PREFACE

This thesis is devoted to the study of some of the topological properties with respect to an ideal.

Given a topological space $(X, \tau)$ and an ideal $\mathcal{I}$ on $X$, the set
\[ \beta(\mathcal{I}, \tau) = \{ V - 1 : V \in \tau \text{ and } I \in \mathcal{I} \} \]
is a basis for a topology $\tau^*(\mathcal{I})$ which is finer than $\tau$. The first unified and extensive study on $\tau^*(\mathcal{I})$-topology was done by Jankovic and Hamlett.

Given ideals $\mathcal{I}$ and $\mathcal{J}$ on two topological spaces $(X, \tau_1)$ and $(Y, \tau_2)$ respectively, there is a natural way to define an ideal $\mathcal{I} \times \mathcal{J}$, called the product ideal of $\mathcal{I} \times \mathcal{J}$ on $X \times Y$ and we get the topology $(\tau_1 \times \tau_2)^*(\mathcal{I} \times \mathcal{J})$ on the space $X \times Y$. There is another topology $\tau_1^*(\mathcal{I}) \times \tau_2^*(\mathcal{J})$, the product topology on $X \times Y$, using the topologies $\tau_1^*(\mathcal{I})$ and $\tau_2^*(\mathcal{J})$ of $X$ and $Y$ respectively. In the second chapter, inclusion relations between $\tau_1^*(\mathcal{I}) \times \tau_2^*(\mathcal{J})$ and $(\tau_1 \times \tau_2)^*(\mathcal{I} \times \mathcal{J})$ have been investigated. Necessary and sufficient conditions on $\mathcal{I}$ and $\mathcal{J}$ under which these two topologies on $X \times Y$ coincide. These results are extended for infinite products.

Extensive study on various topological properties with respect to ideals has been carried out by T.R. Hamlett, Dragan Jankovic and D. Rose. But connectedness property has not been discussed by them. The connectedness property with respect to an ideal has been dealt in the third chapter. The concept of $\mathcal{I}$-connected spaces and strongly $\mathcal{I}$-connected spaces have been introduced and the properties of such spaces are discussed. Conditions on $\mathcal{I}$ are obtained under which connectedness, $\mathcal{I}$-connectedness, strongly $\mathcal{I}$-connectedness are all equivalent.
Viglino has introduced a class of topological spaces called C-compact spaces. Newcomb has introduced the concept of compactness with respect to an ideal in his Ph.D. thesis. The fourth chapter introduces 3C- compact spaces and nearly 3C- compact spaces and investigates their properties. A theorem which gives several characterizations of 3C- compact spaces has been proved. A sufficient condition is obtained, under which 3C- compactness with respect to \( \tau \) and 3C- compactness with respect to \( \tau^* \) coincide.

In the fifth chapter, generalized closed sets with respect to an ideal \( I \) is introduced and a study on the properties of \( I \)-g closed sets and \( I \)-g open sets has been carried out.

In the sixth chapter, a weak form of ideals namely pre ideals has been introduced. It is shown that their behaviours, in some extent resemble that of ideals.

In the first chapter, the necessary background works have been presented.