CHAPTER 4

OPTIMAL RELIABILITY FOR TRANSMISSION FLOWMIXED SERIES-PARALLEL SYSTEM

This chapter considers a problem of mixed system of reliability. The mixed series and parallel configuration of the components are considered, which are connected in series and parallel. The objective is to maximize the reliability subject to the system cost. The cost constraints are taken as interval valued weighted trapezoidal fuzzy number. Few previous models have been limited to the mixed series situation which is a certain case of this model. A numerical example is given to illustrate the model for non linear programming and to evaluate the mixed system reliability.

4.1 PRELIMINARIES

In this chapter, the transmission system network is considered for mixed series and parallel configuration. In General, the transmission system design is series-parallel configuration or mixed series and parallel configuration. In series-parallel configuration, there are ‘m’ subsystem connected in series and those subsystems consisting of ‘n’ components in parallel. In mixed series and parallel configuration the components are connected in series and parallel. Almost the problem of series, parallel or series-parallel system of reliability may be formed as a typical non-linear programming with cost functions in fuzzy environment. This chapter presents a solution for transmission system network in mixed series-parallel configuration and to evaluate the maximum reliability subject to the system
cost. The cost functions are taken as the interval valued for generalized trapezoidal fuzzy number.

4.1.1 Notations

In order to formulate the problem in Series, Parallel and Mixed series-parallel system configuration the following notations have been developed.

\[ R_s(R_1, R_2, ..., R_n) \] - Reliability of Series system with reliability \( R_i \),
for \( i = 1, 2, ..., n \)

\[ R_p(R_1, R_2, ..., R_m) \] - Reliability of Parallel system with reliability \( R_j \),
for \( j = 1, 2, ..., m \)

\[ C_s(R_1, R_2, ..., R_n) \] - Cost of Series system with reliability \( R_i \), for \( i = 1, 2, ..., n \)

\[ C_p(R_1, R_2, ..., R_m) \] - Cost of Parallel system with reliability \( R_j \), for \( j = 1, 2, ..., m \)

4.2 RELIABILITY OPTIMIZATION MODEL

Reliability optimization is described as the selection of components and a system design to maximize the reliability. The reliability of the components measured for the system is generally not known in practice and must be predictable from data. Therefore there is some uncertainty associated with the reliability estimate of the component. In practice, the problem of series, parallel or its combination system reliability may be formed as a usual non-linear programming with cost functions in fuzzy environment.

4.2.1 Formulation of System Reliability

In series system reliability the ‘n’ subsystem is connected in series as in Figure 1.5, in parallel system reliability the ‘m’ subsystem is connected
in parallel as in Figure 1.6 and mixed system is combination of series and parallel system.

Then, the system of reliability in the form of reliability of subsystem is defined as

\[
R = \begin{cases} 
\prod_{i=1}^{n} R_i & \text{for series system} \\
1 - \prod_{j=1}^{m} (1 - R_j) & \text{for parallel system} \\
or combination of series and parallel system
\end{cases} \tag{4.1}
\]

In reliability optimization problems, we have to find the maximization of reliability subject to several constraints. For instance designer is required to minimize the system cost simultaneously maximizing the system of reliability.

The series system model with n components is taken, then the maximization of reliability \( R_S(R_1, R_2, ..., R_n) \) subject to the limited available cost \( c_i \), for \( i=1,2,\ldots,n \) and the cost constraint \( C_S \) is

Maximize \( R_S(R_1, R_2, R_3, ..., R_n) = \prod_{i=1}^{n} R_i \)

Subject to

\[
C_S(R_1, R_2, R_3, ..., R_n) = \sum_{i=1}^{n} c_i R_i \leq C_S \tag{4.2}
\]

The parallel system model with m components is taken, then the maximization of reliability \( R_P(R_1, R_2, ..., R_m) \) subject to the limited available cost \( c_j \), for \( j=1,2,\ldots,m \) and the cost constraint \( C_P \) is
Maximize \( R_p(R_1, R_2, R_3, ..., R_m) = 1 - \prod_{j=1}^{m} (1 - R_j) \)

Subject to

\[
C_p(R_1, R_2, R_3, ..., R_m) = \sum_{j=1}^{m} c_j R_j \leq C_p \quad (4.3)
\]

To perform the objective of mixed system, both series and parallel systems are required in action together.

4.2.2 System Reliability Optimization in Fuzzy Environment

To analyze the fuzzy system reliability, the system reliability of cost component and cost constraints can be involved in uncertain factors and is represented as fuzzy number. So the constraint of system reliability becomes fuzzy in a reliability optimization problem. Therefore it can be represented as fuzzy non-linear programming with fuzzy number.

Then maximization of series system of reliability \( R_s(R_1, R_2, ..., R_n) \) is

Maximize \( R_s(R_1, R_2, R_4, ..., R_n) = \prod_{i=1}^{n} R_i \)

Subject to

\[
C_s(R_1, R_2, R_3, ..., R_n) = \sum_{i=1}^{n} \bar{c}_i R_i \leq \bar{C}_s \text{ where } 0 \leq R_i \leq 1, \bar{c}_i \text{ and } \bar{C}_s \geq 0 \quad (4.4)
\]

The maximization of parallel system reliability \( R_p(R_1, R_2, ..., R_m) \) is

Maximize \( R_p(R_1, R_2, R_3, ..., R_m) = 1 - \prod_{j=1}^{m} (1 - R_j) \)

Subject to

\[
C_p(R_1, R_2, R_3, ..., R_m) = \sum_{j=1}^{m} \bar{c}_j R_j \leq \bar{C}_p \text{ where } 0 \leq R_j \leq 1, \bar{c}_j \text{ and } \bar{C}_p \geq 0 \quad (4.5)
\]
4.3 MATHEMATICAL ANALYSIS

Consider a non-linear programming problem having one inequality constraint of the type is

Maximize \( Z = f( x_1, x_2 \ldots x_k) \)

Subject to

\[ h(x) \leq 0 \quad \text{where} \quad h(x) = g(x_1, x_2 \ldots x_k) - b, h(x) \geq 0 \]  \hspace{1cm} (4.6)

The objective function and constraint of (4.6) in fuzzy non-linear programming problem is

Maximize \( Z = f( x_1, x_2 \ldots x_k) \)

Subject to

\[ \tilde{h}(x) \leq 0 \]  \hspace{1cm} (4.7)

4.4 SOLUTION PROCEDURE FOR RELIABILITY OPTIMIZATION PROBLEMS

Step 1: Let the cost parameter be \( \tilde{c}_i = (c_{i_1}, c_{i_2}, c_{i_3}, c_{i_4} : \omega) \) and the cost constraint be \( \tilde{C} = (C_1, C_2, C_3, C_4 : \omega) \), \( i = 1, 2 \ldots n \), are taken as weighted trapezoidal fuzzy number.

Step 2: Using \( \alpha \)-cut membership fuzzy function of cost parameters and cost constraints are given by

\[ \tilde{c}_i = [c_{i_1} + \frac{\alpha(c_{i_2} - c_{i_1})}{\omega}, c_{i_4} - \frac{\alpha(c_{i_4} - c_{i_3})}{\omega}], \quad i = 1, 2, 3, \ldots, n \]  and

\[ \tilde{C}_s = [C_1 + \frac{\alpha(C_2 - C_1)}{\omega}, C_4 - \frac{\alpha(C_4 - C_3)}{\omega}], \quad i = 1, 2, 3, \ldots, n \] respectively.
Step 3: Applying the Kuhn - Tucker condition in a fuzzy non-linear programming problem for given models for \( i = 1, 2 \ldots n \) of left and right interval \( \alpha \)-cut is expressed as

Maximize \( R_s(r_i^L) = 1 - \prod_{i=1}^{n} (1 - r_i^L) \)

Subject to

\[ \sum_{i=1}^{n} c_i^L r_i - \tilde{C}_s^L \leq 0, \text{ where } 0 < r_i \leq 1, C_s^L \geq 0 \]

Maximize \( R_s(R_i^R) = 1 - \prod_{i=1}^{m} (1 - r_i^R) \)

Subject to

\[ \sum_{i=1}^{n} c_i^R r_i - \tilde{C}_s^R \leq 0, \text{ where } 0 < r_i \leq 1, C_s^R \geq 0 \]

Step 4: To find the optimal solution of \( R_s^L(r_i) \) and \( R_s(R_i^R), i = 1, 2 \ldots n \) for each membership value of \( \alpha \) from step-2,3.

Step 5: To calculate the system of reliability \( R_s^L = \prod_{i=1}^{n} R_s(r_i^L) \) and \( R_s^R = \prod_{i=1}^{n} R_s(r_i^R) \), for \( i=1,2,\ldots,n \) for series system reliability.

Step 6: The above same procedure is applied to calculate the system of reliability \( R_p^L = 1 - \prod_{j=1}^{m} (1 - R_p(r_j^L)) \) and \( R_p^R = 1 - \prod_{j=1}^{m} (1 - R_s(r_j^R)) \), \( j=1, 2\ldots m \) for parallel system of reliability.

Step 7: Using series and parallel system of reliability, maximum of reliability of mixed system is found for each membership values of \( \alpha \).
4.5 ILLUSTRATIVE EXAMPLE

The transmission system network for mixed system reliability is considered, which can be maximized for system of reliability subject to available cost. The design configuration of transmission system network is presented in Figure 4.1 as follows.

It consists of three main units (subsystems) connected in mixed system. The first unit consists of 2 components in parallel, second unit consists of single component connected with series of first system and third component consists of 2 components in series connected with parallel to first two units.

![Figure 4.1 Design Configuration of Transmission System Network](image)

In reliability optimization model, the cost components, cost constraints are assumed and system weight is fuzzy in nature, where the cost components and cost constraints are taken as weighted trapezoidal fuzzy number.
**Step 1:** For Unit 1, the cost components are $c_{i1} = (18, 18.5, 19.0, 19.5: 0.7)$, $c_{i2} = (29.25, 30.0, 31.0, 31.75: 0.8)$ and cost constraint is $C_1 = (43.0, 43.75, 44.5, 45:0.85)$

For Unit 2, the cost component is $c_{i1} = (25.0, 25.50, 26.5, 27.0: 0.75)$ and cost constraint is $C_2 = (24, 24.5, 25.0, 25.5:0.8)$

For Unit 3, the cost components are $c_{i1} = (16, 16.5, 17.5, 18: 0.9)$, $c_{i2} = (33, 33.75, 34.4, 35:0.8)$ and cost constraint is $C_3 = (51.0, 51.5, 52.5, 53:0.85)$

**Step 2:** The cost components and cost constraints of left and right interval $\alpha$-cut membership function which are represented Table 4.1 as follows.

**Table 4.1  Cost components and cost constraints of left and right interval $\alpha$-cut membership function**

<table>
<thead>
<tr>
<th>Unit (i)</th>
<th>Cost components ($c_i$)</th>
<th>Cost constraints ($C_i$)</th>
</tr>
</thead>
</table>
| 1        | $c_{i1} = (c_{i1L}, c_{i1R})$  
= $(18.0+0\text{.}714286\alpha , 19.5-0\text{.}714286\alpha )$ 
$c_{i2} = (c_{i2L}, c_{i2R})$  
= $(29.5+0\text{.}937500\alpha , 31.5-0\text{.}937500\alpha ) $ | $C_1 = (C_{1L}, C_{1R})$  
= $(43.0 +0.882353\alpha , 45.0-0.588235\alpha)$ |
| 2        | $c_{i1} = (c_{i1L}, c_{i1R})$  
= $(25+0\text{.}666667\alpha , 27.0-0\text{.}666667\alpha )$ | $C_2 = (C_{2L}, C_{2R})$  
= $(24 +0.625000\alpha , 25.5 - 0.625000\alpha)$ |
| 3        | $c_{i1} = (c_{i1L}, c_{i1R})$  
= $(16+0\text{.}555555\alpha , 18.0 -0\text{.}555556\alpha)$ 
$c_{i2} = (c_{i2L}, c_{i2R})$  
= $(17.5+0\text{.}625000\alpha , 19.5-0\text{.}937500\alpha)$ | $C_3 = (C_{3L}, C_{3R})$  
= $(30.0+0.588235\alpha , 32.5- 0.588235\alpha)$ |
Step 3: Kuhn-Tucker condition in the optimal solution of a fuzzy non-linear programming problem with i\textsuperscript{th} unit $i = 1, 2, 3$ of left and right interval $\alpha$-cut can be expressed as

Maximize $R_i^L(r_{ij}^L) = 1 - \prod_{j=1}^{m} (1 - r_{ij}^L)$

Subject to

$$\sum_{j=1}^{m} c_{ij}^L r_{ij}^L - \bar{C}_i^L \leq 0 \text{ and}$$

Maximize $R_i^R(r_{ij}^R) = 1 - \prod_{j=1}^{m} (1 - r_{ij}^R)$

Subject to

$$\sum_{j=1}^{m} c_{ij}^R r_{ij}^R - \bar{C}_i^R \leq 0 \quad (4.8)$$

In the first Unit,

The left interval $\alpha$-cut is

Maximize $R_i^L(r_{ij}^L) = 1 - \prod_{j=1}^{2} (1 - r_{ij}^L) = 1 - ((1 - r_{11}^L)(1 - r_{12}^L))$ \quad (4.9)

Subject to

$$(\bar{e}_{11}^L r_{11}^L + \bar{e}_{12}^L r_{12}^L - \bar{C}_1^L) \leq 0 \quad (4.10)$$

The necessary condition for this maximization problem is

$$\frac{\partial}{\partial r_{ij}^L} [1 - (1 - r_{11}^L)(1 - r_{12}^L)] = \lambda \frac{\partial}{\partial r_{ij}^L} (\bar{e}_{11}^L r_{11}^L + \bar{e}_{12}^L r_{12}^L - C_1^L) \quad \text{where } j = 1, 2 \quad (4.11)$$

$$(\bar{e}_{11}^L r_{11}^L + \bar{e}_{12}^L r_{12}^L) - C_1^L = 0 \quad (4.12)$$
Similarly, the right interval $\alpha$-cut is

$$\frac{\partial}{\partial r_{ij}} [1 - (1 - r_{ij}^L)(1 - r_{ij}^R)] = \lambda \left[ \frac{\partial}{\partial r_{ij}^R} (\tilde{c}_{11}^R r_{11}^R + \tilde{c}_{12}^R r_{12}^R) - C_1^R \right] \text{ where } j=1,2 \quad (4.13)$$

$$\tilde{c}_{11}^R r_{11} + \tilde{c}_{12}^R r_{12} - \tilde{C}_1^R = 0 \quad (4.14)$$

**In the Second Unit,**

The left interval value of $\alpha$-cut is

Maximize $R_2^L (r_{21}^L) = r_{21}^L \quad (4.15)$

Subject to

$$\tilde{c}_{21}^L r_{21}^L - \tilde{C}_2^L \leq 0$$

The necessary condition for this maximization problem is

$$\tilde{c}_{21}^L r_{21}^L = \tilde{C}_2^L \quad (4.16)$$

The right interval $\alpha$-cut is

Maximize $R_2^R (r_{21}^R) = r_{21}^R \quad (4.17)$

Subject to

$$\tilde{c}_{21}^R r_{21}^R - \tilde{C}_2^R \leq 0$$

The necessary condition for this maximization problem is

$$\tilde{c}_{21}^R r_{21}^R = \tilde{C}_2^R \quad (4.18)$$
In the third Unit,

The left interval $\alpha$-cut is

Maximize $R^L_3(r^L_{3j}) = \prod_{j=1}^{2} r^L_{3j} = r^L_{31} r^L_{32}$

Subject to

$$(\tilde{c}^L_{31} r^L_{31} + \tilde{c}^L_{32} r^L_{32}) - \tilde{C}^L_3 \leq 0$$

The necessary condition for this maximization problem is

$$\frac{\partial}{\partial r^L_{3j}} [r^L_{31} r^L_{32}] = \lambda \left[ \frac{\partial}{\partial r^L_{3j}} (\tilde{c}^L_{31} r^L_{31} + \tilde{c}^L_{32} r^L_{32}) - C^L_3 \right]$$ (4.19)

$$\tilde{c}^L_{31} r^L_{31} + \tilde{c}^L_{32} r^L_{32} = \tilde{C}^L_3$$ (4.20)

The right interval $\alpha$-cut is

Maximize $R^R_3(r^R_{3j}) = \prod_{j=1}^{2} r^R_{3j} = r^R_{31} r^R_{32}$

Subject to

$$\tilde{c}^R_{31} r^R_{31} + \tilde{c}^R_{32} r^R_{32} = \tilde{C}^R_3$$

The necessary condition for this maximization problem is

$$\frac{\partial}{\partial r^R_{3j}} [r^R_{31} r^R_{32}] = \lambda \left[ \frac{\partial}{\partial r^R_{3j}} (\tilde{c}^R_{31} r^R_{31} + \tilde{c}^R_{32} r^R_{32}) - C^R_3 \right] \text{ where } j=1,2$$ (4.21)

$$\tilde{c}^R_{31} r^R_{31} + \tilde{c}^R_{32} r^R_{32} = \tilde{C}^R_3$$ (4.22)
Step 4:

In the First Unit,

Left Interval Division value

Applying left interval values of cost co-efficient and cost constraint

in the equations (4.11) and (4.12) become

\[
\frac{\partial}{\partial r_{ij}^L} [1 - (1 - r_{ij}^L)(1 - r_{ij}^L)] = \lambda \frac{\partial}{\partial r_{ij}^L} ((18 + 0.714286\alpha)r_{i1}^L + (29.5 + 0.937500\alpha)r_{i2}^L - (43 + 0.882353\alpha))
\]

where \(i=1, 2\) (4.23)

\[
(18 + 0.714286\alpha)r_{i1}^L + (29.5 + 0.937500\alpha)r_{i2}^L = 43 + 0.882353\alpha \quad (4.24)
\]

Substituting \(\alpha = 0\) in the equation (4.23) and (4.24) become

\[
\frac{\partial}{\partial r_{ij}^L} [1 - (1 - r_{ij}^L)(1 - r_{ij}^L)] = \lambda \frac{\partial}{\partial r_{ij}^L} (18r_{i1}^L + 29.5r_{i2}^L - 43.0) \quad \text{where } i=1,2 \quad (4.25)
\]

\[
18r_{i1}^L + 29.5r_{i2}^L = 43.0 \quad (4.26)
\]

From the equation (4.25),

\[
1 - r_{i2}^L = 18\lambda \quad (4.27)
\]

\[
1 - r_{i1}^L = 29.5\lambda \quad (4.28)
\]
Using the equations (4.27) and (4.28)

\[(1 - r_{12}^L) = 0.610169 (1 - r_{11}^L)\]

\[r_{12}^L = 0.389831 + 0.610169 r_{11}^L \]  
\hspace{3cm} (4.29)

Substituting the value of \( r_{12}^L \) in the equation (4.26),

\[36.0 r_{11}^L + 11.5 = 43.0\]

\[r_{11}^L = 0.875000 \]  
\hspace{3cm} (4.30)

From the equation (4.29)

\[r_{12}^L = 0.923729 \]  
\hspace{3cm} (4.31)

For \( \alpha = 0.2 \), the equation (4.23) and (4.24) become

\[\frac{\partial}{\partial r_{ij}^L} [1 - (1 - r_{11}^L)(1 - r_{12}^L)] = \lambda \left[ \frac{\partial}{\partial r_{ij}^L} (18.142857 r_{11}^L + 29.687500 r_{12}^L) - 43.176471 \right] \]

where \( j = 1, 2 \) \hspace{3cm} (4.32)

\[18.142857 r_{11}^L + 29.687500 r_{12}^L = 43.176471 \]  
\hspace{3cm} (4.33)

From the equation (17),

\[1 - r_{12}^L = 18.142857 \lambda \]  
\hspace{3cm} (4.34)

\[1 - r_{11}^L = 29.687500 \lambda \]  
\hspace{3cm} (4.35)

Solving the equation (4.33), (4.34) and (4.35)

\[r_{11}^L = 0.871743 \text{ and } r_{12}^L = 0.921619\]
Continuing this process, for $\alpha = 0.2, 0.4, 0.6, 0.8$ and $1.0$ respectively in the equations (4.23) and (4.24) then the reliability values are

\[ r_{11}^R = 0.919535 \text{ and } r_{12}^R = 0.868647 \]

\[ r_{11}^R = 0.865381 \text{ and } r_{12}^R = 0.917478 \]

\[ r_{11}^R = 0.862274, \text{ and } r_{12}^R = 0.915446 \]

\[ r_{11}^R = 0.859214, \text{ and } r_{12}^R = 0.913438 \]

**Right Interval Division Value**

Applying right interval value of cost co-efficient and cost constraint in the above equations,

\[
\frac{\partial}{\partial r_{ij}^R}[1 - (1 - r_{11}^R)(1 - r_{12}^R)] = \lambda \left[ \frac{\partial}{\partial r_{ij}^R}((19.5 - 0.714286\alpha)r_{11}^R + (31.75 - 0.937500\alpha)r_{12}^R - (45 - 0.588235\alpha)) \right] \\
\text{where } j = 1, 2 \quad (4.36)
\]

\[(19.5 - 0.714286\alpha)r_{11}^R + (31.75 - 0.937500\alpha)r_{12}^R = (45 - 0.588235\alpha) \quad (4.37)\]

For $\alpha = 0$, the equation (4.36) and (4.37) become

\[
\frac{\partial}{\partial r_{ij}^R}[1 - (1 - r_{11}^R)(1 - r_{12}^R)] = \lambda \left[ \frac{\partial}{\partial r_{ij}^R}(19.5r_{11}^R + 31.75r_{12}^R) - 45.0 \right] \\
\text{where } j = 1, 2, 3 \quad (4.38)
\]

\[19.5r_{11}^R + 31.75r_{12}^R = 45.0 \quad (4.39)\]
From the equation (4.38),

\[ 1 - r_{12}^R = 19.5\lambda \]  
(4.40)

\[ 1 - r_{11}^R = 31.75\lambda \]  
(4.41)

Using the equations (4.40) and (4.41)

\[ (1 - r_{12}^R) = 0.614173(1 - r_{11}^R) \]

\[ r_{12}^R = 0.385827 + 0.614173 r_{11}^R \]  
(4.42)

Substituting the value of \( r_{12}^R \) in the equation (4.39)

\[ r_{11}^R = 0.839744 \]  
(4.43)

From the equation (4.42)

\[ r_{12}^R = 0.901575 \]  
(4.44)

For \( \alpha = 0.2 \), the equation (4.36) and (4.37) become

\[ \frac{\partial}{\partial r_{1j}^R} [1 - (1 - r_{11}^R)(1 - r_{12}^R)] = \lambda \left( \frac{\partial}{\partial r_{1j}^R} (19.357143r_{11}^R + 31.562500 r_{12}^R) - 44.882353 \right) \]

where \( j = 1,2 \)  
(4.45)

\[ 19.357143 r_{11}^R + 31.562500 r_{12}^R = 44.882353 \]  
(4.46)

From the equation (4.45),

\[ 1 - r_{12}^R = 19.357143\lambda \]  
(4.47)

\[ 1 - r_{11}^R = 31.562500\lambda \]  
(4.48)
Solving the equation (4.46), (4.47) and (4.48)

\[ r_{11}^R = 0.844067 \text{ and } r_{12}^R = 0.904360 \]

Continuing this process, for \( \alpha = 0.2, 0.4, 0.6, 0.8 \text{ and } 1.0 \) respectively in the equations (4.36) and (4.37) then the reliability values are

\[ r_{11}^R = 0.848431 \text{ and } r_{12}^R = 0.907178 \]
\[ r_{11}^R = 0.852872 \text{ and } r_{12}^R = 0.910030 \]
\[ r_{11}^R = 0.857381, \text{ and } r_{12}^R = 0.912917 \]
\[ r_{11}^R = 0.861958, \text{ and } r_{12}^R = 0.915839 \]

**In the Second Unit,**

**Left Interval Division value**

Applying left interval value of cost co-efficient and cost constraint in the equation (4.16) become

\[ (25.0 + 0.666667\alpha)r_{21}^L = 24.0 + 0.625\alpha \quad (4.49) \]

Substituting \( \alpha = 0 \) in the above equations

\[ 25.0 r_{21}^L = 24 \]
\[ r_{21}^L = 0.960000 \quad (4.50) \]

Continuing this process, for \( \alpha = 0.2, 0.4, 0.6, 0.8 \text{ and } 1.0 \) respectively in the equations (4.49) then the reliability values are
\[ r_{21}^L = 0.959881 \]
\[ r_{21}^L = 0.959763 \]
\[ r_{21}^L = 0.959646 \]
\[ r_{21}^L = 0.959530 \]
\[ r_{21}^L = 0.959416 \]

**Right Interval Division value**

Applying right interval value of cost co-efficient and cost constraint in the equation (4.18) become

\[ (27.0 - 0.666667 \alpha ) r_{21}^R = 25.5 - 0.625 \alpha \]  
(4.51)

Substituting \( \alpha = 0 \) in the above equations

\[ 27.0 \ r_{21}^R = 25.5 \]
\[ r_{21}^R = 0.944444 \]  
(4.52)

Continuing this process, for \( \alpha = 0.2, 0.4, 0.6, 0.8 \) and 1.0 respectively in the equations (4.51) then the reliability values are

\[ r_{21}^R = 0.944479 \]
\[ r_{21}^R = 0.944514 \]
\[ r_{21}^R = 0.944549 \]
\[ r_{21}^R = 0.944584 \]
\[ r_{21}^R = 0.944620 \]
In the Third Unit,

Left Interval Division value

Applying left interval value of cost co-efficient and cost constraint in the equations (4.19) and (4.20) become

\[
\frac{\partial}{\partial r_{3j}^L}[r_{31}^L r_{32}^L] = \lambda \left[ \frac{\partial}{\partial r_{3j}^L} ((16 + 0.55555\alpha) r_{31}^L + (17.5 + 0.62500\alpha) r_{32}^L) - (30.5 + 0.588235\alpha) \right]
\]

where \( j = 1, 2 \)  \hspace{1cm} (4.53)

\[(16 + 0.55555\alpha) r_{31}^L + (17.5 + 0.62500\alpha) r_{32}^L = 30.5 + 0.588235\alpha \]  \hspace{1cm} (4.54)

Substituting \( \lambda = 0 \) in the above equations

\[
\frac{\partial}{\partial r_{3j}^L}[r_{31}^L r_{32}^L] = \lambda \left[ \frac{\partial}{\partial r_{3j}^L} (16.0 r_{31}^L + 17.5 r_{32}^L) - 30.5 \right] \quad \text{where } j = 1, 2
\]

\[16 r_{31}^L + 17.5 r_{32}^L = 30.5 \]  \hspace{1cm} (4.56)

From the equation (4.55),

\[ r_{32}^L = 16.0 \lambda \]  \hspace{1cm} (4.57)

\[ r_{31}^L = 17.5 \lambda \]  \hspace{1cm} (4.58)

From the equations (4.57) and (4.58),

\[ r_{32}^L = 0.914286 r_{31}^L \]  \hspace{1cm} (4.59)
From the equations (4.56) \[ 32 r^L_{31} = 30.5 \]

\[ \therefore r^L_{31} = 0.953125 \] (4.60)

\[ r^L_{32} = 0.871429 \] (4.61)

Continuing this process, for \( \alpha = 0.2, 0.4, 0.6, 0.8 \) and 1.0 respectively in the equations (4.53) and (4.54) then the reliability values are

\[ r^L_{31} = 0.950203 \text{ and } r^L_{32} = 0.868586 \]

\[ r^L_{31} = 0.947321 \text{ and } r^L_{32} = 0.865783 \]

\[ r^L_{31} = 0.944478 \text{ and } r^L_{32} = 0.863019 \]

\[ r^L_{31} = 0.941673 \text{ and } r^L_{32} = 0.860294 \]

\[ r^L_{31} = 0.938907 \text{ and } r^L_{32} = 0.857606 \]

**Right Interval Division value for the Third Unit**

Applying right interval value of cost co-efficient and cost constraint in the equations (4.21) and (4.22) become

\[ \frac{\partial}{\partial r^R_{3j}} [r^R_{31} r^R_{32}] \]

\[ = \lambda \left[ \frac{\partial}{\partial r^R_{3j}} ((18 - 0.555556 \alpha) r^R_{31} + (19.5 - 0.9375 \alpha) r^R_{32}) - (32.5 - 0.588235 \alpha) \right] \]

where \( j = 1, 2 \) (4.62)

\[ (18 - 0.555556 \alpha) r^R_{31} + (19.5 - 0.9375 \alpha) r^R_{32} = 32.5 - 0.588235 \alpha \] (4.63)
Substituting $\alpha = 0$ in the above equations

$$\frac{\partial}{\partial \beta_j} [r_{31} r_{32}] = \lambda \left[ \frac{\partial}{\partial \beta_j} (18.0 r_{31}^R + 19.5 r_{32}^R) - 32.5 \right] \text{ where } j = 1, 2 \quad (4.64)$$

(18.0 $r_{31}^R$ + 19.5 $r_{32}^R$) = 32.5 \quad (4.65)

From the equation (4.64),

$$r_{32}^R = 18.0 \lambda \quad (4.66)$$

$$r_{31}^R = 19.5 \lambda \quad (4.67)$$

From the equations (4.66) and (4.67),

$$r_{32}^R = 0.923077 r_{31}^R \quad (4.68)$$

From the equations (4.65)

$$36 r_{31}^R = 32.5$$

$$\therefore r_{31}^R = 0.902778 \quad (4.69)$$

$$r_{32}^R = 0.833333 \quad (4.70)$$

Continuing this process for $\alpha = 0.2, 0.4, 0.6, 0.8$ and 1.0 respectively in the equations (4.62) and (4.63) then the reliability values are

$$r_{31}^R = 0.905096 \text{ and } r_{32}^R = 0.838377$$

$$r_{31}^R = 0.907445 \text{ and } r_{32}^R = 0.843522$$

$$r_{31}^R = 0.909822 \text{ and } r_{32}^R = 0.848767$$

$$r_{31}^R = 0.912230 \text{ and } r_{32}^R = 0.854118$$

$$r_{31}^R = 0.914669 \text{ and } r_{32}^R = 0.859576$$
**Step 5:** For $\alpha = 0$

In the first unit, the left interval value of reliability is

$$r_{11}^L = 0.875000, \quad r_{12}^L = 0.923729$$

$$R_1^L = R_1^L (r_{1j}^L) = 1 - \prod_{j=1}^{2} (1 - r_{1j}^L) = 1 - ((1 - r_{11}^L)(1 - r_{12}^L))$$

$$= 0.990466$$

In the second unit, the left interval value of reliability is

$$r_{21}^L = 0.960000$$

$$R_2^L = R_2^L (r_{21}^L) = r_{21}^L = 0.960000$$

In the third unit, the left interval value of reliability is

$$r_{31}^L = 0.953125 \quad \text{and} \quad r_{32}^L = 0.871429$$

$$R_3^L = \prod_{j=1}^{2} r_{3j}^L = r_{31}^L r_{32}^L = 0.830581$$

**Step 6:** Since the first and second units are connected in series system, the left interval system reliability of first two units is

$$R_3^L (r_{11}^L, r_{12}^L, r_{21}^L) = \prod_{i=1}^{2} R_i^L = R_1^L R_2^L$$

$$= 0.950857$$
Step 7: The transmission flow mixed series-parallel system reliability of left interval value is

\[ R_M^L = R_{Mixed}^L = 1 - (1 - \prod_{i=1}^{2} R_i^L)(1 - R_3^L) = 1 - (1 - R_1^L R_2^L)(1 - R_3^L) \]

\[ = 0.991673 \]

Continuing this process, the left interval transmission flow with mixed series-parallel system reliability \( R_M^L \) for \( \alpha = 0.2, 0.4 \ldots 1.0 \) respectively has been found out.

Similarly, the right interval transmission flow with mixed series-parallel system reliability \( R_M^R \) for \( \alpha = 0, 0.2, 0.4 \ldots 1.0 \) respectively has been found out.

The following Table 4.2 and Table 4.3 show the left and right interval optimal solution of transmission network reliability system through fuzzy parametric non-linear programming.

**Table 4.2 Left interval optimal solution of fuzzy membership value of \( \alpha \)**

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( r_{11}^L )</th>
<th>( r_{12}^L )</th>
<th>( R_1^L )</th>
<th>( R_2^L )</th>
<th>( r_{31}^L )</th>
<th>( r_{32}^L )</th>
<th>( R_3^L )</th>
<th>( R_M^L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.875000</td>
<td>0.923729</td>
<td>0.990466</td>
<td>0.960000</td>
<td>0.953125</td>
<td>0.871429</td>
<td>0.830581</td>
<td>0.991673</td>
</tr>
<tr>
<td>0.2</td>
<td>0.871743</td>
<td>0.921619</td>
<td>0.989947</td>
<td>0.959881</td>
<td>0.950203</td>
<td>0.868586</td>
<td>0.825333</td>
<td>0.991307</td>
</tr>
<tr>
<td>0.4</td>
<td>0.868647</td>
<td>0.919535</td>
<td>0.989431</td>
<td>0.959763</td>
<td>0.947321</td>
<td>0.865783</td>
<td>0.820174</td>
<td>0.990940</td>
</tr>
<tr>
<td>0.6</td>
<td>0.865381</td>
<td>0.917478</td>
<td>0.988891</td>
<td>0.959646</td>
<td>0.944478</td>
<td>0.863019</td>
<td>0.815102</td>
<td>0.990568</td>
</tr>
<tr>
<td>0.8</td>
<td>0.862274</td>
<td>0.915446</td>
<td>0.988355</td>
<td>0.959530</td>
<td>0.941673</td>
<td>0.860294</td>
<td>0.810116</td>
<td>0.990194</td>
</tr>
<tr>
<td>1.0</td>
<td>0.859214</td>
<td>0.913438</td>
<td>0.987813</td>
<td>0.959416</td>
<td>0.938907</td>
<td>0.857606</td>
<td>0.805212</td>
<td>0.989817</td>
</tr>
</tbody>
</table>
Table 4.3 Right interval optimal solution of fuzzy membership value of $\alpha$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$r_{11}^R$</th>
<th>$r_{12}^R$</th>
<th>$R_1^R$</th>
<th>$R_2^R$</th>
<th>$r_{31}^R$</th>
<th>$r_{32}^R$</th>
<th>$R_3^R$</th>
<th>$R_M^R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.839744</td>
<td>0.901575</td>
<td>0.984227</td>
<td>0.944444</td>
<td>0.902778</td>
<td>0.833333</td>
<td>0.752315</td>
<td>0.982550</td>
</tr>
<tr>
<td>0.2</td>
<td>0.844067</td>
<td>0.904360</td>
<td>0.985087</td>
<td>0.94479</td>
<td>0.905096</td>
<td>0.838377</td>
<td>0.758812</td>
<td>0.983212</td>
</tr>
<tr>
<td>0.4</td>
<td>0.848431</td>
<td>0.907178</td>
<td>0.985931</td>
<td>0.94451</td>
<td>0.907445</td>
<td>0.843522</td>
<td>0.765450</td>
<td>0.983854</td>
</tr>
<tr>
<td>0.6</td>
<td>0.852872</td>
<td>0.910030</td>
<td>0.986763</td>
<td>0.944549</td>
<td>0.909822</td>
<td>0.848767</td>
<td>0.772227</td>
<td>0.984522</td>
</tr>
<tr>
<td>0.8</td>
<td>0.857381</td>
<td>0.912917</td>
<td>0.987580</td>
<td>0.944584</td>
<td>0.912230</td>
<td>0.852118</td>
<td>0.777328</td>
<td>0.985048</td>
</tr>
<tr>
<td>1.0</td>
<td>0.861958</td>
<td>0.915839</td>
<td>0.988382</td>
<td>0.944620</td>
<td>0.914669</td>
<td>0.859576</td>
<td>0.786228</td>
<td>0.985815</td>
</tr>
</tbody>
</table>

Figure 4.2 Interval valued reliability with weighted trapezoidal fuzzy number
4.6 CONCLUSION

The reliability evaluation of a system design with high reliability and low requirement, reveals that the system designer can adopt series, parallel and mixed system technique to improve system reliability with available constraints such as cost, weight etc. Transmission network system is a good example of mixed system reliability. This chapter attempts to provide the reliability optimization problem of the transmission system network under the fuzzy environment, where it maximizes the reliability subject to the available cost and the cost component and cost constraint of each subsystem is taken as weighted Trapezoidal fuzzy number. Kuhn-Tucker conditions are taken into concern to solve the non-linear programming problem with fuzzy components to find out the optimal solutions for left and right interval valued membership function of $\alpha$. This can be maximized for the system reliability subject to the available cost. Table 4.1, 4.2 shows that the left and right interval optimal solutions of transmission network system reliability, which identify that maximum reliability of the membership function.