INTRODUCTION

Sequence Space Theory occupies a very prominent position in several branches of analysis, for instance in the structural theory of topological vector spaces, Schauder basis theory, summability theory and theory of functions. The impact and importance of the study of sequence spaces can be appreciated when one sees the construction of numerous examples of locally convex spaces and other spaces like FK, BK-spaces etc., obtained as a consequence of the dual structure displayed by several pairs of distinct sequence spaces; thus, reflecting in depth the distinguishing structural features of the spaces in quest.

The theory of sequence spaces (especially of solid sequence spaces), topological in a variety of ways has been developed in considerable detail, in particular by Köthe and Toeplitz [29] and subsequently by Köthe. These results have been generalized by several mathematicians, to function spaces, to vector valued sequence spaces or to algebras. In other direction, another general class of sequence spaces was introduced by K. Zeller (FK-räume in der Funktionentheorie-I Math. Z. 58 (1953) 288-305. II, Math. Z 58(1953) 414-435). Mazur and Orlicz have obtained useful and important results (with applications to summability) when the spaces considered are Fréchet or Banach spaces. (See S. Mazur and W. Orlicz, On linear methods of summability, Studia Math. 14 (1955) 129-160 and also see K. Zeller, Theorie der Limitierungsverfahren, Berlin, 1958). The theory of sequence spaces has made remarkable advances in recent times in enveloping summability theory via unified techniques effecting matrix transformations from one sequence space into another. Further, being the ‘space’ approach (so called soft analysis) is used in the
theory of sequence spaces, the theory itself has established now-a-days as a powerful
and pervading tool in almost all branches of Mathematics with several important
applications. Therefore a study is made on this subject matter.

There are five chapters in this dissertation.

The first chapter contains two sections. The first section deals with the diagonal
mappings of sequence spaces in which the properties of the 'diagonally transformed'
sequence spaces are studied in relation to the given sequence spaces. Let $\lambda$ be a
fixed sequence $\{\lambda_n\}$ of scalars and $X$ be a given sequence space. Then the 'diagonally
transformed' sequence space of the sequence space $X$ is the sequence space

$$X_\lambda = \{x = \{x_n\} : \lambda \times e X\}$$

where $\lambda x = \{\lambda_n x_n\}$. If a determining set of a BK - space $X$ is known, then the
corresponding determining set of the transformed space $X_\lambda$ is found. In the second
section, 'difference sequence spaces' are generalized to some extent. Determining sets
of subspaces of difference sequence space are evaluated.

Ruckle (Sequence Spaces, Pitman 1981) and others have discussed multiplier
spaces - a generalization of the well known Köthe - Toeplitz duals. In the first section
of the second chapter some further properties of multiplier spaces are examined. We
give some typical results below:

(i) If $Y$ is solid, then $M(X,Y)$ is solid,

(ii) If $X$ and $Y$ are symmetric, then $M(X,Y)$ is symmetric,

(iii) Let $X, Y$ be BK-spaces with $X \supset \emptyset$. If $Y$ has monotone norm, then
     $M(X,Y)$ has monotone norm.
An attempt has been made to unify the concepts of multiplier spaces and summability theory. The second section of this chapter is devoted to distinguished spaces and their generalizations. The following are also proved:

(i) Suppose that $X$ and $E$ are sequence spaces and that $E$ has AD property such that $E^f \supset M(E,Y)$. Then $X \supset E$ if and only if $D^+(X,Y) \supset E$.

(ii) Suppose that a sequence space $X$ has AK property.

Then $B = F = C = X$.

(iii) Suppose that a sequence space $X$ has AK property and is also $\alpha$-perfect. Then $B^+ = F^+ = C^+ = X$.

The third chapter introduces the concept of semi-replete spaces (denoted by $sr(\mu)$) and deals with a few properties of them. For instance, it is proved that, for an FK space $X$ containing $\varphi$, the following are equivalent:

(i) $X_A$ is a $sr(\mu)$ space whenever the infinite matrix, $A \in (X:X)$, that is $A x \in X$ for all $x \in X$.

(ii) $X$ is a $sr(\mu)$ space.

As a consequence of the cited result we prove: if $\lambda$ is a $\beta$-perfect space with $\lambda^B$ having AK-property, then $\lambda^B$ is the only space such that

$X_A$ is $sr(\mu)$ if and only if $A \in (X:X)$.

The fourth chapter deals with summability theory in which 'soft analysis' approach is made. Several known results are obtained as corollaries here. We have computed a table of matrix transformations between sequence spaces using the results that we obtained.
In the fifth chapter, we are interested in dealing with properties of \((\ell_1 : \ell_2), (\Lambda:\Lambda)\) and \((\Gamma:\Gamma)\); respectively, we may call them as classes of sectional absolute methods, analytic methods and entire (matrix) methods. We replace the Brown conditions on entire methods. Some results obtained are presented below:

(i) Let \(A\) be a triangular matrix which is absolutely regular. If \(M_\infty(A)\) holds, then \(A\) is a triangle.

(ii) If \(A\) and \(B\) are analytical methods, then their product \(AB\) is also an analytical method,

(iii) If \(AB\) is of type \(M\) entire method, then \(A\) is of type \(M\) entire method.

Further, \(\Gamma\)-Mercerian matrices are defined and some of their properties are discussed.