CHAPTER 0

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In this chapter we give the brief outline of the Thesis and the historical background of the concepts related to the Thesis.

0.1. INTRODUCTION

After the works of Stone[88] in 1937, on regular open sets, various mathematicians turned their attention to the nearly open sets which replace open sets to generalize the various concepts of topology. While open sets are replaced by nearly open sets, some results have been obtained in some occasions and in other occasions generalizations have been established. In this Dissertation we define a strongly locally finite family which is used to characterize functions weaker than continuous functions.

Using the concept of strongly locally finite family, a new class of paracompact spaces which is a subclass of the class of paracompact spaces, has been introduced and its properties have been discussed here.

Further we introduce here the concepts of p-sets and q-sets whose definitions depend on the relationships among the interiors and closures and we discuss their properties.
In 1970, Levine[47] introduced the concept of generalized closed set and investigated its properties. Since then, various mathematicians worked in this direction which leads to other of types of generalized sets. In 1997, Gnanambal[32] introduced generalized preregular closed sets and investigated their properties.

As an application to algebra, we introduce irresolute topological groups as a generalization of semi-topological group introduced by Elwood Bohn and Jong Lee[26] in 1965. We use irresolute topological groups to characterize continuous functions and other functions nearer to continuous functions.

0.2. HISTORICAL NOTES

Paracompact Spaces

In 1944, Dieudonne[16] introduced the class of paracompact spaces. He proved that every metric separable space is paracompact, that every paracompact Hausdorff space is normal, and that if $S_1$ is compact and $S_2$ is paracompact, then $S_1 \times S_2$ is paracompact. He left an open question as to whether the topological product of two paracompact spaces is paracompact. In 1947, Sorgenfrey[86] answered this question in the negative.

Dieudonne conjectured that every metric space is paracompact. In 1948, Stone[87] proved that every metric space is paracompact. In 1951, Dowker[23] introduced the class of countable
paracompact spaces which contains the classes of compact spaces, paracompact spaces and countably compact spaces. He obtained some analogous results of paracompact spaces. In 1953, Michael[55] characterized paracompact spaces using partition of unity and in 1959 Michael[57] established the famous metrization theorem which was also proved by Nagata[59] in 1957.


Nearly open sets and nearly continuous functions.

In 1947, Sorgenfrey[86] used the concept of semi-open interval of the form $a \leq x < b$ to construct a topological space which is used to answer the open question posed by Dieudonne[16]. In 1963, Levine[46] generalized semi-open interval to define a semi-open set in an arbitrary topological space and he initiated the study of semi-continuous functions. Since then, various topological properties have been studied by Crossley and Hildebrand, Biswas, Noiri, Hamlett and Dorsett using semi-open sets.

In 1965, Njaståd[60] introduced the concepts of $\alpha$-sets and $\beta$-sets and investigated their properties. It has been observed that the $\beta$-sets of Njaståd[60] are precisely the semi-open sets of Levine[46].
Hamlett[34] in 1976, established how a semi-continuous function can be constructed by "pasting" together a collection of locally semi-continuous functions.

In 1982, Mashhour[53] introduced the concept of preopen sets and precontinuous functions. Following this, Andrijevic[3], in 1984, introduced the concept of semi-preopen sets and investigated their relationship with other nearly open sets. In 1980, Jain [40] introduced the concept of totally continuous functions. Following this, in 1995, Nour[72] initiated the study of totally semi-continuous functions.

In this Thesis we further investigate the functions which are nearer to continuous functions and also the sets which are nearer to open sets.

Generalized sets

generalized preregular closed (resp. open) sets to characterize locally indiscrete spaces.

In 1999, Jin Han Park[43], investigated the properties of generalized semi-open sets. In this Thesis we characterize further properties of generalized sets.

0.3. OUTLINE OF THESIS

The Thesis consists of 1-6 chapters. Each chapter begins with an introduction followed by preliminaries needed for the development of the chapter and ends with certain open problems related to the key concept of the chapter.

Chapter 1 is devoted to strongly locally finite families by which we characterize semi-closed sets, semi-continuous functions, irresolute functions and almost continuous functions. The important result of this chapter is the generalization of "PASTING LEMMA" of continuous functions which has been proved in Section 1.4.

The object of Chapter 2 is to introduce a new class of spaces called S-paracompact spaces. The class of paracompact spaces contains properly the class of S-paracompact spaces. The important result of this chapter is that in a Hausdorff, Lindelöf and Extremally Disconnected spaces the concepts of regularity, paracompactness and S-paracompactness are equivalent.
In Chapter 3 Extremally Disconnected spaces are characterized by p-sets and p-continuity. Decomposition of totally continuous functions is an important aspect of this chapter which has been proved in Section 3.5. Also p-sets are characterized by using the concepts of locally finite and strongly locally finite families.

Chapter 4 is concerned with q-sets and q-continuity by which we characterize various types of nearly open sets and nearly continuous functions.

Chapter 5 deals with generalized sets by which we characterize complemented spaces, semi-pre $T_{1/2}$ and preregular $T_{1/2}$ spaces.

Chapter 6 is devoted to irresolute maps. In this chapter we introduce irresolute topological groups to characterize irresolute functions. A salient feature of this chapter is to find a necessary and sufficient condition for a linear functional to be irresolute which has been obtained in Section 6.3.
0.4 Symbols and Abbreviations

\[ \Rightarrow \quad \text{implies} \]
\[ \Leftrightarrow \quad \text{is implied by} \]
\[ \Leftarrow \quad \text{implies and is implied by} \]
\[ \subseteq \quad \text{is a subset of} \]
\[ \supseteq \quad \text{is a superset of} \]
\[ \square \quad \text{end of proof} \]
\[ : \quad \text{such that} \]
\[ 2^X \quad \text{the power set of } X \]
\[ \mathbb{R} \quad \text{the set of all real numbers} \]
\[ \mathbb{R}^1 \quad \text{the euclidean space.} \]
\[ \mathbb{Z} \quad \text{the set of all integers} \]
\[ \mathbb{Z}^+ \quad \text{the set of all positive integers.} \]
\[ f: X \to Y \quad \text{function from } X \text{ to } Y \]
\[ f|_A \quad \text{restriction of } f \text{ to } A \]
\[ f^{-1} \quad \text{inverse of } f \]
\[ \psi_A \quad \text{characteristic function of } A \]
\[ A \setminus B \quad \text{set theoretic difference} \]
\[ \mathfrak{S}, \sigma \quad \text{topology on a set.} \]
\[ A^0 \text{ or int } A \quad \text{interior of } A \]
\[ A^c \text{ or cl } A \quad \text{closure of } A \]
\[ \mathfrak{S}_Y \quad \text{subspace topology of } Y \subseteq X \]
\[ \text{int}_Y A \quad \text{interior of } A \text{ in the subspace } Y. \]
\[ \text{cl}_Y A \quad \text{closure of } A \text{ in the subspace } Y. \]
\[ \text{Fr} A \quad \text{boundary of } A. \]
RO(3) the collection of all regular open sets with respect to $\mathcal{J}$
RC(3) the collection of all regular closed sets with respect to $\mathcal{J}$
$\mathcal{J}^\alpha$ the collection of all $\alpha$-open sets with respect to $\mathcal{J}$
$\chi^\alpha$ $\alpha$-space
acl $A$ $\alpha$-closure of $A$
aint $A$ $\alpha$-interior of $A$
SO(3) the collection of all semi-open sets with respect to $\mathcal{J}$
SC(3) the collection of all semi-closed sets with respect to $\mathcal{J}$.
SR(3) the collection of all semi-regular sets with respect to $\mathcal{J}$.
$A_-$ or scl $A$ semi-closure of $A$
$A_0$ or sint $A$ semi-interior of $A$
PO(3) the collection of all preopen sets with respect to $\mathcal{J}$
PC(3) the collection of all preclosed sets with respect to $\mathcal{J}$
pcl $A$ preclosure of $A$
pint $A$ preinterior of $A$
SPO(3) the collection of all semi-preopen sets with respect to $\mathcal{J}$
SPC(3) the collection of all semi-preclosed sets with respect to $\mathcal{J}$
spint $A$ semi-preinterior of $A$
spcl $A$ semi-preclosure of $A$
ED Extremally Disconnected space
$\{A_\alpha : \alpha \in \mathcal{A}\}$ family of sets with index set $\mathcal{A}$.
p(3) the collection of all p-sets with respect to $\mathcal{J}$
q(3) the collection of all q-sets with respect to $\mathcal{J}$
g-closed generalized closed
g-open generalized open
sg-closed  semi-generalized closed
sg-open    semi-generalized open
\g\alpha\text{-closed}  \alpha\text{-generalized closed}
\g\alpha\text{-open}    \alpha\text{-generalized open}
\alpha\text{-closed} \alpha\text{-generalized closed}
\alpha\text{-open}    \alpha\text{-generalized open}
gs\text{-closed}  generalized semi-closed
gs\text{-open}    generalized semi-open
rg\text{-closed}  regular generalized closed
rg\text{-open}    regular generalized open
gp\text{-closed}  generalized preclosed
gp\text{-open}    generalized preopen
gsp\text{-closed}  generalized semi-preclosed
gsp\text{-open}    generalized semi-preopen
gpr\text{-closed}  generalized preregular closed
gpr\text{-open}    generalized preregular open
$L(X,Y)$ the vector space of all linear maps from vector space $X$ to vector space $Y$
$Irr(X,Y)$ the subset of $L(X,Y)$, consisting of irresolute maps.