,24), x_{332} = (17,19,21))</td>
<td>(x_{111} = 2.5, x_{121} = 21.25, x_{221} = 6, x_{231} = 2.75, x_{331} = 8, x_{212} = 23, x_{332} = 19)</td>
</tr>
<tr>
<td>(0.1,0.9)</td>
<td>(x_{111} = (0,3,4), x_{121} = (21,21,22), x_{221} = (4,6,8), x_{231} = (2,3,3), x_{331} = (8,8,8), x_{212} = (22,23,24), x_{332} = (17,19,21))</td>
<td>(x_{111} = 2.5, x_{121} = 21.25, x_{221} = 6, x_{231} = 2.75, x_{331} = 8, x_{212} = 23, x_{332} = 19)</td>
</tr>
<tr>
<td>(0.2,0.8)</td>
<td>(x_{111} = (0,3,4), x_{121} = (19,19,20), x_{221} = (6,8,10), x_{331} = (10,10,12), x_{132} = (2,2,2), x_{212} = (22,23,24), x_{332} = (15,17,19))</td>
<td>(x_{111} = 2.5, x_{121} = 19.25, x_{221} = 8, x_{331} = 10, x_{132} = 2, x_{212} = 23, x_{332} = 17)</td>
</tr>
<tr>
<td>(0.3,0.7)</td>
<td>(x_{111} = (0,2,3), x_{121} = (19,19,19), x_{221} = (6,8,10), x_{331} = (10,12,12), x_{132} = (0,0,0.5), x_{212} = (22,24,24), x_{332} = (15,17,17))</td>
<td>(x_{111} = 1.875, x_{121} = 19, x_{221} = 8.125, x_{331} = 11.5, x_{132} = 0.125, x_{212} = 2.75, x_{212} = 23.62, x_{332} = 15.5)</td>
</tr>
<tr>
<td>(0.4,0.6)</td>
<td>(x_{111} = (0,2,3.5), x_{121} = (4,4,4), x_{221} = (6,8,10), x_{331} = (25,27,27), x_{132} = (2,3,3), x_{212} = (22,24,24), x_{332} = (0,0,2))</td>
<td>(x_{111} = 1.875, x_{121} = 4, x_{221} = 8, x_{331} = 26.5, x_{132} = 15, x_{132} = 2.75, x_{212} = 23.625, x_{332} = 0.5)</td>
</tr>
<tr>
<td>(0.5,0.5)</td>
<td>(x_{111} = (0,2,3.5), x_{121} = (4,4,4), x_{221} = (6,8,10), x_{331} = (25,27,27), x_{132} = (2,3,3), x_{212} = (22,24,24), x_{332} = (0,0,2))</td>
<td>(x_{111} = 1.875, x_{121} = 4, x_{221} = 8, x_{331} = 26.5, x_{132} = 15, x_{132} = 2.75, x_{212} = 23.625, x_{332} = 0.5)</td>
</tr>
</tbody>
</table>
| (0,6,0.4) | $x_{11}=(7.7,9.7,10.7)$, $x_{221}$  
=($13.7,15.7,17.7$), $x_{331}=(13.7,15.7,16.7)$,  
$x_{132}=(13.3,14.3,15.3)$, $x_{212}$  
=$(14.3,16.3,17.3)$, $x_{322}=(11.3,11.3,12.3)$ | $x_{111}=9.417$, $x_{221}=15.67$,  
x_{331}=15.417, $x_{132}=14.34$,  
x_{212}=16.09, $x_{322}=11.58$ |
| (0,7,0.3) | $x_{11}=(7.7,9.7,10.7)$, $x_{221}$  
=($13.7,15.7,17.7$), $x_{331}=(13.7,15.7,16.7)$,  
$x_{132}=(13.3,14.3,15.3)$, $x_{212}$  
=$(14.3,16.3,17.3)$, $x_{322}=(11.3,11.3,12.3)$ | $x_{111}=9.417$, $x_{221}=15.67$,  
x_{331}=15.417, $x_{132}=14.34$,  
x_{212}=16.09, $x_{322}=11.58$ |
| (0,8,0.2) | $x_{11}=(7.7,9.7,10.7)$, $x_{221}$  
=($13.7,15.7,17.7$), $x_{331}=(13.7,15.7,16.7)$,  
$x_{132}=(13.3,14.3,15.3)$, $x_{212}$  
=$(14.3,16.3,17.3)$, $x_{322}=(11.3,11.3,12.3)$ | $x_{111}=9.417$, $x_{221}=15.67$,  
x_{331}=15.417, $x_{132}=14.34$,  
x_{212}=16.09, $x_{322}=11.58$ |
| (0,9,0.1) | $x_{11}=(7.7,9.7,10.7)$, $x_{221}=(13.7,15.7,17.7)$  
$x_{331}=(13.7,15.7,16.7)$, $x_{132}$  
=$(13.3,14.3,15.3)$, $x_{212}=(14.3,16.3,17.3)$,  
x_{322}=(11.3,11.3,12.3) | $x_{111}=9.417$, $x_{221}=15.67$,  
x_{331}=15.417, $x_{132}=14.34$,  
x_{212}=16.09, $x_{322}=11.58$ |
| (1,0) | $x_{11}=(7.7,9.7,10.7)$, $x_{221}$  
=($13.7,15.7,17.7$), $x_{331}=(13.7,15.7,16.7)$,  
$x_{132}=(13.3,14.3,15.3)$, $x_{212}$  
=$(14.3,16.3,17.3)$, $x_{322}=(11.3,11.3,12.3)$ | $x_{111}=9.417$, $x_{221}=15.67$,  
x_{331}=15.417, $x_{132}=14.34$,  
x_{212}=16.09, $x_{322}=11.58$ |
Table 6.7 Compromise optimal scheme for the transportation of goods with integer restrictions

<table>
<thead>
<tr>
<th>Weights ((\lambda, 1 - \lambda))</th>
<th>Fuzzy compromise optimal transportation policy ((\tilde{x}_{ij}^*))</th>
<th>Crisp equivalent compromise optimal values ((x_{ij}^*))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.1)</td>
<td>(x_{111} = (0.3, 4), x_{121} = (21, 21, 22), x_{221} = (4, 6, 8)), (x_{231} = (2, 3, 3)), (x_{331} = (8, 8, 8)), (x_{212} = (22, 23, 24)), (x_{332} = (17, 19, 21))</td>
<td>(x_{111} = 2.5), (x_{121} = 21.25), (x_{221} = 6), (x_{231} = 2.75), (x_{331} = 8), (x_{212} = 23), (x_{332} = 19)</td>
</tr>
<tr>
<td>(0.1,0.9)</td>
<td>(x_{111} = (0.3, 4), x_{121} = (21, 21, 22), x_{221} = (4, 6, 8)), (x_{231} = (2, 3, 3)), (x_{331} = (8, 8, 8)), (x_{212} = (22, 23, 24)), (x_{332} = (17, 19, 21))</td>
<td>(x_{111} = 2.5), (x_{121} = 21.25), (x_{221} = 6), (x_{231} = 2.75), (x_{331} = 8), (x_{212} = 23), (x_{332} = 19)</td>
</tr>
<tr>
<td>(0.2,0.8)</td>
<td>(x_{111} = (0.3, 4), x_{121} = (19, 19, 20), x_{221} = (6, 8, 10)), (x_{331} = (10, 10, 10), x_{132} = (2, 2, 2)), (x_{212} = (22, 23, 24)), (x_{332} = (15, 17, 19))</td>
<td>(x_{111} = 1.75), (x_{121} = 19), (x_{221} = 8), (x_{331} = 10), (x_{132} = 2), (x_{212} = 23), (x_{332} = 17)</td>
</tr>
<tr>
<td>(0.3,0.7)</td>
<td>(x_{111} = (0.2, 3), x_{121} = (19, 19, 19), x_{221} = (6, 8, 10)), (x_{331} = (10, 12, 12)), (x_{132} = (2, 3, 3)), (x_{212} = (22, 24, 24)), (x_{332} = (15, 15, 17))</td>
<td>(x_{111} = 1.75), (x_{121} = 19), (x_{221} = 8), (x_{331} = 11.5), (x_{132} = 2.75), (x_{212} = 23.75), (x_{332} = 15.5)</td>
</tr>
<tr>
<td>(0.4,0.6)</td>
<td>(x_{111} = (0.2, 3), x_{121} = (4, 4, 4), x_{221} = (6, 8, 10), x_{331} = (25, 27, 27)), (x_{132} = (2, 3, 3)), (x_{132} = (13, 14, 15)), (x_{212} = (22, 24, 24)), (x_{332} = (0, 0, 2))</td>
<td>(x_{111} = 1.75), (x_{121} = 4), (x_{221} = 8), (x_{331} = 26.5), (x_{132} = 15), (x_{132} = 2.75), (x_{212} = 23.5), (x_{332} = 0.5)</td>
</tr>
<tr>
<td>(0.5,0.5)</td>
<td>(x_{111} = (0.2, 3), x_{121} = (4, 4, 4), x_{221} = (6, 8, 10), x_{331} = (25, 27, 28)), (x_{132} = (2, 3, 4)), (x_{212} = (22, 24, 25)), (x_{332} = (0, 0, 1))</td>
<td>(x_{111} = 1.75), (x_{121} = 4), (x_{221} = 8), (x_{331} = 26.75), (x_{132} = 15), (x_{132} = 3), (x_{212} = 23.75), (x_{332} = 0.25)</td>
</tr>
<tr>
<td>(0.6,0.4)</td>
<td>(x_{111} = (7, 9, 10), x_{111} = (1, 1, 1), x_{221} = (13, 15, 17)), (x_{331} = (14, 16, 17)), (x_{132} = (13, 14, 15)), (x_{212} = (15, 17, 18)), (x_{332} = (8, 8, 8))</td>
<td>(x_{111} = 8.75), (x_{111} = 1), (x_{221} = 15), (x_{331} = 15.75), (x_{132} = 14), (x_{212} = 16.75), (x_{332} = 11.25)</td>
</tr>
</tbody>
</table>
In Table (6.6), (6.7), the fuzzy as well as their equivalent crisp values for the decision variables are given. From Table (6.4), (6.5), (6.6) and (6.7), it is clear that the compromise optimal solution for some of the weight sets is same. This is because of the reason that for weighted sum technique uniformly distributed set of weights does not guarantee a uniformly distributed set of Pareto-optimal solutions. This may be considered as a drawback of the weighted sum approach. However, because of its simplicity and the advantage of finding solutions on the entire pareto-optimal set for convex problems, we have used the weighted sum approach.

### 6.7.1 Conclusions

In this chapter, we have discussed a solution approach for solving the fully fuzzy solid transportation problems with more than one objective functions. This problem
is formulated by taking into account the presence of vagueness in the input data and this vagueness in parameters of STP are defined by the triangular fuzzy numbers. The MOSTP problem is solved taking into consideration two cases i.e., one without integer restrictions and another with integer restriction. The decision maker can use any of the two cases taking into consideration the nature of his problem and the requirement of the solution to be integer or non-integer. Moreover, as weighted sum approach is used so a decision maker can give preferences or weights to objective functions as per his choice. The fuzzy set theory approach is widely used to model many real life scenarios such as supply chain modelling, information theory problems, inventory operations problems etc., hence, in future the proposed method can be extended for solving such optimization problems. The main advantage of this method is that it not only provides the easiest way to solve the fully fuzzy MOSTP but also gives the result in fuzzy form. However, the crisp form can also be obtained as per the requirement.