CHAPTER 5

INFLUENCE OF HEAT SOURCE, THERMAL RADIATION AND INCLINED MAGNETIC FIELD ON PERISTALTIC FLOW OF A HYPERBOLIC TANGENT NANOFLUID IN A TAPERED ASYMMETRIC CHANNEL

5.1 SCOPE AND OBJECTIVE

In the present analytic thinking, we have modeled the governing equations of a two dimensional peristaltic transport of a Hyperbolic tangent nanofluid in the presence of a heat source/sink with the combined effects of thermal radiation and inclined magnetic field in a tapered asymmetric channel. The propagation of waves on the non-uniform walls to have different amplitudes and phase but the same wave speed is produced by the tapered asymmetric channel. The equations of dimensionless temperature and nanoparticle concentration are solved analytically under the assumptions of long wavelength and low Reynolds number. The governing equations of momentum of a hyperbolic tangent nanofluid for the tapered asymmetric channel have also been solved analytically using the regular perturbation method. The expression for average rise in pressure has been figured using numerical integrations. The effects of various physical parameters entering into the problem are discussed numerically and graphically. The phenomenon of trapping is also investigated. Furthermore, the received results show that the maximum pressure rise gets increased in case of non-Newtonian fluid when equated with Newtonian fluid.
5.2 MATHEMATICAL FORMULATION

We consider a two dimensional MHD flow of an electrically conducting hyperbolic tangent nanofluid in a vertical tapered asymmetric channel under the effects of radiation and heat source sink parameters. The nanofluid is electrically conducting in the presence of a uniform magnetic field $B_0$ applied in the transverse direction. Heat transfer along with nanoparticle phenomena has been taken into description. Let $Y = H_1$ and $Y = H_2$ be right hand side wall and left hand side wall boundaries and the medium is considered to be induced by a sinusoidal wave train propagating with a constant speed $c$ along the asymmetric trapped channel wall as shown in Figure 3.1. The right side wall of the channel is sustained at temperature $T_0$ and nanoparticle volume fraction $C_0$ while the left wall has temperature $T_1$ and nanoparticle volume fraction $C_1$.

The geometry of the tapered asymmetric surface is defined as

$$H_1(X, t') = -d - m' X - a_1 \sin \left( \frac{2\pi}{\lambda} (X - ct') + \phi \right).$$  \hspace{1cm} (5.1a)$$

$$H_2(X, t') = d + m' X + a_2 \sin \left( \frac{2\pi}{\lambda} (X - ct') \right).$$ \hspace{1cm} (5.1b)

where $a_1$ and $a_2$ are the amplitudes of right and left walls respectively, $d$ is the half-width of the channel, $c$ is the phase speed of the wave, $m' (<< 1)$ is the non-uniform parameter, $\lambda$ is the wave length, the phase difference $\phi$ varies in the range $0 \leq \phi \leq \pi$, $\phi = 0$ corresponds to symmetric channel with waves out of phase and $\phi = \pi$ the waves are in phase, and further $a_1, a_2, d$ and $\phi$ satisfy the following condition at the inlet of divergent channel.
\[ a_1^2 + a_2^2 + 2a_1a_2 \sin(\phi) \leq (2d)^2. \]

The governing equations for an incompressible, hyperbolic tangent nanofluid under the effect of inclined magnetic field and radiation parameter are given by (Akram and Nadeem 2014).

The continuity equation is

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \quad (5.2)
\]

The equation of the \( X \) momentum is

\[
\rho_f \left[ \frac{\partial}{\partial t'} + U \frac{\partial}{\partial X} + V \frac{\partial}{\partial Y} \right] U \\
= \frac{\partial P}{\partial X} + \frac{\partial}{\partial X} \left( \overline{\tau}_{XX} \right) + \frac{\partial}{\partial Y} \left( \overline{\tau}_{XY} \right) - \sigma B_0^2 \cos \xi (U \cos \xi - V \sin \xi) \\
+ \rho_f g \alpha (1-C_0) + (\rho_p - \rho_f) g \beta' (C - C_0) + \rho_g \sin \Omega, \quad (5.3)
\]

The \( Y \) momentum equation is

\[
\rho_f \left[ \frac{\partial}{\partial t'} + U \frac{\partial}{\partial X} + V \frac{\partial}{\partial Y} \right] V \\
= -\frac{\partial P}{\partial Y} + \frac{\partial}{\partial X} \left( \overline{\tau}_{XY} \right) - \sigma B_0^2 \sin \xi (U \cos \xi - V \sin \xi) - \rho_g \cos \Omega, \quad (5.4)
\]

The equation of temperature is

\[
(\rho c_p') \left[ \frac{\partial T}{\partial t'} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} \right] = \kappa \left[ \frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right] + (\rho c_p) D_B \left[ \frac{\partial C}{\partial X} \frac{\partial T}{\partial X} + \frac{\partial C}{\partial Y} \frac{\partial T}{\partial Y} \right] \\
- \left( \frac{\partial q_r}{\partial X} + \frac{\partial q_r}{\partial Y} \right) + Q_0 + \frac{D_T (\rho c_p)}{T_m} \left[ \left( \frac{\partial T}{\partial X} \right)^2 + \left( \frac{\partial T}{\partial Y} \right)^2 \right], \quad (5.5)
\]
and the nanoparticle volume fraction phenomena is

\[
\left[ \frac{\partial C}{\partial \tau'} + U \frac{\partial C}{\partial X} + Y \frac{\partial C}{\partial Y} \right] = D_n \left[ \frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} \right] + \frac{D_T}{T_m} \left[ \frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right].
\] (5.6)

in which

\[
\tau_{XX} = 2\eta_0 \left[ 1 + n \left( \Gamma \bar{\gamma} - 1 \right) \right] \frac{\partial U}{\partial X}, \quad \tau_{XY} = \eta_0 \left[ 1 + n \left( \Gamma \bar{\gamma} - 1 \right) \right] \left( \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right),
\]

\[
\tau_{YY} = 2\eta_0 \left[ 1 + n \left( \Gamma \bar{\gamma} - 1 \right) \right] \frac{\partial V}{\partial Y}, \quad \bar{\gamma} = \sqrt{2 \left( \frac{\partial U}{\partial Y} \right)^2 + \left( \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right)^2 + 2 \left( \frac{\partial V}{\partial Y} \right)^2}.
\]

where \( U, V \) are the components of velocity along \( X \) and \( Y \) directions respectively, \( \tau' \) is the dimensional time, \( g \) is the acceleration due to gravity, \( P \) is the pressure, \( \sigma \) is the electrical conductivity of the fluid, \( B_0 \) is the uniform applied magnetic field, \( \rho_f \) is the constant density of the base fluid, \( \rho_p \) is the density of the particle, \( V \) is the velocity vector, \( \eta_0 \) is the coefficient of viscosity of the fluid, \( \kappa \) is the thermal conductivity, \( C \) is the nanoparticle concentration, \( Q_0 \) is the constant heat addition/absorption, \( \xi \) is the inclination of the magnetic field, \( \Omega \) is the angle of the inclination, \( D_B \) is the Brownian diffusion coefficient, \( T_m \) is the fluid mean temperature, \( D_T \) is the thermophoretic diffusion coefficient, \( \tau = \frac{(\rho c')_p}{(\rho c')_f} \) is the ratio of the effective heat capacity of nanoparticle material, \( \alpha \) is the thermal expansion coefficient, \( \beta' \) is the coefficient of expansion with concentration and heat capacity of the fluid with \( \rho \) being the density.
In order to describe the fluid flow in a non-dimensional form, we introduce the following quantities in Equations (5.1) - (5.6),

\[
\bar{x} = \frac{X}{\lambda}, \quad \bar{y} = \frac{Y}{d}, \quad t = \frac{ct}{\lambda}, \quad \bar{u} = \frac{U}{c}, \quad \bar{v} = \frac{V}{\lambda}, \quad \delta = \frac{d}{\lambda}, \quad h_1 = \frac{H_1}{d}, \quad M = \sqrt{\frac{\sigma}{\mu} dB_0},
\]

\[
h_2 = \frac{H_2}{d}, \quad p = \frac{d^2 P}{c^2 \eta_0}, \quad \theta = \frac{T - T_0}{T_1 - T_0}, \quad a = \frac{a_1}{d}, \quad b = \frac{a_2}{d}, \quad m = \frac{\lambda m_1}{d}, \quad Pr = \frac{\eta_0 c_f}{\kappa},
\]

\[
\sigma = \frac{C - C_0}{C_1 - C_0}, \quad \tau_{xx} = \frac{\lambda}{\eta_0 c}, \quad \tau_{xy} = \frac{d}{\eta_0 c}, \quad \tau_{yy} = \frac{\tau_{yy}}{\eta_0 c}, \quad \gamma = \frac{\gamma d}{c}, \quad Fr = \frac{c^2}{gd},
\]

\[
R = \frac{\rho_f c d}{\eta_0}, \quad \beta = \frac{Q_0 d^2}{(T_1 - T_0) w_p}, \quad Gr = \frac{\rho \alpha d^2 (T_1 - T_0)}{c n_0}, \quad Br = \frac{\rho \alpha d^2 (C_1 - C_0)}{c n_0},
\]

\[
We = \frac{\Gamma c}{d}, \quad Nb = \frac{\pi D_B (C_1 - C_0)}{\nu}, \quad Nt = \frac{\pi D_r (T_1 - T_0)}{T_m \nu}, \quad Rn = \frac{16 \sigma T_0^3}{3 k \eta_0 c_f}. \quad (5.7)
\]

and the stream function \( u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \) and omitting bar, we obtain,

\[
R \delta \left[ \psi_{yy} + \psi_x \psi_{xy} - \psi_y \psi_{xy} \right] = -\frac{\partial p}{\partial x} + \delta \frac{\partial}{\partial x} \left( \tau_{xx} \right) + \delta \frac{\partial}{\partial y} \left( \tau_{xy} \right) - M^2 \cos \xi (\psi_y \cos \xi + \delta \psi_x \sin \xi)
\]

\[
+ Gr \theta + Br \sigma + \frac{R}{Fr} \sin \Omega, \quad (5.8)
\]

\[
R \delta^3 \left[ \psi_{xx} - \psi_y \psi_{xy} + \psi_x \psi_{yy} \right] = \frac{\partial^2 p}{\partial y^2} + \delta \frac{\partial^2}{\partial x^2} \left( \tau_{xx} \right) + \delta \frac{\partial}{\partial y} \left( \tau_{yy} \right) + M^2 \delta \sin \xi (\psi_y \cos \xi + \delta \psi_x \sin \xi)
\]

\[
- \delta \frac{R}{Fr} \cos \Omega. \quad (5.9)
\]
$$R\delta \left[ \frac{\partial \theta}{\partial t} + \psi_y \frac{\partial \theta}{\partial x} \delta \psi_x \frac{\partial \theta}{\partial y} \right] = \frac{1}{\text{Pr}} \delta^2 \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) + R\eta \left( \delta \frac{\partial^2 \theta}{\partial x \partial y} + \frac{\partial^2 \theta}{\partial y^2} \right) + \beta$$

$$+ Nb \left[ \delta^2 \frac{\partial \sigma}{\partial x} \frac{\partial \theta}{\partial x} + \frac{\partial \sigma}{\partial y} \frac{\partial \theta}{\partial y} \right] + Nt \left[ \delta^2 \left( \frac{\partial \theta}{\partial x} \right)^2 + \left( \frac{\partial \theta}{\partial y} \right)^2 \right],$$

\hspace{1cm} \text{(5.10)}

$$R\delta \left[ \frac{\partial \sigma}{\partial t} + \psi_y \frac{\partial \sigma}{\partial x} + \delta \psi_x \frac{\partial \sigma}{\partial y} \right] = \delta^2 \left( \frac{\partial^2 \sigma}{\partial x^2} + \frac{\partial^2 \sigma}{\partial y^2} \right) + \frac{Nt}{Nb} \left[ \delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right].$$

\hspace{1cm} \text{(5.11)}

where

$$\tau_{xx} = 2[1 + n(\text{We} \dot{\gamma} - 1)] \frac{\delta^2 \psi}{\partial x \partial y}, \quad \tau_{xy} = [1 + n(\text{We} \dot{\gamma} - 1)] \left( \frac{\delta^2 \psi}{\partial y^2} - \delta^2 \frac{\partial^2 \psi}{\partial x^2} \right).$$

$$\tau_{yy} = -2[1 + n(\text{We} \dot{\gamma} - 1)] \frac{\delta^2 \psi}{\partial x \partial y}.$$

$$\dot{\gamma} = 2\delta \left( \frac{\delta^2 \psi}{\partial x \partial y} \right)^2 + \left( \frac{\delta^2 \psi}{\partial y^2} - \delta^2 \frac{\partial^2 \psi}{\partial x^2} \right) + 2\delta \left( \frac{\delta^2 \psi}{\partial x \partial y} \right)^{1/2}.$$

where \( x \) is non-dimensional axial coordinate, \( y \) is non-dimensional transverse coordinate, \( t \) is dimensionless time, \( u \) and \( v \) are non-dimensional axial and transverse velocity components, \( p \) is dimensionless pressure, \( a \) and \( b \) are amplitudes of left and right walls respectively, \( \delta \) is wave number, \( m \) is the non-uniform parameter, \( R \) is the Reynolds number, \( \nu \) is the nanofluid kinematic viscosity, \( \theta \) is the dimensionless temperature, \( \sigma \) is the dimensionless rescaled nanoparticle volume fraction, \( \text{Pr} \) is the Prandtl number, \( M \) is the Hartmann number, \( Fr \) is the Froude number, \( \beta \) is the non-dimensional heat source/sink parameter, \( Gr \) is the local temperature Grashof number, \( Br \) is the local nanoparticle Grashof number, \( Nb \) is the Brownian motion parameter,
thermophoresis \( N_t \) is the thermophoresis parameter and \( R_n \) is the radiation parameter.

Under the assumptions of long wavelength and low-Reynolds number and neglecting the terms of order \( \delta \) and higher, Equations (5.8) – (5.11) become

\[
\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left(1 + n\left(We \frac{\partial^2 \psi}{\partial y^2} - 1\right)\right)\frac{\partial^2 \psi}{\partial y^2} - M^2 \frac{\partial \psi}{\partial y} \cos^2 \xi + \frac{R}{Fr} \sin \Omega + Gr \theta + Br \sigma, \tag{5.12}
\]

\[
\frac{\partial p}{\partial y} = 0, \tag{5.13}
\]

\[
\left(1 + \frac{RnPr}{Pr}\right)\frac{\partial^2 \theta}{\partial y^2} + Nb \left(\frac{\partial \sigma \partial \theta}{\partial y^2}\right) + Nt \left(\frac{\partial \theta}{\partial y}\right)^2 + \beta = 0, \tag{5.14}
\]

\[
\frac{\partial^2 \sigma}{\partial y^2} + \frac{Nt \partial^2 \theta}{Nb \partial y^2} = 0. \tag{5.15}
\]

Elimination of pressure from Equation (5.12) and (5.13), gives

\[
\frac{\partial^2}{\partial y^2} \left(1 + n\left(We \frac{\partial^2 \psi}{\partial y^2} - 1\right)\right)\frac{\partial^2 \psi}{\partial y^2} - M^2 \frac{\partial^2 \psi}{\partial y^2} \cos^2 \xi + Gr \frac{\partial \theta}{\partial y} + Br \frac{\partial \sigma}{\partial y} = 0, \tag{5.16}
\]

The corresponding boundary conditions in terms of stream function are given as

\[
\psi = \frac{F}{2}, \quad \frac{\partial \psi}{\partial y} = 0, \text{ at } y = h_2 = 1 + mx + b \sin[2\pi(x-t)]. \tag{5.17a}
\]
\[
\psi = -\frac{F}{2} \frac{\partial \psi}{\partial y} = 0, \text{ at } y = h_1 = -1 - mx - a\sin[2\pi(x-t) + \phi]. \quad (5.17b)
\]

\[
\theta = 1 \text{ and } \sigma = 1 \text{ at } y = h_2, \quad \theta = 0 \text{ and } \sigma = 0 \text{ at } y = h_1, \quad (5.18)
\]

which satisfy, at the inlet of channel,

\[
a^2 + b^2 + 2absin(\phi) \leq 4.
\]

It is observed that the instantaneous average volume rate of the flow \( F(x, t) \) is periodic in \((x - t)\) and the detailed discussion is given in chapter-2 as

\[
F(x, t) = \Theta + b\sin[2\pi(x-t)] + a\sin[2\pi(x-t) + \phi]. \quad (5.19)
\]

in which

\[
F = \int_{h_1}^{h_2} \eta dy.
\]

### 5.3 EXACT SOLUTION OF THE PROBLEM

Integration of Equation (5.15) with respect to \( y \) and implementing in Equation (5.14) by using boundary conditions of Equation (5.18), the dimensionless temperature field is obtained as

\[
\theta = \frac{\beta(h_1 - y)}{A_1Nb} + \frac{(\beta(h_1 - h_2) - A_1Nb)(e^{-A_1Nb\phi y} - e^{-A_1Nb\phi h_1})}{A_1Nb(e^{-A_1Nb\phi h_1} - e^{-A_1Nb\phi h_2})}. \quad (5.20)
\]

Substituting Equation (5.20) into Equation (5.15), moreover integrating Equation (5.15) with respect to \( y \) and using proper boundary conditions of Equation (5.18), the nanoparticle fraction field is received as
\[ \sigma = (h_1 - y) \left( \frac{A_1 N b^2 - N_t (\beta (h_1 - h_2) - A_1 N b)}{A_1 N b^2 (h_1 - h_2)} \right) \]
\[ + \frac{(e^{-A_1 N b h_1 A_2} - e^{-A_1 N b h_2 A_2}) N_t (\beta (h_1 - h_2) - A_1 N b)}{A_1 N b^2 (e^{-A_1 N b h_1 A_2} - e^{-A_1 N b h_2 A_2})} \]  \hspace{1cm} (5.21)

5.4 PERTURBATION SOLUTION

We obtain the solution for the stream function as a perturbation method in terms of the small parameter \( \text{We} \) (Weissenberg number), by expanding \( \psi \), \( p \) and \( F \) in the following form:

\[ \psi = \psi_0 + \text{We} \psi_1 + o(\psi_3), \]
\[ p = p_0 + \text{We} p_1 + o(p_3), \]
\[ F = F_0 + \text{We} F_1 + o(F_2). \]

Substituting above expressions in Equations (5.12) and (5.16) and collecting the powers \( \text{We} \), we get the following system

5.4.1 For the system of order \( (\text{We}^0) \)

\[ \frac{\partial^4 \psi_0}{\partial y^4} - M^2 \cos^2 \frac{x}{\xi} \frac{\partial^2 \psi_0}{\partial y^2} = \frac{Gr}{n-1} \theta_y + \frac{Br}{n-1} \sigma_y, \]
\[ \frac{\partial p_0}{\partial x} = (1 - n) \frac{\partial^3 \psi_0}{\partial y^3} - M^2 \cos^2 \frac{x}{\xi} \frac{\partial \psi_0}{\partial y} \]
\[ + \frac{R}{F_r} \sin \Omega + Gr \theta + Br \sigma, \]
\[ \psi_0 = \frac{F_0}{2} \frac{\partial \psi_0}{\partial y} = 0 \text{ at } y = h_2, \]  \hspace{1cm} (5.27a)
\[ \psi_0 = -\frac{F_0}{2}, \quad \frac{\partial \psi_0}{\partial y} = 0 \text{ at } y = h_1, \] (5.27b)

**5.4.2 For the system of order \( (We^1) \)**

\[ \frac{\partial^4 \psi_1}{\partial y^4} - \frac{M^2 \cos^2 \xi \ \partial^2 \psi_1}{1 - n \ \partial^2} = \frac{n}{n - 1} \ \partial^2 \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^2, \] (5.28)

\[ \frac{\partial p_1}{\partial x} = (1 - n) \frac{\partial^3 \psi_1}{\partial y^3} - M^2 \cos^2 \xi \ \partial^2 \psi_1 + n \ \partial \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^2, \] (5.29)

\[ \psi_1 = \frac{F_1}{2}, \quad \frac{\partial \psi_1}{\partial y} = 0 \text{ at } y = h_2, \] (5.30a)

\[ \psi_1 = -\frac{F_1}{2}, \quad \frac{\partial \psi_1}{\partial y} = 0 \text{ at } y = h_1. \] (5.30b)

**5.4.3 Solution for system of order \( (We^0) \)**

Solution of Equation (5.25) satisfying the boundary conditions (5.27a) – (5.27b) can be written as

\[ \psi_0(y) = A_{18} + A_{17}y + A_{16}e^{A_1y} + A_1 e^{A_1y} + \frac{Br(Nb+Ni)}{2M^2Nb(h_1-h_1)} y^2 \]

\[ + \frac{(NbGr-BrN_l)A_1}{Nb(e^{-A_1h_1} - e^{-A_1h_2})(A_1^4 - M^2 A_1^2)} e^{-A_1y}, \] (5.31)
\[ \frac{\partial p_0}{\partial x} = \left( A_4 \left( A_{15} e^{-A_{14} y} - A_{16} e^{A_{14} y} \right) - A_{17} + \frac{A_{6} y}{A_4} - \frac{A_5 A_3 e^{-A_3 y}}{A_4 h_1^2 - A_3^2} \right) M^2 \cos^2 \xi + Gr \left( \frac{\beta(h_1 - y)}{A_1 N b} + \left( \frac{\beta(h_1 - h_2) - A_1 N b(e^{-A_Nb A_{2/y}} - e^{-A_Nb A_2 h_2})}{A_1 N b(e^{-A_Nb A_2 h_1} - e^{-A_Nb A_2 h_2})} \right) \right) \\
+ Br \left( \frac{(h_1 - y) \left( A_1 N b^2 - N_f(\beta(h_1 - h_2) - A_1 N b) \right)}{A_1 N b^2(h_1 - h_2)} \right) \\
+ \frac{(e^{-A_Nb h_2} - e^{-A_Nb h_2 A_2 y}) N_f(\beta(h_1 - h_2) - A_1 N b)}{A_1 N b^2(e^{-A_Nb A_2 h_1} - e^{-A_Nb A_2 h_2})} \right) \\
+ (1 - n) \left( A_4^2 (A_{16} e^{A_{14} y} - A_{15} e^{-A_{14} y}) + \frac{A_5 A_3 e^{-A_3 y}}{A_4 - A_3^2} \right) + \frac{R}{Fr} \sin \Omega. \tag{5.32} \]

### 5.4.4 Solution for system of order \((We^1)\)

Substituting Equation 5.31 into Equation 5.28 and solving the resulting equation subject to the boundary conditions in Equations (5.30a) – (5.30b), we obtain

\[ \psi_1(y) = A_{22} + A_{21} y - \frac{A_{5} e^{-2 \gamma A_3}}{A_4 - A_3^2 (A_4 - 4 A_3^2)} - \frac{2 A_6 (A_{16} e^{A_{14} y} + A_{15} e^{-A_{14} y})}{A_4 - 1} \\
+ A_{20} e^{A_{14} y} + A_{19} e^{-A_{14} y} + \frac{A_{16} A_3 A_{24} e^{2 A_{14} y}}{4 A_4 - 1} + \frac{A_{15} A_4 e^{-2 A_{14} y}}{4 A_4 - 1} - \frac{2 A_5 A_6 e^{-\gamma A_3}}{A_4 (A_4 - A_3^2)^2} \\
+ \frac{2 A_{16} A_4^2 A_4 e^{-(A_4 - A_3)^2}}{(A_4 - A_3^2)(A_4 - (A_4 - A_3)^2)} + \frac{2 A_{15} A_4^2 A_4 e^{-(A_4 + A_3)}}{(A_4 - A_3^2)(A_4 - (A_4 + A_3)^2)}, \tag{5.33} \]
\[
\frac{\partial p_1}{\partial x} = \left( \frac{2A_6(A_{16}A_{4}e^{A_{4y}} - A_{15}A_{4}e^{-A_{4y}})}{A_4 - 1} - A_{21} - A_{20}A_{4}e^{A_{4y}} + A_{19}A_{4}e^{-A_{4y}} \right) - 2A_4^2(A_{16}^2e^{2A_{4y}} + A_{15}^2e^{-2A_{4y}}) - \frac{2A_5^2A_3e^{-2A_{3y}}}{(A_4 - A_3^2)^2} - \frac{2A_5A_6A_3e^{-A_{3y}}}{A_4(A_4 - 4A_3^2)^2} \\
+ 2A_{15}A_4^2A_5e^{-y(A_4 + A_3)}(A_4 + A_3) - \frac{2A_{16}A_4^2A_5e^{y(A_4 - A_3)}(A_4 - A_3)}{(A_4 - A_3^2)^2} \right) M^2 \cos^2 \xi \\
- (n - 1) \left( A_{20}A_4^3e^{A_{4y}} - \frac{2A_6A_4^3(A_{16}e^{A_{4y}} - A_{15}e^{-A_{4y}})}{A_4 - 1} - A_{19}A_4^3e^{-A_{4y}} \right) \\
+ \frac{8A_4^6(A_{16}^2e^{2A_{4y}} - A_{15}^2e^{-2A_{4y}})A_4^3e^{-3A_{3y}}}{4A_4 - 1} + \frac{8A_5^2A_3e^{-3A_{3y}}}{(A_4 - A_3^2)^2} + \frac{2A_5A_6A_3e^{-3A_{3y}}}{A_4(A_4 - A_3^2)} \\
+ 2A_{15}A_4^2A_5e^{-y(A_4 + A_3)}(A_4 + A_3)^3 + \frac{2A_{16}A_4^2A_5e^{y(A_4 - A_3)}(A_4 - A_3)^3}{(A_4 - A_3^2)(A_4 - (A_4 - A_3)^2)} \\
- 2n \left( A_{16}^3e^{A_{4y}} - A_{15}A_4^3e^{-A_{4y}} \right) + \frac{A_6A_4^3e^{-A_{3y}}}{A_4^2A_3^2 - A_4^2} \right) \\
(5.34)
\]

Defining

\[
F = F_0 + WeF_1. 
(5.35)
\]

Summarizing the perturbation solutions up to first order for \( \psi, dp/dx \) and \( \Delta p \) as

\[
\psi = \psi_0 + We\psi_1, \quad \frac{dp}{dx} = \frac{dp_0}{dx} + We\frac{dp_1}{dx}, \quad \Delta p = \Delta p_0 + We\Delta p_1, \quad (5.36)
\]

Using \( F_0 = F - o(WeF_1) \) and then neglecting the terms greater than \( o(We) \), Equation (5.36) can be expressed up to first order.
The average rise in pressure $\Delta P_\lambda$ over one period of wave is given as follows:

$$\Delta P_\lambda = \int_0^1 \left( \frac{\partial p}{\partial x} \right)_{y=0} \, dx \, dt .$$

(5.37)

The constant expressions are described in the Appendix section.

### 5.5 GRAPHICAL RESULTS AND DISCUSSION

The perturbation solution of Equations (5.12) – (5.15) subject to the boundary conditions (5.17) – (5.18) has been computed. The graphical results of axial velocity, average rise in pressure, temperature, nanoparticle concentration and streamlines are displayed in Figures 2–5. Nowadays, the nanoparticles and nanofluids have enormous applications in field of science and technology. Now, attention is focused on such a new weapon in the arsenal to fight cancer, exploring the advances in areas such as improved drug delivery, new therapies, energy conversion, material properties, and fluid flow and heat transfer. Magnetic nanofluids are to be utilized with magnets to direct the particles up the bloodstream to the tumor. This will allow doctors to distribute high local doses of drugs or radiation without damaging in close proximity healthy tissue, which may be a substantial side impression of traditional cancer treatment methods (Foster 2000, Zhu et al. 1998).

#### 5.5.1 Flow characteristics

To study the influences of Brownian motion ($N\beta$), Hartmann number ($M$), heat source/sink parameter ($\beta$), inclined magnetic angle ($\xi$), non-uniform parameter ($m$), power law index number ($n$), Weissenberg number ($We$) and radiation parameter ($Rn$) on the axial velocity ($u$), we have plotted Figures 5.1 - 5.7 for fixed values of other parameters. The effects of varying Brownian
motion ($Nb$) on the axial velocity distribution can be seen in Figure 5.1. It can be noticed that the axial velocity decreases in the region $y \in [-1.08, 0.16]$, otherwise it increases as Brownian motion increases. Figure 5.2 is drawn to study the effect of the transverse magnetic field parameter ($M$) on the axial velocity ($u$). It is obvious that the axial velocity field decreases with the increase of magnetic field parameter $M$. This suggests that the axial velocity is diluted by increasing the magnetic field and supports the concept that the purpose of a magnetic field to an electrically conducting fluid develops a dragline force which induces a reduction in the nanofluid axial velocity. Figure 5.3 illustrates the effect of the heat source/sink parameter on the axial velocity for the fixed values of other parameters. It is clear that the axial velocity increases as $\beta$ increases at $y \in [-1.5, -0.37]$ and opposite situation is noticed in the rest of the channel. Moreover the axial velocity profiles are parabolic in nature. The result of the inclination of the magnetic field parameter on the axial velocity of nanofluid is shown in Figure 5.4. It is mentioned that the performance of $\xi$ on axial velocity profile is quite opposite as compared to those of Brownian motion parameter ($Nb$). Figure 5.5 is plotted to see the effects of non-uniform channel parameter $m$ on the axial velocity profile. It is expressed that the behavior of axial velocity near the channel walls and at the centre is not similar, also the maximum velocities are always occurred at the heart part of the channel, decaying smoothly to zero at the periphery (channel wall). Perhaps, the velocity of the nanofluid in an uniform channel is higher than the non-uniform channel. The effect of power law index parameter on the axial velocity is shown in Figure 5.6. It is considered that the axial velocity decreases at $y \in [-1.49, 0.15]$ after that it increases with increasing $n$. The axial velocity for the Weissenberg number ($We$) has been plotted in Figure 5.7. It is found that the axial velocity field increases in the region $y \in [-1.46, -0.18]$, wherein $y \in [-0.18, 1.5]$ gets increases.
\[ a = 0.1, \quad b = 0.2, \quad \Theta = 1.9, \quad m = 0.3, \]
\[ \phi = 3\pi/2, \quad We = 0.25, \quad M = 1, \quad Pr = 3, \]
\[ Nt = 0.5, \quad Gr = 1.2, \quad Br = 1.8, \quad Rn = 0.5, \]
\[ \xi = \pi/4, \quad n = 0.6, \quad \beta = 0.2, \quad x = 0.4, \quad t = 0.2. \]

Figure 5.1  Axial velocity profile  \( u(y) \) for  \( Nb \)

\[ a = 0.5, \quad b = 0.3, \quad \Theta = 1.4, \quad m = 0.1, \]
\[ \phi = \pi/4, \quad We = 0.05, \quad Pr = 2, \quad Nt = 0.4, \]
\[ Nb = 0.8, \quad Gr = 2, \quad Br = 0.5, \quad Rn = 4, \]
\[ \xi = \pi/6, \quad n = 3, \quad \beta = 0.2, \quad x = 0.4, \quad t = 0.2. \]

Figure 5.2  Axial velocity profile  \( u(y) \) for  \( M \)
Figure 5.3 Axial velocity profile $u(y)$ for $\beta$

Figure 5.4 Axial velocity profile $u(y)$ for $\xi$
Figure 5.5  Axial velocity profile $u(y)$ for $m$

Figure 5.6  Axial velocity profile $u(y)$ for $n$
Figure 5.7  Axial velocity profile  $u(y)$ for  $We$

Figure 5.8  Average rise in pressure  $\Delta p_\lambda$ for  $We$
Figure 5.9  Average rise in pressure $\Delta p_{\phi}$ for $m$

Figure 5.10  Average rise in pressure $\Delta p_{\phi}$ for $M$
5.5.2 Peristaltic Pumping Characteristics

MATHEMATICA and MATLAB are used to evaluate the integrals in Equation (5.37) and later generated all the plots for various values of the parameters of interest. Figures 5.8 – 5.10 illustrate the variation of the average rise in pressure ($\Delta p_\lambda$) versus time – averaged flow rate ($\Theta$). The pumping regions, peristaltic pumping ($\Theta > 0, \Delta p_\lambda > 0$), Augmented pumping ($\Theta > 0, \Delta p_\lambda < 0$) and retrograde pumping ($Q < 0, \Delta p > 0$) are also shown in Figures 5.8 – 5.10. Figure 5.8 is a graph of variation $\Delta p_\lambda$ versus $\Theta$ for different values of Weissenberg number ($We$). It is observed that the retrograde pumping rate increases with increase of $We$ and also noticed the non-linear relation amongst $\Theta$ and $\Delta p_\lambda$. Figure 5.9 elucidated the variations of average rise in pressure and time-average flow rate $\Theta$ for different value of non-uniform parameter. It is observed that the peristaltic free pumping rate decreases with an increase in non-uniform parameter. The variation of pressure rise with the mean flow for different values of $M$ is shown in Figure 5.10. It is clear that the retrograde pumping increases as $M$ increases.

5.5.3 Heat transfer and nanoparticle mass transfer distributions

The effects of $Nb$, $\beta$, $m$ and $Rn$ on heat transfer and nanoparticle mass transfer distributions are plotted in Figures 5.11-5.18. Figures 5.11 – 5.12, show that the heat transfer increases with the increase of $Nb$ and $\beta$. The effects of non-uniform parameter and radiation parameter on the heat transfer are considered in Figures 5.13 – 5.14. It is illustrated that the heat transfer increases with the increase of $m$ and decreases with $Rn$. Figures 5.15 – 5.18 represent the variation of the nanoparticle mass transfer $\sigma$ with non-dimensional transverse coordinate $y$ with different parameters. It is examined that the nanoparticle mass transfer $\left(\sigma\right)$ has reverse behaviour compared to heat transfer distribution.
\( a = 0.3, b = 0.2, m = 0.2, Rn = 0.6, \)
\( \phi = \pi/4, Nt = 0.5, Pr = 2, \)
\( \beta = 0.5, x = 0.5, t = 0.2. \)

**Figure 5.11** Temperature profile \( \theta(y) \) for \( N_b \)

\( a = 0.3, b = 0.5, m = 0.4, Rn = 0.6, \)
\( \phi = \pi/6, Nt = 0.4, N_b = 0.6, \)
\( Pr = 3, x = 0.5, t = 0.2. \)

**Figure 5.12** Temperature profile \( \theta(y) \) for \( \beta \)
Figure 5.13  Temperature profile $\theta(y)$ for $m$

Figure 5.14  Temperature profile $\theta(y)$ for $Rn$
Figure 5.15  Nanoparticle volume fraction $\sigma(y)$ for $Nb$

Figure 5.16  Nanoparticle volume fraction $\sigma(y)$ for $\beta$
Figure 5.17 Nanoparticle volume fraction $\sigma(y)$ for $m$

Figure 5.18 Nanoparticle volume fraction $\sigma(y)$ for $Rn$
Figure 5.19 Streamlines for (a) \( m = 0, \xi = \pi / 6, n = 0.2 \); (b) \( m = 0.2, \xi = \pi / 6, n = 0.2 \); (c) \( m = 0.12, \xi = \pi / 3, n = 0.2 \); (d) \( m = 0.2, \xi = \pi / 3, n = 3 \); and fixed other parameters are \( a = 0.3, b = 0.2, \phi = \pi / 4, We = 0.1, M = 0.7, Pr = 1, Nt = 0.4, Nb = 0.2, Gr = 0.5, Br = 0.2, Rn = 2 \).
5.5.4 Trapping phenomena

The formation of an inside circulating bolus of fluid as a result of closed streamlines is named trapping and this trapped bolus is pulled ahead along with the peristaltic wave. The stream lines for non-uniform parameter \( m \), inclination of the magnetic field \( \gamma \) and power law index number \( n \) are plotted in Figure 5.19. The effect of non-uniform parameter \( m \) on trapping is analyzed through Figures 5.19(a) – 5.19(b). It is indicated that the size of the trapped bolus increases as the channel changes from uniform asymmetric channel \( (m=0) \) to non-uniform asymmetric channel \( (m>0) \). From the result of the inclination of the magnetic field on trapping, we have prepared Figures 5.19(b) – 5.19(c). We observe that the increase in the inclination of the magnetic field increases the size of the trapped bolus of the tapered asymmetric channel. To see the influences of power law index number on trapping we have
prepared Figures 5.19(c) – 5.19(d). We note that an increase in the power law index number decreases the size of trapped bolus.

5.6 CONCLUDING REMARKS

In the present analysis, we have discussed an inclination of the magnetic field of a hyperbolic tangent nanofluid in the tapered asymmetric channel with the presence of heat source/sink and radiation parameters. The exact form solutions for temperature and nanoparticle volume fraction have been achieved and the perturbation technique in the Weissenberg number was utilized to get explicit solution for axial velocity, stream function and pressure gradient. The main findings are summarized as follows: The geometric parameters like, non-uniform parameter, amplitudes and phase difference control the fluid transport phenomena.

i. Figure 5.20 represents the comparison of analytical and numerical solutions for the stream function \( \psi \). We find a very good agreement between the results obtained by the perturbation method and numerical solution for all values of \( y \).

ii. The axial velocity increases in the left side of channel and the converse of this behavior occur in the right side wall of tapered asymmetric channel with increase of the Brownian motion inclination of the magnetic field.

iii. The nanoparticles mass transfer \( \sigma \) has reverse behavior compared to temperature distribution.

iv. The size of trapped bolus occurring in the tapered asymmetric channel increases with increasing \( m \) as it decreases with increasing \( n \).