CHAPTER 3

(τ₁, τ₂) – REGULAR GENERALIZED LOCALLY CLOSED SETS

In this Chapter we define new class of sets namely

(τ₁, τ₂) – regular generalized locally closed sets, (τ₁, τ₂) - rglc sets.

(τ₁, τ₂) - rglc" sets. Also we define new functions on them. Their properties are investigated.

3.1 INTRODUCTION

K. Balachandran, P. Sundaram and H. Maki [5] introduced the concept of

generalized locally closed sets and GLC continuous functions.

Regular generalized locally closed sets in unital topological spaces are

introduced by I. Arockiarani, K. Balachandran and M. Ganster [3]. In this chapter it is shown that some of the results may be extended to bitopological spaces.

We recall

DEFINITION 3.1.1

Let (X, τ₁, τ₂) be a bitopological space. A subset A of X is said to be

(τ₁, τ₂) – regular open if A = τ₁ - int ( τ₂ - cl ( A ) ).
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**DEFINITION 3.1.2**

Let \((X, \tau_1, \tau_2)\) be a bitopological space. A subset \(A\) of \(X\) is said to be \((\tau_1, \tau_2)\) - regular closed if \(A = \tau_2 \cap \text{int} (\tau_1 \cap \text{cl}(A))\).

**DEFINITION 3.1.3**

Let \((X, \tau_1, \tau_2)\) be a bitopological space. A subset \(A\) of \(X\) is said to be \((\tau_1, \tau_2)\) - regular generalized closed \([\text{briefly } (\tau_1, \tau_2)\) - rg closed\] \([2.2.5]\).

if \(\tau_i \cap \text{cl}(A) \subseteq U\) for \(i = 1\) or \(2\) whenever \(A \subseteq U\) and \(U\) is \((\tau_1, \tau_2)\) - regular open.

**DEFINITION 3.1.4**

The complement of \((\tau_1, \tau_2)\) - regular generalized closed set is called \((\tau_1, \tau_2)\) - regular generalized open set \([\text{briefly } (\tau_1, \tau_2)\) - rg open\].

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**DEFINITION 3.2.1**

A subset \(S\) of \((X, \tau_1, \tau_2)\) is said to be \((\tau_1, \tau_2)\) - regular generalized locally closed \([\text{briefly } (\tau_1, \tau_2)\) rg lc\] if \(S = G \cap F\) where \(G\) is \((\tau_1, \tau_2)\) - rg open and \(F\) is \((\tau_1, \tau_2)\) - rg closed in \((X, \tau_1, \tau_2)\).
EXAMPLE 3.2.2

Let $X = \{a, b, c\}$.

Let $\tau_1 = \{X, \phi, \{a\}, \{a, b\}\}$ and $\tau_2 = \{\phi, X, \{c\}, \{a, c\}\}$ be topologies on $X$. Take $G = \{a, b\}$ and $F = \{b\}$. Then $G$ is $(\tau_1, \tau_2)$-rg open and $F$ is $(\tau_1, \tau_2)$-rg closed. Then $\{b\}$ is $(\tau_1, \tau_2)$-rgc.

DEFINITION 3.2.3

A subset $B$ of $(X, \tau_1, \tau_2)$ is said to be $(\tau_1, \tau_2)$-rgc if there exists a $(\tau_1, \tau_2)$-rg open set $G$ and a $\tau_2$-closed set $F$ of $(X, \tau_1, \tau_2)$ such that $B = G \cap F$.

EXAMPLE 3.2.4

Let $X = \{a, b, c\}$.

Let $\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$ and $\tau_2 = \{\phi, X, \{b\}, \{a, b\}\}$ be topologies on $X$.

Take $G = \{a, b\}$ and $F = \{a, c\}$.

Then $G$ is $(\tau_1, \tau_2)$-rg open and $F$ is $\tau_2$-closed.

Then $G \cap F = \{a\}$ is $(\tau_1, \tau_2)$-rgc.
DEFINITION 3.2.5

A subset $S$ of $(X, \tau_1, \tau_2)$ is said to be $(\tau_1, \tau_2)$-rglc if there exist $\tau_1$-open set $G$ and $(\tau_1, \tau_2)$-rg closed set $F$ such that $S = G \cap F$.

EXAMPLE 3.2.6

In Example 3.2.2, Take $G = \{a, b\}$ which is $\tau_1$-open and $F = \{b\}$ which is $(\tau_1, \tau_2)$-rg closed. Then $G \cap F = \{b\}$ is $(\tau_1, \tau_2)$-rglc.

DEFINITION 3.2.7

A set $A$ is said to be $(\tau_1, \tau_2)$-locally closed if $A = S \cap F$ where $S$ is $\tau_1$-open and $F$ is $\tau_2$-closed.

EXAMPLE 3.2.8

In Example 3.2.2,

Take $S = \{a\}$ which is $\tau_1$-open and $F = \{a, b\}$ which is $\tau_2$-closed.

Then $A = S \cap F = \{a\}$ is $(\tau_1, \tau_2)$-locally closed.

THEOREM 3.2.9

For a subset $S$ of $(X, \tau_1, \tau_2)$ the following are equivalent:
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(i) $S \in (\tau_1, \tau_2) - \text{RGLC}^* (X, \tau_1, \tau_2)$.

(ii) $S = P \cap \tau_2 - \text{cl}(S)$ for some $(\tau_1, \tau_2) - \text{rg open}$ set $P$.

(iii) $\tau_2 - \text{cl}(S) \setminus S$ is $(\tau_1, \tau_2) - \text{rg closed}$.

(iv) $S \cap (X \setminus \tau_2 - \text{cl}(S))$ is $(\tau_1, \tau_2) - \text{rg open}$.

Proof (i) $\Rightarrow$ (ii): Let $S \in (\tau_1, \tau_2) - \text{RGLC}^* (X, \tau_1, \tau_2)$.

$\Rightarrow S = P \cap F$ where $P$ is $(\tau_1, \tau_2) - \text{rg open}$ and $F$ is $\tau_2$ - closed.

Since $S \subseteq P$ and $S \subseteq \tau_2 - \text{cl}(S)$ we have $S \subseteq P \cap \tau_2 \text{cl}(S)$.

Conversely let $S \subseteq F$.

Then $\tau_2 \text{cl}(S) \subseteq \tau_2 - \text{cl}(F) = F$.

Therefore $\tau_2 - \text{cl}(S) \subseteq F$.

And hence $P \cap \tau_2 - \text{cl}(S) \subseteq P \cap F = S$.

So $P \cap \tau_2 - \text{cl}(S) \subseteq S$.

From (1) and (2) $S = P \cap \tau_2 - \text{cl}(S)$.

(ii) $\Rightarrow$ (i): Since $P$ is $(\tau_1, \tau_2) - \text{rg open}$ and $\tau_2 - \text{cl}(S)$ is $\tau_2$ - closed, we have

$P \cap \tau_2 - \text{cl}(S) \in (\tau_1, \tau_2) - \text{RGLC}^* (X, \tau_1, \tau_2)$.

(iii) $\Rightarrow$ (iv): Let $F = \tau_2 - \text{cl}(S) \setminus S$. Then $F$ is $(\tau_1, \tau_2) - \text{rg closed}$.
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\[ X \setminus F = X \cap [X \setminus (\tau_2 \cdot \text{cl}(S)) \setminus S] \]

\[ = S \cup [X \setminus (\tau_2 \cdot \text{cl}(S))] \tag{3} \]

\(F\) is \((\tau_1, \tau_2)\)-rg closed implies \(X \setminus F\) is \((\tau_1, \tau_2)\)-rg open and hence by (3)

\[ S \cup (X \setminus \tau_2 \cdot \text{cl}(S)) \text{ is } (\tau_1, \tau_2)\text{-rg open.} \]

(iv) \(\Rightarrow\) (iii): Let \(U = S \cup (X \setminus \tau_2 \cdot \text{cl}(S))\). Then \(U\) is \((\tau_1, \tau_2)\)-rg open.

That implies \(X \setminus U\) is \((\tau_1, \tau_2)\)-rg closed.

And \(X \setminus U = (X \setminus [S \cup (X \setminus \tau_2 \cdot \text{cl}(S))])\).

\[ = \tau_2 \cdot \text{cl}(S) \cap (X \setminus S) \]

\[ = \tau_2 \cdot \text{cl}(S) \setminus S. \]

Therefore \(\tau_2 \cdot \text{cl}(S) \setminus S\) is \((\tau_1, \tau_2)\)-rg closed.

(iv) \(\Rightarrow\) (ii): Let \(U = S \cup (X \setminus \tau_2 \cdot \text{cl}(S))\). Then \(U\) is \((\tau_1, \tau_2)\)-rg open.

We have to prove \(S = U \cap \tau_2 \cdot \text{cl}(S)\) for some \((\tau_1, \tau_2)\)-rg open set \(U\).

Now \(U \cap \tau_2 \cdot \text{cl}(S) = \{S \cup (X \setminus \tau_2 \cdot \text{cl}(S))\} \cap \tau_2 \cdot \text{cl}(S)\).

\[ = [\tau_2 \cdot \text{cl}(S) \cap S] \cup [\tau_2 \cdot \text{cl}(S) \cap (X \setminus \tau_2 \cdot \text{cl}(S))] \]

\[ = S \cup \emptyset = S. \]

That implies \(S = U \cap \tau_2 \cdot \text{cl}(S).\)
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\((ii) \Rightarrow (iv): S = P \cap \tau_2 - \text{cl}(S)\) for some \((\tau_1, \tau_2)\)-rg open set \(P\). Then we have to prove \(S \cup (X \setminus \tau_2 - \text{cl}(S))\) is \((\tau_1, \tau_2)\)-rg open.

Now \(S \cup (X \setminus \tau_2 - \text{cl}(S)) = (P \cap \tau_2 - \text{cl}(S)) \cup (X \setminus \tau_2 - \text{cl}(S))\).

\[= P \cap X = P \text{ which is } (\tau_1, \tau_2)\text{-rg open.}\]

And hence \(S \cup (X \setminus \tau_2 - \text{cl}(S))\) is \((\tau_1, \tau_2)\)-rg open.

**THEOREM 3.2.10**

For a subset \(S\) of \((X, \tau_1, \tau_2)\), if \(S \in (\tau_1, \tau_2)\)-RGLC* \((X, \tau_1, \tau_2)\), then there exists an \(\tau_1\)-open set \(P\) such that \(S = P \cap \text{cl}_r(S)\) where \(\text{cl}_r(S)\) is the \((\tau_1, \tau_2)\)-rg closure of \(S\).

**PROOF**

Let \(S \in (\tau_1, \tau_2)\)-RGLC* \((X, \tau_1, \tau_2)\).

Then there exists an \(\tau_1\)-open set \(P\) and \((\tau_1, \tau_2)\)-rg closed set \(F\) such that

\[S = P \cap F.\]

Since \(S \subseteq P\) and \(S \subseteq \text{cl}_r(S)\) we have \(S \subseteq P \cap \text{cl}_r(S)\) --- (4)

Conversely since \(S \subseteq F\), we have

\[\text{cl}_r(S) \subseteq \text{cl}_r(F) = F.\]

Therefore \(\text{cl}_r(S) \subseteq F\). That implies
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P \cap \text{cl}_r(S) \subseteq P \cap F = S \text{ and hence}

P \cap \text{cl}_r(S) \subseteq S \quad \text{(5)}

From (4) and (5) we have

S = P \cap \text{cl}_r(S).

**THEOREM 3.2.11**

Let A and B are subsets of \((X, \tau_1, \tau_2)\). If A, B are \((\tau_1, \tau_2)\)-rglc',

then \(A \cap B\) is also \((\tau_1, \tau_2)\)-rglc'.

**PROOF**

Let A and B are \((\tau_1, \tau_2)\)-rglc' sets of \((X, \tau_1, \tau_2)\). Then there exist \((\tau_1, \tau_2)\)-rg open sets P and Q such that \(A = P \cap \tau_2 - \text{cl}(A)\) and

\(B = Q \cap \tau_2 - \text{cl}(B)\).

But then \(A \cap B = (P \cap \tau_2 - \text{cl}(A)) \cap (Q \cap \tau_2 - \text{cl}(B))\). That implies

\(A \cap B = (P \cap Q) \cap (\tau_2 - \text{cl}(A) \cap \tau_2 - \text{cl}(B))\).

Since \(P\) and \(Q\) are \((\tau_1, \tau_2)\)-rg open sets, we have that \(P \cap Q\) is also \((\tau_1, \tau_2)\)-rg open set.

Also \(\tau_2 - \text{cl}(A) \cap \tau_2 - \text{cl}(B)\) is again \(\tau_2\)-closed set. Hence \(A \cap B\) is
THEOREM 3.2.12

If $A$ is $(\tau_1, \tau_2)$-rglc" and $B$ is $\tau_1$-open, then $A \cap B$ is also $(\tau_1, \tau_2)$-rglc" in $(X, \tau_1, \tau_2)$.

PROOF

Let $A$ is $(\tau_1, \tau_2)$-rglc".

Then there exist $\tau_1$-open set $G$ and a $(\tau_1, \tau_2)$-rg closed set $F$ such that $A = G \cap F$.

Now $A \cap B = G \cap F \cap B$.

$$= (G \cap B) \cap F.$$ 

But $G$ and $B$ are $\tau_1$-open sets. Hence $G \cap B$ is also $\tau_1$-open set.

Also $F$ is $(\tau_1, \tau_2)$-rg closed set. Hence $A \cap B$ is $(\tau_1, \tau_2)$-rglc".

THEOREM 3.2.13

If $A$ is $(\tau_1, \tau_2)$-rg lc and $B$ is $(\tau_1, \tau_2)$-rg open, then $A \cap B$ is $(\tau_1, \tau_2)$-rglc in $(X, \tau_1, \tau_2)$. 

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**PROOF**

Let \( A \) is \((\tau_1, \tau_2)\)-rglc in \((X, \tau_1, \tau_2)\).

Then \( A = G \cap F \) where \( G \) is \((\tau_1, \tau_2)\)-rg open and \( F \) is \((\tau_1, \tau_2)\)-rg closed. Accordingly

\[ A \cap B = (G \cap F) \cap B. \]

\[ = (G \cap B) \cap F. \]

Now \( G \) and \( B \) are \((\tau_1, \tau_2)\)-rg open. Hence \( G \cap B \) is \((\tau_1, \tau_2)\)-rg open. Also \( F \) is \((\tau_1, \tau_2)\)-rg closed. Therefore \( A \cap B \) is \((\tau_1, \tau_2)\)-rglc in \((X, \tau_1, \tau_2)\).

**THEOREM 3.2.14**

If \( A \) is \((\tau_1, \tau_2)\)-rglc in \((X, \tau_1, \tau_2)\) and \( B \) is \((\tau_1, \tau_2)\)-generalized closed \(\tau_1\)-open subset of \( X \), then \( A \cap B \) is \((\tau_1, \tau_2)\)-rglc in \( X \).

**PROOF**

Let \( A \) is \((\tau_1, \tau_2)\)-rglc* in \((X, \tau_1, \tau_2)\)

Then \( A = G \cap F \) where \( G \) is \((\tau_1, \tau_2)\)-rg open and \( F \) is \(\tau_2\)-closed. Accordingly

\[ A \cap B = (G \cap F) \cap B = \emptyset \cap (F \cap B). \]
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Now G is (τ₁, τ₂) - rg open.

We have to prove F ∩ B is (τ₁, τ₂) - rg closed.

Let F ∩ B ⊆ U is (τ₁, τ₂) - regular open.

Now U is (τ₁, τ₂) - regular open. Hence U is τ₁ - open.

Since B is (τ₁, τ₂) - generalized closed, we have

τ₂ - cl (B) ⊆ U whenever B ⊆ U and U is τ₁ - open.

Now F ∩ B ⊆ B. That implies

τ₂ - cl (F ∩ B) ⊆ τ₂ - cl (B) ⊆ U. And hence

τ₂ - cl (F ∩ B) ⊆ U and U is (τ₁, τ₂) - regular open. Therefore

F ∩ B is (τ₁, τ₂) - rg closed and hence A ∩ B is (τ₁, τ₂) - rg lc in

(X, τ₁, τ₂).

THEOREM 3.2.15

Let A and Z be subsets of (X, τ₁, τ₂) and let A ⊆ Z. If Z is

(τ₁, τ₂) - rg open in (X, τ₁, τ₂) and A ∈ (τ₁, τ₂) - RGLC⁺ of

(Z, τ₁|Z, τ₂|Z), then A ∈ (τ₁, τ₂) - RGLC⁺ (X, τ₁, τ₂).
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PROOF

Suppose A is (τ₁, τ₂) - rglc* (Z, τ₁ | Z, τ₂ | Z). Then there exists a

(τ₁, τ₂) - rg open set G of (Z, τ₁ | Z, τ₂ | Z) such that A = G ∩ τ₂ - clε (A).

But τ₂ - clε (A) = Z ∩ τ₂ - cl (A).

Therefore A = (G ∩ Z) ∩ τ₂ - cl (A).

Now G is (τ₁, τ₂) - rg open and Z is (τ₁, τ₂) - rg open. That implies

G ∩ Z is (τ₁, τ₂) - rg open and hence A is (τ₁, τ₂) - rglc* in (X, τ₁, τ₂).

THEOREM 3.2.16

If Z is (τ₁, τ₂) - generalized closed τ₁ -open set in (X, τ₁, τ₂) and

A ∈ (τ₁, τ₂) - RGLC* (Z, τ₁ | Z, τ₂ | Z), then A is (τ₁, τ₂) - RGLC in

(X, τ₁, τ₂).

PROOF

Let A ∈ (τ₁, τ₂) - RGLC* (Z, τ₁ | Z, τ₂ | Z).

Then A = G ∩ F where G is (τ₁, τ₂) - rg open and F is τ₂ - closed in

(Z, τ₁ | Z, τ₂ | Z). Therefore

F = B ∩ Z for some τ₂ - closed set B of (X, τ₁, τ₂).
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Hence \(A = G \cap (B \cap Z)\).

Now \(Z\) is \((\tau_1, \tau_2)\) - generalized closed and \(B\) is \(\tau_2\) - closed. Therefore

\(B \cap Z\) is \((\tau_1, \tau_2)\) - rg closed [by 2.2.11]. Hence

\(A\) is \((\tau_1, \tau_2)\) - RGLC in \((X, \tau_1, \tau_2)\). \(\square\)

**THEOREM 3.2.17**

If \(Z\) is \((\tau_2, \tau_1)\) - generalized closed and \(\tau_2\) - open subset of \((X, \tau_1, \tau_2)\) and \(A\) is \((\tau_2, \tau_1)\) - RGLC" set in \((Z, \tau_1 | Z, \tau_2 | Z)\), then \(A\) is \((\tau_2, \tau_1)\) - RGLC" in \((X, \tau_1, \tau_2)\).

**PROOF**

Let \(A\) be \((\tau_2, \tau_1)\) - RGLC" set in \((Z, \tau_1 | Z, \tau_2 | Z)\).

Then \(A = G \cap F\) where \(G\) is \(\tau_2\) - open and \(F\) is \((\tau_2, \tau_1)\) - rg closed in \((Z, \tau_1 | Z, \tau_2 | Z)\).

Now \(Z\) is \((\tau_2, \tau_1)\) generalized closed \(\tau_2\) - open set and \(F\) is \((\tau_2, \tau_1)\) - rg closed in \(Z\). Then by [2.2.10] \(F\) is \((\tau_2, \tau_1)\) - rg closed in \((X, \tau_1, \tau_2)\).

Now \(A = G \cap F\) where \(G\) is \(\tau_2\) - open and \(F\) is \((\tau_2, \tau_1)\) - rg closed in \((X, \tau_1, \tau_2)\).
Theorem 3.2.18

If \( A \) is \((\tau_1, \tau_2)\)-rg open and \( B \) is \(\tau_1\)-open, then \( A \cap B \) is

\((\tau_1, \tau_2)\)-rg open.

Proof

Let \( A \) be \((\tau_1, \tau_2)\)-rg open set in \((X, \tau_1, \tau_2)\).

Then \( X - A \) is \((\tau_1, \tau_2)\)-rg closed in \((X, \tau_1, \tau_2)\).

Let \( U \) be any \((\tau_1, \tau_2)\)-regular open set.

Let \( X - A \subset U \).

Then \( \tau_1 - \text{cl}(X - A) \subset U \). \hfill (6)

Let \( (X - (A \cap B)) \subset U \).

That is, \( (X - A) \cup (X - B) \subset U \).

By hypothesis \( B \) is \(\tau_1\)-open.

Accordingly \( X - B \) is \(\tau_1\)-closed set.

Hence \( \tau_1 - \text{cl}(X - B) = X - B \subset U \) \hfill (7)

From (6) and (7) we obtain
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\[ \{\tau_1 - \text{cl} (X - A)\} \cup \{\tau_1 - \text{cl} (X - B)\} \subseteq U \]

Therefore \(\tau_1 - \text{cl} \{X - (A \cap B)\} \subseteq U\)

Hence \(X - (A \cap B)\) is \((\tau_1, \tau_2)\)-rg closed.

Therefore \((A \cap B)\) is \((\tau_1, \tau_2)\)-rg open.

\[ \square \]

**THEOREM 3.2.19**

Let \((X, \tau_1, \tau_2)\) and \((Y, \sigma_1, \sigma_2)\) be bitopological spaces. Then

(i) If \(A\) is \((\tau_1, \tau_2)\)-RGLC in \((X, \tau_1, \tau_2)\) and \(B\) is \((\sigma_1, \sigma_2)\)-RGLC in \((Y, \sigma_1, \sigma_2)\), then \(A \times B\) is \((\tau_1 \times \sigma_1, \tau_2 \times \sigma_2)\)-RGLC in \((X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)\).

(ii) If \(A\) is \((\tau_1, \tau_2)\)-RGLC* in \((X, \tau_1, \tau_2)\) and \(B\) is \((\sigma_1, \sigma_2)\)-RGLC* in \((Y, \sigma_1, \sigma_2)\), then \(A \times B\) is \((\tau_1 \times \sigma_1, \tau_2 \times \sigma_2)\)-RGLC* in \((X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)\).

(iii) If \(A\) is \((\tau_1, \tau_2)\)-RGLC** in \((X, \tau_1, \tau_2)\) and \(B\) is \((\sigma_1, \sigma_2)\)-RGLC** in \((Y, \sigma_1, \sigma_2)\), then \(A \times B\) is \((\tau_1 \times \sigma_1, \tau_2 \times \sigma_2)\)-RGLC** in \((X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)\).
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PROOF

(i) Let \(A\) is \((\tau_1, \tau_2)\)-RGLC in \((X, \tau_1, \tau_2)\) and \(B\) is \((\sigma_1, \sigma_2)\)-RGLC in \((Y, \sigma_1, \sigma_2)\). Then there exist \((\tau_1, \tau_2)\)-rg open set \(V\) and \((\tau_1, \tau_2)\)-rg closed set \(W\) such that \(A = V \cap W\), and \((\sigma_1, \sigma_2)\)-rg open set \(V'\) and \((\sigma_1, \sigma_2)\)-rg closed set \(W'\) such that \(B = V' \cap W'\). Now \(A \times B = (V \cap W) \times (V' \cap W') = (V \times V') \cap (W \times W')\)

where \(V \times V'\) is \((\tau_1 \times \sigma_1, \tau_2 \times \sigma_2)\)-rg open set and \(W \times W'\) is \((\tau_1 \times \sigma_1, \tau_2 \times \sigma_2)\)-rg closed set.

Therefore \(A \times B\) is \((\tau_1 \times \sigma_1, \tau_2 \times \sigma_2)\)-RGlc in \((X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)\).

Similarly (ii) and (iii) can be proved.

THEOREM 3.2.20

Every \((\tau_1, \tau_2)\) - locally closed set is \((\tau_1, \tau_2)\)-rglc in \((X, \tau_1, \tau_2)\)

PROOF

Let \(A\) is \((\tau_1, \tau_2)\)-locally closed. Then

\[A = S \cap F\]

where \(S\) is \(\tau_1\)-open and \(F\) is \(\tau_2\)-closed.

But \(S\) is \(\tau_1\)-open \(\Rightarrow \tau_1\)-int \(S = S\).
Let $S \supset U$ where $U$ is $(\tau_1, \tau_2)$-regular closed.

Now $\tau_1 - \text{int} \ S = S \supset U$. That implies

$\tau_1 - \text{int} \ S \supset U$, $S \supset U$ and $U$ is $(\tau_1, \tau_2)$-regular closed.

Therefore $S$ is $(\tau_1, \tau_2)$-rg open.

Similarly we can prove $F$ is $(\tau_1, \tau_2)$-rg closed.

And hence $A = S \cap F$ where $S$ is $(\tau_1, \tau_2)$-rg open and $F$ is

$(\tau_1, \tau_2)$-rg closed. Therefore $A$ is $(\tau_1, \tau_2)$-rglc.

The converse of the above theorem need not be true as shown from the following example

**EXAMPLE 3.2.21.**

Let $X = \{a, b, c\}$.

Let $\tau_1 = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}\}$ and

$\tau_2 = \{\phi, X, \{a\}, \{a, b, c\}\}$ be topologies on $X$.

$\tau_2$-closed sets are $\phi, X, \{b, c, d\}, \{d\}$ and

$(\tau_1, \tau_2)$-locally closed sets are $\phi, X, \{a\}, \{a, b, c\}, \{b\}, \{a, b\}, \{b, c, d\}, \{b, c\}, \{d\}$. 

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Take \( G = \{ b, c, d \} \). Then \( G \) is \((τ₁, τ₂)\) - rg open set.

Take \( F = \{ c \} \) which is \((τ₁, τ₂)\) - rg closed set.

Then \( A = G \cap F = \{ c \} \) which is \((τ₁, τ₂)\) - rglc but it is not \((τ₁, τ₂)\) - locally closed in \((X, τ₁, τ₂)\).

**THEOREM 3.2.22**

Every \((τ₁, τ₂)\) - rglc* set or \((τ₁, τ₂)\) - rglc** set is \((τ₁, τ₂)\) - rglc.

**PROOF**

Let \( A \) be \((τ₁, τ₂)\) - rglc*. Then

\[ A = G \cap F \text{ where } G \text{ is } (τ₁, τ₂) - \text{rg open and } F \text{ is } τ₂ - \text{closed}. \]

We have to prove \( F \) is \((τ₁, τ₂)\) - rg closed.

Let \( U \) be any \((τ₁, τ₂)\) - regular open set containing \( F \).

Since \( F \) is \( τ₂ \) - closed we have \( τ₂ - \text{cl}(F) = F \).

Now \( F \subseteq U \) implies \( τ₂ - \text{cl}(F) \subseteq U \) and \( U \) is \((τ₁, τ₂)\) - regular open.

Therefore \( F \) is \((τ₁, τ₂)\) - rg closed. And hence \( A \) is \((τ₁, τ₂)\) - rglc.

Similarly we can prove if \( A \) is \((τ₁, τ₂)\) - rglc** then it is \((τ₁, τ₂)\) - rglc.
EXAMPLE 3.2.23

The converse of the above theorem need not be true as shown by the following example.

Let $X = \{a, b, c\}$.

Let $\tau_1 = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{c\}, \{a, c\}\}$ be topologies on $X$.

Now $\{b, c\}$ is $(\tau_1, \tau_2)$-rglc but $\{b, c\}$ is not $(\tau_1, \tau_2)$-rglc*.

3.3. $(\tau_1, \tau_2)$ - RGL CONTINUITY

DEFINITION 3.3.1

A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be rgl continuous [3] (rgl* continuous, rgl** continuous) if $f^1(V) \in \text{RGLC}(x, \tau)$

$[f^1(V) \in \text{RGLC}^*(x, \tau), f^1(V) \in \text{RGLC}^{**}(x, \tau)]$ for every $V \in \sigma$.

DEFINITION 3.3.2

A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be LC- continuous [11] if $f^1(V)$ is locally closed in $(x, \tau)$ for every $V \in (Y, \sigma)$. 
(\tau_1, \tau_2) \text{ Regular generalized locally closed sets}

**DEFINITION 3.3.3**

A function \( f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \) is called pairwise \( \text{rgl} \)-continuous (pairwise \( \text{rgl}' \)-continuous, pairwise \( \text{rgl}'' \)-continuous) if

\( f: (X, \tau_1) \to (Y, \sigma_1) \) and \( f: (X, \tau_2) \to (Y, \sigma_2) \) are \( \text{rgl} \)-continuous (\( \text{rgl}' \)-continuous, \( \text{rgl}'' \)-continuous).

**DEFINITION 3.3.4**

A function \( f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \) is called pairwise \( \text{LC} \)-continuous if \( f: (X, \tau_1) \to (Y, \sigma_1) \) and \( f: (X, \tau_2) \to (Y, \sigma_2) \) are \( \text{LC} \)-continuous.

**THEOREM 3.3.5**

Let \( f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \) be a function.

(i) If \( f \) is pairwise \( \text{LC} \), then it is pairwise \( \text{rgl} \)-continuous.

(ii) If \( f \) is pairwise \( \text{rgl}' \)-continuous, then it is pairwise \( \text{rgl} \)-continuous.

**PROOF**

(i) Given \( f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2) \) be pairwise \( \text{LC} \) continuous.

Then \( f: (X, \tau_1) \to (Y, \sigma_1) \) and \( f: (X, \tau_2) \to (Y, \sigma_2) \) are
\((\tau_1, \tau_2)\) Regular generalized locally closed sets

LC - continuous.

Then by theorem 17 of [3] we have

\[ f: (X, \tau_1) \rightarrow (Y, \sigma_1) \text{ and } \]

\[ f: (X, \tau_2) \rightarrow (Y, \sigma_2) \text{ are rgl - continuous. Therefore } \]

\[ f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \text{ is pairwise rgl-continuous.} \]

In a similar way ( ii ) can be proved.