CHAPTER 6

PERFORMANCE COMPARISONS OF PLATE BENDING ELEMENT (SFCFP)

6.1 Introduction

The development of a simple, four node quadrilateral plate bending element-SFCFP, in which the assumed displacement functions satisfy the differential equations of stress field equilibrium was described in chapter 3. SFCFP is a four node quadrilateral element with three degrees of freedom $w$, $\partial w/\partial x$ and $\partial w/\partial y$ at each node. The element geometry of SFCFP was shown in fig.3.3 The numerical investigation of the element performance has been carried out by testing it in standard test problems available finite element literatures and the results are compared with exact solutions and the results obtained with other established displacement based finite elements.

6.2 Eigenvalue test

The eigenvalue test is one of the important test for element quality. The test can detect zero energy deformation modes, lack of invariance and absence of rigid body motion capability. It can also be used to estimate the relative quality of different elements. An unrestrained element is considered for the eigenvalue test, so that $[k]$ is complete element stiffness matrix.
A beam of size 10 x 1 x 1 is modeled with a single element as shown in Fig. 6.1. Material properties are considered as $E = 1500$ and $v = 0.25$. The element stiffness matrix and global stiffness matrix are same for this problem. The element has 3 dof per node and 12 dof in total. Stiffness matrix and eigenvalues of unrestricted element are calculated using the computer code plbe.for and are given in Table 6.1. All the 12 eigenvalues are real and positive. Among them, three zeros or near zero values are obtained, showing that the element is exhibiting three rigid body modes.

<table>
<thead>
<tr>
<th>Sl No.</th>
<th>Eigenvalues</th>
<th>Sl No.</th>
<th>Eigenvalues</th>
<th>Sl No.</th>
<th>Eigenvalues</th>
</tr>
</thead>
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<tr>
<td>1</td>
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<td>5</td>
<td>0.27911E+04</td>
<td>9</td>
<td>0.14213E+03</td>
</tr>
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<td>6</td>
<td>0.20888E+04</td>
<td>10</td>
<td>0.19256E-13</td>
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<tr>
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<td>7</td>
<td>0.20888E+04</td>
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<td>0.32422E-13</td>
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<tr>
<td>4</td>
<td>0.35714E+04</td>
<td>8</td>
<td>0.19231E+04</td>
<td>12</td>
<td>0.13224E-12</td>
</tr>
</tbody>
</table>

When the element is reoriented in global co-ordinates by changing the node numbering sequence, the eigenvalues do not change, indicating the geometric invariance of the
element. Hence, it is inferred that the stiffness matrix is real, positive semi definite and the element is geometrically isotropic.

6.3 Single element test

Most plate-bending element, including the present one, behave well in bending. The critical test for a single element is usually the twist moment with one edge fully camped, which activates differential (bilinear) bending (Robinson 1976). In this test, a one edge fully clamped cantilever plate of unit width is modelled with a single element as shown in fig.6.2

![Fig. 6.2 Single element results, twist case using differential loads](image)

A unit twisting moment is applied on the free edge of the plate. The plate thickness is taken as $t = 0.05$ units and material properties are $E = 10^7$ units and $v = 0.25$. The vertical displacements $w$ of a free corner is plotted against element aspect ratio $l$ in the

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same figure. The results are also compared with the benchmark analysis results, which were obtained using sixteen high precision rectangles (STRAP5) (Hubka 1972). The figure also compares the results of LORA (Robinson and Haggenmacher 1977), a four nodded plate element based on stress assumptions, old NASTRAN plate bending element (QDPLT) (MacNeal 1969) and the new MSC/NASTRAN element (QUAD4) (MacNeal 1976). The STRAP5, QDPLT and QUAD4 elements are all based on displacement assumptions. The plate-bending element PIAN (Pian 1965) is also based on stress assumptions with nine independent force variables. It is emphasized that there are three degrees of freedom per node for LORA, QDPLT, QUAD4 and PIAN, and four degrees of freedom per node for STRAP5. All elements have four nodes. Results obtained with SFCFP, LORA and STRAP5 are close to the benchmark analysis and almost unaffected by the variation in aspect ratio. Elements QUAD4 and QDPLT showed poor performance in this test.

6.4 Convergence test

Fig. 6.3 Rectangular plate structure parameters for convergence test

\[ E = 1000.0 \text{ LBS}, \ E = 30 \times 10^6 \text{ LB/IN}^2, \ \nu = 0.3, \ \ t = 0.1 \text{ IN} \]
The convergence test (Clough and Tocher 1965, Robinson 1978) presented here, consists of investigating the central displacement of a rectangular plate with a centrally applied discrete load for two boundary conditions, simply supported and clamped. The plate problem is shown in fig. 6.3. To investigate the convergence characteristics of the plate element the variation of central displacement is studied using four different quarter plates meshes as shown in fig. 6.4. For each mesh, the aspect ratio of the element and hence that of the plate is varied from 1 to 3.

The convergence results (central displacement w) obtained with SFCFP are compared with that of LORA, STRAP5, QDPLT, QUAD4 and PIAN elements in fig. 6.5 - 6.10. Theoretical solutions given by Timoshenko are also plotted in the respective figures for comparison purpose.
Fig. 6.5 Convergence results for a simply supported plate with a central point load and aspect ratio one

Fig. 6.6 Convergence results for a simply supported plate with a central point load and aspect ratio two
Fig. 6.7 Convergence results for a simply supported plate with a central point load and aspect ratio three

Fig. 6.8 Convergence results for a clamped plate with a central point load and aspect ratio one
Fig. 6.9 Convergence results for a clamped plate with a central point load and aspect ratio two

Fig. 6.10 Convergence results for a clamped plate with a central point load and aspect ratio three
It can be seen that almost all elements considered except QDPLT converged to exact solutions for all the three aspect ratios and boundary conditions. When the aspect ratio increases beyond 1, the accuracy of the solutions obtained from SFCFP improves and at higher aspect ratios, it out performed all the other elements including the elements like PIAN and LORA, which are used in commercially available finite element packages.

6.5 Remarks

The performance of the Plate bending element is evaluated using a. Single element test using cantilever with aspect ratios ranging from 1 to 12 under twist load. b. Convergence test using simply supported plate under central load with different aspect ratios. c. Convergence test using clamped plate under central load with different aspect ratios. The performance of element was found to be encouraging.