CHAPTER 3

MHD EFFECTS ON MOVING ISOTHERMAL VERTICAL PLATE WITH DIFFUSION OF A CHEMICALLY REACTIVE SPECIES

3.1 INTRODUCTION

Magnetocoonvection plays an important role in various industrial applications. Examples include magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi-conducting materials. It is of importance in connection with many engineering problems, such as sustained plasma confinement for controlled thermonuclear fusion, liquid-metal cooling of nuclear reactors and electromagnetic casting of metals. MHD finds applications in electromagnetic pumps, controlled fusion research, crystal growing, MHD couples and bearings, plasma jets and chemical synthesis.

Boundary layer flow on moving horizontal surfaces was studied by Sakiadis [84]. Chambre and Young [11] analysed a first order chemical reaction in the neighbourhood of a horizontal plate. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar et al [107]. The dimensionless governing equations were solved using Laplace transform technique.

An exact solution of flow past an impulsively started infinite isothermal vertical plate with variable mass diffusion was discussed by Muthucumaraswamy et al [68]. Muthucumaraswamy and Ganesan [72] presented an exact solution
of the flow past an impulsively started infinite vertical plate with variable
temperature and uniform mass diffusion in the presence of first order chemical
reaction using Laplace transform technique.

It is proposed to study the flow past an impulsively started infinite
vertical plate with uniform temperature and variable mass diffusion in the
presence of transversely applied magnetic field. The governing equations are
solved by Laplace transform technique. The effect of velocity and skin-friction
for different magnetic field parameter, Schmidt number and chemical reaction
parameter is studied graphically.

3.2 ANALYSIS

The Hydromagnetic flow of a viscous incompressible fluid past an
impulsively started infinite vertical isothermal plate with variable mass diffusion
is studied. Here the x-axis is taken along the plate in the vertically upward
direction and the y-axis is taken normal to the plate. Initially the plate and
fluid are at the same temperature and concentration. At time $t' > 0$, the plate is
given an impulsive motion in the vertical direction against gravitational field with
constant velocity $u_0$. The plate temperature is raised to $T_w$ and the concentration
level near the plate is raised linearly with time. A transverse magnetic field of
uniform strength $B_0$ is assumed to be applied normal to the plate. The induced
magnetic field and viscous dissipation is assumed to be negligible. It is also
assumed that there exists a homogeneous first order chemical reaction between
the fluid and species concentration. Then by usual Boussinesq approximation,
the flow is governed by the following equations:
with the following initial and boundary conditions:

\[t' \leq 0: \quad u = 0, \quad T = T_{\infty}, \quad C' = C'_{\infty} \quad \text{for all} \quad y\]

\[t' > 0: \quad u = u_0, \quad T = T_w, \quad C' = C'_{\infty} + (C'_{w} - C'_{\infty}) A \quad \text{at} \quad y = 0\]

\[u = 0, \quad T \rightarrow T_{\infty}, \quad C' \rightarrow C'_{\infty} \quad \text{as} \quad y \rightarrow \infty\]

where \(A = \frac{u_0^2}{\nu}\). On introducing the following non-dimensional quantities:

\[U = \frac{u}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad Y = \frac{y u_0}{\nu}, \quad \theta = \frac{T - T_{\infty}}{T_w - T_{\infty}}\]

\[Gr = \frac{g \beta \nu (T_w - T_{\infty})}{u_0^3}, \quad C = \frac{C' - C'_{\infty}}{C'_{w} - C'_{\infty}}, \quad Gc = \frac{\nu g \beta (C'_{w} - C'_{\infty})}{u_0^3}, \quad Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{\nu}{D}, \quad M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}, \quad K = \frac{\nu K_l}{u_0^2}\]

in equations (3.1) to (3.4), leads to

\[\frac{\partial U}{\partial t} = Gr \theta + Gc C + \frac{\partial^2 U}{\partial Y^2} - M U\]

\[\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2}\]

\[\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} - K C\]
The last term in (3.8) represents homogeneous first order chemical reaction. The initial and boundary conditions in dimensionless form are

\[ U = 0, \; \theta = 0, \; C = 0 \quad \text{for all} \quad Y, t \leq 0 \]
\[ t > 0: \; U = 1, \; \theta = 1, \; C = t \quad \text{at} \quad Y = 0 \]
\[ U = 0, \; \theta \to 0, \; C \to 0 \quad \text{as} \quad Y \to \infty \]

All the physical variables are defined in the List of symbols. The equations (3.6) to (3.8), subject to the boundary conditions (3.9), are solved by the usual Laplace transform technique and the solutions are derived as follows:

\[ \theta = \text{erfc}(\eta \sqrt{Pr}) \] (3.10)

\[ C = \frac{t}{2} \left[ \exp(2\eta \sqrt{KtSc}) \text{erfc}(\eta \sqrt{Sc} + \sqrt{Kt}) ight. \\
+ \exp(-2\eta \sqrt{KtSc}) \text{erfc}(\eta \sqrt{Sc} - \sqrt{Kt}) \\
- \frac{\eta \sqrt{Sc} t}{2 \sqrt{K}} \left[ \exp(-2\eta \sqrt{KtSc}) \text{erfc}(\eta \sqrt{Sc} - \sqrt{Kt}) \\
- \exp(2\eta \sqrt{KtSc}) \text{erfc}(\eta \sqrt{Sc} + \sqrt{Kt}) \right] \] (3.11)
\[
U = \frac{1}{2} \left( 1 + \frac{Gr}{a(1 - Pr)} + \frac{Gc(1 + bt)}{b^2(1 - Sc)} \right) \left[ \exp(2 \eta \sqrt{Mt}) \text{erfc}(\eta + \sqrt{Mt}) \\
+ \exp(-2 \eta \sqrt{Mt}) \text{erfc}(\eta - \sqrt{Mt}) \right] \\
- \frac{Gc \eta \sqrt{t}}{2b(1 - Sc) \sqrt{M}} \left[ \exp(-2 \eta \sqrt{Mt}) \text{erfc}(\eta - \sqrt{Mt}) \\
- \exp(2 \eta \sqrt{Mt}) \text{erfc}(\eta + \sqrt{Mt}) \right] - \frac{Gr}{a(1 - Pr)} \text{erfc}(\eta \sqrt{Pr}) \\
- \frac{Gr \exp(at)}{2a(1 - Pr)} \left[ \exp(2 \eta \sqrt{(M + a)t}) \text{erfc}(\eta + \sqrt{(M + a)t}) \\
+ \exp(-2 \eta \sqrt{(M + a)t}) \text{erfc}(\eta - \sqrt{(M + a)t}) \right] \\
- \frac{Gc \exp(bt)}{2b^2(1 - Sc)} \left[ \exp(2 \eta \sqrt{(M + b)t}) \text{erfc}(\eta + \sqrt{(M + b)t}) \\
+ \exp(-2 \eta \sqrt{(M + b)t}) \text{erfc}(\eta - \sqrt{(M + b)t}) \right] \\
+ \frac{Gr \exp(at)}{2a(1 - Pr)} \left[ \exp(2 \eta \sqrt{a Pr t}) \text{erfc}(\eta \sqrt{Pr} + \sqrt{at}) \\
+ \exp(-2 \eta \sqrt{a Pr t}) \text{erfc}(\eta \sqrt{Pr} - \sqrt{at}) \right] \\
- \frac{Gc(1 + bt)}{2b^2(1 - Sc)} \left[ \exp(2 \eta \sqrt{Kt Sc}) \text{erfc}(\eta \sqrt{Sc} + \sqrt{Kt}) \\
+ \exp(-2 \eta \sqrt{Kt Sc}) \text{erfc}(\eta \sqrt{Sc} - \sqrt{Kt}) \right] \\
+ \frac{Gc \eta \sqrt{t} \sqrt{Sc}}{2b(1 - Sc) \sqrt{K}} \left[ \exp(-2 \eta \sqrt{Kt Sc}) \text{erfc}(\eta \sqrt{Sc} - \sqrt{Kt}) \\
- \exp(2 \eta \sqrt{Kt Sc}) \text{erfc}(\eta \sqrt{Sc} + \sqrt{Kt}) \right] \\
+ \frac{Gc \exp(bt)}{2b^2(1 - Sc)} \left[ \exp(2 \eta \sqrt{Sc(K + b)t}) \text{erfc}(\eta \sqrt{Sc} + \sqrt{(K + b)t}) \\
+ \exp(-2 \eta \sqrt{Sc(K + b)t}) \text{erfc}(\eta \sqrt{Sc} - \sqrt{(K + b)t}) \right] \tag{3.12}
\]
where, \( a = \frac{M}{Pr - 1} \), \( b = \frac{M - KSc}{Sc - 1} \), and \( \eta = \frac{Y}{2\sqrt{t}} \).

### 3.3 RESULTS AND DISCUSSION

The numerical values of the velocity and skin-friction are computed for different parameters like magnetic field parameter, Chemical reaction parameter, Schmidt number, thermal Grashof number, mass Grashof number, \( t = 0.2 \) and \( Pr = 7 \). The purpose of the calculations given here is to study the effects of the parameters \( M, K, Gr \) and \( Gc \) upon the nature of the flow and transport.

The numerical solutions for the velocity are obtained from equation (3.12). The velocity profiles for different values of the magnetic field parameter in the presence of the chemical reaction are shown in Fig. 3.1. It is observed that the velocity increases with the decreasing magnetic field parameter.

Fig. 3.2 exhibits the effect of chemical reaction parameter in the presence of the magnetic field parameter. In this case, the velocity decreases with increasing chemical reaction parameter. Fig. 3.3 represents the velocity profiles for different thermal Grashof number and mass Grashof number. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number. It is interesting to note that the velocity increases slightly in the variation of thermal Grashof number.

Fig. 3.4 depicts temperature profiles for different Prandtl number \( Pr = 7.0, 0.71 \). It is observed that temperature falls due to the decrease in Prandtl number. The concentration effects for different Schmidt number \( (Sc = 0.3, 0.6, 2.0) \), chemical reaction parameter \( (K = 0.2, 2.0) \) and time \( (t = 0.4, 0.6, 0.8, 1.0) \) are studied from Fig. 3.5. It is clear that the concentration decreases with the raise in chemical reaction parameter but increases with decreasing Schmidt number and time.
Fig. 3.2 Velocity profiles for different $K$

- $K = 0.2$
- $K = 2.0$

- $Pr = 7.0$
- $Sc = 2.01$
- $Gr = 5$
- $Cc = 5$
- $t = 0.2$
- $M = 2$
Fig. 3.3 Velocity profiles for different $Gr$ and $Gc$

- $Pr = 7.0$
- $Sc = 2.01$
- $M = 2$
- $K = 0.2$
- $t = 0.2$
Fig. 3.4 Temperature profiles

Pr = 7.0
Pr = 0.71
Fig. 3.5 Concentration profiles for different Sc and t
From the velocity field, we now study the changes in the skin-friction due to the magnetic field in the presence of chemical reaction. It is given by

$$\tau = -\left(\frac{dU}{dY}\right)_{y=0} = -\left(\frac{1}{2\sqrt{t}}\right)\left(\frac{dU}{d\eta}\right)_{\eta=0}$$  \hspace{1cm} (3.13)

Hence, from the equations (3.12) and (3.13), the wall shear stress in the presence of magnetic field is as follows:

$$\tau = \frac{1}{\sqrt{\pi} t} \left\{ \left(1 + \frac{Gr}{a(1 - Pr)} + \frac{Gc(1 + bt)}{b^2(1 - Sc)}\right) \left(1 + \sqrt{M\pi t} \operatorname{erf}(\sqrt{Mt})\right) \right. $$

$$+ \frac{Gc\sqrt{\pi} t}{2b(1 - Sc)\sqrt{M}} \operatorname{erf}(\sqrt{Mt}) - \frac{Gr\sqrt{Pr}}{a(1 - Pr)} \frac{Gc\sqrt{\pi} t \operatorname{erf}(\sqrt{Mt})}{2b(1 - Sc)\sqrt{K}}$$

$$- \frac{Gr \exp(at)}{a(1 - Pr)} \left[1 - \sqrt{Pr} + \sqrt{(M + a)\pi t} \operatorname{erf}(\sqrt{(M + a)t}) - \sqrt{Pr\pi at} \operatorname{erf}(\sqrt{at})\right]$$

$$\left. - \frac{Gc \exp(bt)}{b^2(1 - Sc)} \left[1 - \sqrt{Sc} + \sqrt{(M + b)\pi t} \operatorname{erf}(\sqrt{(M + b)t}) \right. 
- \sqrt{Sc(b + K)\pi t} \operatorname{erf}(\sqrt{(K + b)t})\right]$$

$$\left. - \frac{Gc(1 + bt)}{b^2(1 - Sc)} \left(\sqrt{Sc} + \sqrt{\pi t ScK} \operatorname{erf}(\sqrt{Kt})\right)\right\}$$  \hspace{1cm} (3.14)

The numerical values of skin-friction $\tau$ are presented in Table 3.1 for $Gr = 5$, $Gc = 5$ and $Pr = 7$. It is inferred from this table, skin-friction increases with increasing values of the magnetic field parameter. This shows that the wall shear stress increases with increasing magnetic field parameter. It is also observed that the skin-friction decreases with increasing values of the time or chemical reaction parameter. It is interesting to note that the value of the skin-friction remains constant for different values of the Schmidt number.
Table 3.1 Values of the Skin-friction $\tau$

<table>
<thead>
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<th>M</th>
<th>K</th>
<th>Sc</th>
<th>t</th>
<th>$\tau$</th>
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<td>1.1251</td>
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3.4 CONCLUDING REMARKS

An analysis is performed to study the flow past an impulsively started infinite vertical plate with uniform temperature and variable mass diffusion in the presence of transverse applied magnetic field. The dimensionless governing equations are solved by the usual Laplace transform technique. The velocity profiles for different $M, Gr, Gc$ and $K$, the temperature profiles and concentration profiles are studied. The conclusions of the study are as follows:

(i) The velocity decreases with increasing magnetic field parameter or chemical reaction parameter.

(ii) The skin-friction increases with increasing values of magnetic parameter. This trend is just reversed with respect to the chemical reaction parameter.

(iii) The temperatures raise as Prandtl number increases.

(iv) As time increases, we found that there is a fall in concentration and wall shear stress.