CHAPTER 2

ECONOMIC DISPATCH USING EP, PSO AND HPSO ALGORITHMS WITH PROHIBITED OPERATING ZONES AND RAMP-RATE LIMITS CONSTRAINTS

2.1 INTRODUCTION

Economic dispatch (ED) is one of the essential optimization problems in power system that has the objective of sorting out the power demand among the online generators economically while satisfying various constraints (Selvakumar & Thanushkodi 2007). As power demand increases and since the fuel cost of the power generated is overstated, reducing the operating cost of power systems becomes an important topic. ED is one of the most important problems to be solved for smooth and economic operation of a power system. Improvements in scheduling the unit outputs can lead to significant cost savings. A good load dispatch reduces the production cost, increases the system reliability and maximizes the energy capability of thermal units (Grainger & Stevenson 1994). ED is a process for sharing the total load on a power system among various generating plants to achieve greatest economy of operation. The ED problem is a non linear optimization problem is basically solved to generate the optimal amount of generating power from the fossil fuel based generating units in the system by minimizing the fuel cost and satisfying all system constraints of power system network.
(Chowdhury & Rahman 1990). ED problem has been considered to be one of the key functions in electric power system operation which can help to build up an effective generating management plan.

In this chapter, Evolutionary Programming (EP), Particle Swarm Optimization (PSO) and Hybrid Particle Swarm Optimization (HPSO) algorithms have been applied to solve 3, 6, 15 and 20 generating unit systems while considering power balance equation, generator capacity limits and nonlinear characteristics of the generator such as prohibited operating zones and ramp rate limit constraints. The results obtained by the proposed HPSO algorithm are compared with EP, PSO and other methods which are presented in the references.

2.2 PROBLEM FORMULATION

The objective of ED problem is to minimize the total generation cost of thermal generating units, while satisfying various system constraints, including power balance equation, generator power limits, prohibited operating zones and ramp rate limit constraints (Bhattacharya & Chattopadhyay 2010).

The problem of ED is multimodal, non-differentiable and highly nonlinear. Mathematically, the problem can be stated as in (2.1)

\[
\text{Min } F_T = \sum_{i=1}^{N} F_i(P_{Gi}) \tag{2.1}
\]

Where \( i = 1, 2, 3, \ldots, N \)

Where \( F_T \) is the total fuel cost, \( N \) is the number of generating units in the system. \( F_i(P_{Gi}) \) is the fuel cost function of unit \( i \) and \( P_{Gi} \) is the output power of unit \( i \). Generally, the fuel cost of generating unit can be expressed as
\[ F_i(P_{Gi}) = a_iP_{Gi}^2 + b_iP_{Gi} + c_i \text{($/hr$)} \]  
\[ (2.2) \]

Where \( a_i, b_i \) and \( c_i \) are the cost coefficients of the unit \( i \) subjected to the following constraints.

**Real power balance equation:** The total generation must be equal to total demand plus transmission losses

\[ \sum_{i=1}^{N} P_{Gi} = P_D + P_L \]  
\[ (2.3) \]

Where \( P_D \) is real power demand and \( P_L \) is the transmission loss.

The transmission loss \( (P_L) \) can be expressed in a quadratic function of generation (Using B-loss coefficient matrix).

\[ P_L = \sum_{i=1}^{N} \sum_{j=1}^{N} P_{Gi}B_{ij}P_{Gj} + \sum_{i=1}^{N} B_{0i}P_{Gi} + B_{00} \]  
\[ (2.4) \]

Where \( P_{Gi} \) and \( P_{Gj} \) are the power generation of \( i^{th} \) and \( j^{th} \) units and \( B_{ij}, B_{0i}, B_{00} \) are the B – loss coefficients.

**Generator operating limits:** The power output of each unit \( i \) is restricted by its maximum and minimum limits of real power generation and is given by

\[ P_{Gi \text{ min}} \leq P_{Gi} \leq P_{Gi \text{ max}} \]  
\[ (2.5) \]

Where \( P_{Gi \text{ min}} \) and \( P_{Gi \text{ max}} \) are the minimum and maximum generation limits on \( i^{th} \) unit respectively.
Prohibited operating zones constraint: The generators may have the certain range where operation is restricted due to the physical limitation of steam valve, component, and vibration in shaft bearing etc.,. The consideration of Prohibited Operating Zones (POZ) creates a discontinuity in fuel cost curve and converts the constraint as below

\[ P_{Gi} \in \begin{cases} 
  P_{Gi,min} \leq P_{Gi} \leq P_{Gi,1} \\
  P_{Gi,k-1}^u \leq P_{Gi} \leq P_{Gi,k}^l \\
  P_{Gi,z_i}^u \leq P_{Gi} \leq P_{Gi,max} 
\end{cases} \]  

(2.6)

\( k = 2,3,\ldots,z_i \) and \( i = 1,2,\ldots,N \)

Where, \( P_{Gi,k}^l \) and \( P_{Gi,k}^u \) are the lower and upper boundary of \( k^{th} \) prohibited operating zone of unit \( i \), \( k \) is the index of the prohibited operating zone, and \( z_i \) is the number of prohibited operating zones is shown in Figure 2.1.

![Figure 2.1 Cost function with prohibited operating zones](image)

Ramp rate limits constraint: The generator constraints due to ramp rate limits of generating units are given as
(i) As generation increases
\[ P_{Gi(t)} - P_{Gi(t-1)} \leq UR_i \] (2.7)

(ii) As generation decreases
\[ P_{Gi(t-1)} - P_{Gi(t)} \leq DR_i \] (2.8)

Therefore the generator power limit constraints can be modified as
\[ \text{Max}(P_{Gi\text{min}}, P_{Gi(t-1)} - DR_i) \leq P_{Gi(t)} \leq \text{Min}(P_{Gi\text{max}}, P_{Gi(t-1)} + UR_i) \] (2.9)

From equation (2.9), the limits of minimum and maximum output powers of generating units are modified as
\[ P_{Gi\text{min}} = \text{Max}(P_{Gi\text{min}}, P_{Gi(t-1)} - DR_i) \] (2.10)
\[ P_{Gi\text{max}} = \text{Min}(P_{Gi\text{max}}, P_{Gi(t-1)} + UR_i) \] (2.11)

Where \( P_{Gi(t)} \) is the output power of generating unit \( i \) in the time interval \( t \), \( P_{Gi(t-1)} \) is the output power of generating unit \( i \) in the previous time interval \( t-1 \), \( UR_i \) is the up ramp limit of generating unit \( i \) and \( DR_i \) is the down ramp limit of generating unit \( i \). The ramp rate limits of the generating units with all possible cases are shown in Figure 2.2.

Figure 2.2 Ramp-rate limits of generating units
2.3 OVERVIEW OF EP AND PSO

Four-decade past EP was projected for progress of finite state machines, in order to solve a forecast task. Since then, several modifications, enhancements and implementations have been intended and investigated. Mutation is often implemented by adding a random number or a vector from a certain distribution to a parent. The degree of variation of Gaussian mutation is controlled by its standard deviation, which is also known as a ‘strategy parameter’ in an evolutionary search (Nidul et al 2003). The EP is a near global stochastic optimization method starting from multiple points, which placed importance on the behavioral linkage between parents and their offspring rather than seeking to emulate explicit genetic operators as pragmatic in nature to find an optimal solution.

Particle Swarm Optimization (PSO) is a population based stochastic optimization technique which can be effectively used to solve the nonlinear and non-continuous optimization problems. It is stimulated by social behaviour of bird flocking or fish schooling. The PSO algorithm searches in parallel using a group of random particles similar to other AI-based optimization techniques. Eberhart and Kennedy recommended a particle swarm optimization based on the analogy of a swarm of bird and school of fish (Zwe-Lee 2003). PSO is basically developed through simulation of bird flocking in two-dimensional space. The position of each agent is represented by XY axis position, and also the velocity is expressed by $V_x$ (velocity of X axis) and $V_y$ (velocity of Y axis). Modification of the agent (particle) position is realized by the position and velocity information. Bird flocking optimizes a certain objective function. Each agent knows its best value so far (pbest) and its XY position. This information is the similarity of individual experiences of each agent. Moreover, each agent knows the best value so far in the group.
(gbest) among pbests. This sequence is the analogy of knowledge of how other agents around them have performed. The particles are drawn stochastically toward the position of the present velocity of each particle, their earlier best performance and the best previous performance of their neighbours (Park et al 2005 and Jeyakumar et al 2006).

Each agent tries to modify its position using the subsequent information:

1. The current position \((x, y)\),
2. The current velocities \((V_x, V_y)\),
3. The distance between the current position and pbest,
4. The distance between the current position and gbest.

This modification is represented by the concept of velocity. The velocity of each agent could be modified by the following equation (2.12)

\[
V_{id}^{(t+1)} = w \times V_{id}^{(t)} + C_1 \times \text{rand}(\cdot) \times \left(pbest_{id} - P_{Gi d}^{(t)}\right) +
C_2 \times \text{Rand}(\cdot) \times \left(gbest_{id} - P_{Gi d}^{(t)}\right) 
\]

\( i = 1,2,\ldots,n; \ d = 1,2,\ldots,m \)

Where ‘\( n \)’ is the population size, ‘\( m \)’ is the number of units and the ‘\( w \)’ be the inertia weight factor. The appropriate selection of the inertia weight factor provides a balance between global and local explorations, thus requires fewer iteration on average to find an adequately optimal solution (Zwe-Lee 2003). In general, the inertia weight factor \( w \) is set according to equation (2.13)
Where, \( w_{\min} \) and \( w_{\max} \) are the minimum and maximum weighting factors respectively.

\[
w_{\max} = 0.9 \quad \text{and} \quad w_{\min} = 0.4
\]

- \( \text{iter} \) - current number of iterations
- \( \text{iter}_{\max} \) - maximum no of iterations (generations)
- \( C_1, C_2 \) - acceleration constant, equal to 2
- \( \text{rand}(\cdot), \text{Rand}(\cdot) \) - Random number value between 0 and 1
- \( V_{id}^{(t)} \) - velocity of agent \( i \) at iteration \( t \)
- \( P_{Gid}^{(t)} \) - current position of agent \( i \) at iteration \( t \)
- \( \text{pbest}_i \) - pbest of agent \( i \)
- \( \text{gbest} \) - gbest of the group

Using the above equation, a certain velocity, which gradually gets closer to pbest and gbest, can be calculated. The current position can be modified by equation (2.14)

\[
P_{Gid}^{(t+1)} = P_{Gid}^{(t)} + V_{id}^{(t+1)}
\]  

(2.14)

The first term of the right-hand side of the equation (2.12) is corresponding to the diversification in the search procedure. The second and third terms of that are corresponding to intensification in the search procedure. The PSO method has a well-balanced mechanism to utilize the diversification and strengthening in the search procedure efficiently. Figure 2.3 shows the concept of modification of a searching point by PSO.
Figure 2.3 Concept of modification of a searching point by PSO

Where

\( P_t \) - Current searching point

\( P_{t+1} \) - Modified searching point

\( V^t \) - Current velocity

\( V^{t+1} \) - Modified velocity

\( V_{pbest} \) - Velocity based on pbest

\( V_{gbest} \) - Velocity based on gbest

2.3.1 Implementation of the PSO Method for Solving ED Problem

The implementation of the PSO method for solving ED problem is given as follows and the general flowchart of the PSO is shown in Figure 2.4.

Step 1 Generate an initial population of particles with random positions and velocities within the solution space

Step 2 Compute the value of the fitness function for each particle

Step 3 To compare the fitness of each particle with each pbest. If the current solution is better than its pbest, then replace its pbest by the current solution.

Step 4 Compare the fitness of all the particles with gbest. If the fitness of any particle is better than gbest, then replace gbest.
Step 5  Adjust the velocity and position of all particles according to equations (2.12) & (2.14).

Step 6  Replicate the steps 2-5 until a criterion is met.
2.4 STEP BY STEP DEVELOPMENT OF THE PROPOSED HPSO ALGORITHM

combine the meticulous features of the EP and PSO, the proposed HPSO algorithm has been urbanized, and the steps are given as follows.

**Step 1.** Arbitrarily generates the initial searching points of real power generation of generators and velocities within the permissible range. The current searching point is set to pbest for each agent. The best evaluated value of pbest is set to be gbest and gbest value is stored.

**Step 2.** Adaptation of searching point of each agent is changed using equations (2.12), (2.13) and (2.14) and the corresponding evaluation values are calculated.

**Step 3.** If the evaluation value of each agent is improved than the previous pbest, then the value is set to be pbest. If the best pbest is better than the previous gbest, then the value is set to be gbest.

**Step 4.** Adaptation of searching points using Gaussian mutation and the evaluation values are considered.

**Step 5.** If the evaluation value of each agent is better than the previous pbest, then the value is set to be pbest. If the best pbest is better than the previous gbest, then the value is set to be gbest.

**Step 6.** If the present iteration number reaches the prearranged maximum iteration number, then exit. or else, go to step 2.
2.5 IMPLEMENTATION OF THE PROPOSED HPSO ALGORITHM TO SOLVE ED PROBLEM

The step by step procedure of the proposed HPSO algorithm for solving the ED problem is given below and the flow chart is shown in Figure 2.5.

Step 1 Initialize the individuals of the population according to limits of each unit, including velocity, search points and individual dimensions. This initial individual must be a viable candidate solution that satisfies the practical operating constraints. Initial velocity limits of each member in individual is

\[ V_{d_{\text{max}}} = 0.5P_{d_{\text{max}}}; \quad V_{d_{\text{min}}} = -0.5P_{d_{\text{min}}} \]  \hspace{1cm} (2.15)

Where, \( P_{d_{\text{max}}} = \sum_{i=1}^{N} P_{d_{Gi}} \) and \( P_{d_{\text{min}}} = \sum_{i=1}^{N} P_{d_{Gi}} \)

Step 2 For each \( P_{Gi} \) of the population use B-coefficients loss formula given in equation (2.4) to work out the transmission loss

Step 3 Compute the evaluation value of each individual \( P_{Gi} \) in the population using the equation (2.16)

\[ f = \frac{1}{F_{\text{cost}} + P_{\text{pbc}}} \]  \hspace{1cm} (2.16)

Where \( F_{\text{cost}} = 1 + \text{abs} \left( \frac{\sum_{i=1}^{N} F_{i}(P_{Gi}) - F_{\text{min}}}{F_{\text{max}} - F_{\text{min}}} \right) \)

\( P_{\text{pbc}} = 1 + \left[ \sum_{i=1}^{N} P_{Gi} - P_{D} - P_{L} \right]^2 \)
\( F_{\text{max}} \) and \( F_{\text{min}} \) are the maximum and minimum generation cost among all individuals in the initial population correspondingly.

In order to limit the evaluation value of each individual of the population within a feasible range previous to estimating the evaluation value of an individual, the generation output power must satisfy the constraints.

**Step 4** Evaluate each individual’s evaluation value with its pbest values. The best evaluation value among the pbest values is assigned as gbest value.

**Step 5** Revolutionize the member velocity \( V \) of the each individual \( P_{\text{Gi}} \) using equation (2.12)

**Step 6** Ensure the velocity components constraint limits from the following conditions.

If \( V_{\text{id}}^{(t+1)} > V_{\text{d}}^{\text{max}} \) \( \Rightarrow \) then \( V_{\text{id}}^{(t+1)} = V_{\text{d}}^{\text{max}} \)  \hspace{1cm} (2.17)

If \( V_{\text{id}}^{(t+1)} < V_{\text{d}}^{\text{min}} \) \( \Rightarrow \) then \( V_{\text{id}}^{(t+1)} = V_{\text{d}}^{\text{min}} \)  \hspace{1cm} (2.18)

**Step 7** Transform the member position of each individual \( P_{\text{Gi}} \) using the equation (2.14). \( P_{\text{Gid}}^{(t+1)} \) must satisfy the constraints of prohibited operating zone and ramp rate limit constraints.

**Step 8** If the evaluation value of each individual is better than the previous pbest value, then the current value is set to be pbest. If the best pbest is better than gbest, then the pbest is assigned as the gbest.
Step 9  

\[ P_{Gi}^{(t+1)}' = P_{Gi} + N(0,\sigma_i^2) \]  

(2.19)

where \( N(0,\sigma_i^2) \) = Gaussian random variable with mean 0 and standard deviation \( \sigma_i \)

\[ P_{Gi}^{(t+1)}' \] must satisfy the constraints of prohibited operating zones, ramp-rate limits and generator capacity limits

\[ \sigma_i = \beta \frac{f_i}{f_{i\min}} (P_{Gi,max} - P_{Gi,min}) \]  

(2.20)

where \( f_{i\min} \) - minimum cost among ‘n’ trial solutions, \( \beta \) - scaling factor is equal to 0.001 and \( f_i \) - value of the objective function associated with vector \( P_{Gi} \).

Step 10  
If the evaluation value of each individual is improved than the pbest value in step 8 then, the current value is set to be the pbest. If the best pbest among all particles is better than the gbest in step8, then, the value is set to be the gbest.

Step 11  
If the number of iterations reaches the maximum go to the step12. or else go to the step 5.

Step 12  
The individual that generates the most recent gbest is the optimal generation power of each unit with the least amount total generation cost.
Figure 2.5 Flowchart of the proposed HPSO algorithm
2.6 RESULTS AND DISCUSSION

To bear out the feasibility of the proposed algorithm, four different test systems are considered such as three, six, fifteen and twenty units with ramp rate limits and prohibited operating zones constraints. The results of the proposed algorithm are compared with EP, PSO and other methods, which are presented in the literature. 100 trail runs was performed and observed the variations throughout the evolutionary process to reach convergence characteristics and optimal solutions. The B-loss coefficient matrix of power system network is working to calculate the transmission line losses. The software was written in Mat Lab language and executed on the third generation Intel Core i3 processor based personal computer with 4 GB RAM. From the comparison of results, the proposed HPSO algorithm is found to be better in solving the non-linear ED problems.

2.6.1 Test System 1

A 3 unit system (Chen & Chang 1995) is considered. The system load demand is 300MW. The dimension of population is $100 \times 3$ and the numbers of generations are 100. 100 trial runs are conducted, and the best solutions are shown in Table 2.1 that satisfies the system constraints. The results of the proposed HPSO algorithm are compared with EP, PSO, GA (Chen & Chang 1995) and 2PNN (Naresh et al 2004) methods. From the comparison of the results, the fuel cost obtained by the proposed HPSO algorithm is better than the other methods. Figure 2.6 shows the comparison of fuel costs for various methods in a three unit system and Figure 2.7 shows the convergence nature of the EP, PSO and HPSO algorithms. From the convergence property, it is evident that the proposed HPSO algorithm has better convergence characteristics than the EP and PSO methods.
Table 2.1 Results of three unit system

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<tr>
<td>P1 (MW)</td>
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<td>165.00</td>
<td>199.53</td>
<td>190.59</td>
<td>200.18</td>
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<td>P2 (MW)</td>
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<td>38.19</td>
<td>34.80</td>
<td>34.40</td>
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<td>$\sum P_i$ (MW)</td>
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<td>313.40</td>
<td>311.16</td>
<td>310.84</td>
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<td>$P_L$ (MW)</td>
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<td>13.40</td>
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<td>10.84</td>
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<td>Fuel Cost ($/hr)</td>
<td>3737.16</td>
<td>3652.60</td>
<td>3641.70</td>
<td>3631.1</td>
<td>3623.11</td>
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Figure 2.6 Comparison of fuel cost for 3 unit system
2.6.2 Test System 2

The system contain 6 thermal units, 26 buses and 46 transmission lines (Zwe-Lee 2003). The load demand is 1263MW. The losses are calculated using B-loss coefficient matrix. The dimension of the population is $100 \times 6$ and number of generations is taken as 100. 100 trial runs were conducted and the best solutions are shown in Table 2.2. The results obtained by the proposed method are compared with EP, PSO, GA (Zwe-Lee 2003) DSPSO-TSA (Khamsawang & Jiriwibhakorn 2010), BBO (Bhattacharya & Chattopadhyay 2010) HHS (Pandi et al 2011) HIGA (Mir Mahmood et al 2012) and PSO-GSA (Hari Mohan Dubey et al 2013) methods. From the comparison of results, it evidently shows the proposed HPSO algorithm gives minimum fuel cost than the other methods. Figure 2.8 shows the comparison of fuel cost for various methods for a six unit test system and Figure 2.9 shows the convergence nature of EP, PSO and proposed HPSO algorithms.
### Table 2.2 Results of six unit system

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<td>P1(MW)</td>
<td>474.80</td>
<td>439.29</td>
<td>447.3997</td>
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<td>P2(MW)</td>
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<td>138.9289</td>
<td>147.98</td>
<td>131.52</td>
<td>108.83</td>
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<td>P5(MW)</td>
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<td>166.967</td>
<td>165.392</td>
<td>165.3541</td>
<td>182.64</td>
<td>170.50</td>
<td>171.07</td>
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<td>P6(MW)</td>
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<td>87.1269</td>
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<td>1275.404</td>
<td>1275.24</td>
<td>1274.15</td>
<td>1271.98</td>
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**Fuel Cost ($/hr)**

|          | 15459 | 15441.5 | 15443.09 | 15448.37 | 15443.1 | 15442.59 | 15451 | 15433 | 15404 |

**Figure 2.8 Comparison of fuel cost for 6 unit system**
2.6.3 Test System 3

The input data of 15 unit test system are in use from reference (Zwe-Lee Gaing 2003). The load demand of the system is 2630 MW. The prohibited operating zones and ramp-rate limits are considered as the generator constraints. The losses are calculated using B-loss coefficient matrix. The dimension of the population is 100×15 and number of generations is taken as 100. The results obtained by the proposed algorithm are compared with EP, PSO, GA (Zwe-Lee Gaing 2003), PSO-MSAF (Subbaraj et al 2010), GA-AFI (Ciornei & Kyria kides 2012) and TVAC-EPSO (Mohd et al 2013) methods and are shown in Table 2.3. From the comparison of results, it is observed that the proposed HPSO algorithm gives minimum fuel cost than the other methods. Figure 2.10 shows the fuel cost comparison of various methods for a fifteen unit test system and Figure 2.11 shows the convergence nature EP, PSO and proposed HPSO algorithms.
Table 2.3 Results of fifteen unit system

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<td>PSO-MSAF (Subbaraj et al 2010)</td>
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Table 2.3 Results of fifteen unit system

**Figure 2.10 Comparison of fuel cost for 15 unit system**
2.6.4 Test System 4

The input data for 20 unit test system is taken from reference (Ching-Tzong Su & Chen-Tung Lin 2000). The system load demand is 2500 MW. In this test system, the transmission losses, POZ and ramp rate limit constraints are considered. The dimension of the population is 100×20 and the numbers of generations are 100. The results obtained by the proposed algorithm are compared with EP, PSO, Lambda-iteration method, Hopfield neural network (Ching-Tzong Su & Chen-Tung Lin 2000), BBO (Bhattacharya & Chattopadhyay 2010) and EBBO (Vanitha & Thanuskodi 2012) methods and are shown in Table 2.4. On comparison of the results, it is evident that the proposed algorithm can provide significant cost saving than other methods. Figure 2.12 shows the fuel cost comparison of various methods for a 20 unit test system and Figure 2.13 shows the convergence nature EP, PSO and proposed HPSO algorithms. It’s evident from the Figures 2.7, 2.9, 2.11 and 2.13, the proposed HPSO algorithm is free from the shortcoming of premature convergence exhibited by the EP and PSO methods.
Table 2.4 Results of twenty unit system

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Figure 2.12 Comparison of fuel cost for 20 unit system
2.7 CONCLUSION

In this chapter, the EP, PSO, and proposed HPSO algorithms are applied fruitfully to solve the non-linear economic dispatch problems. The proposed HPSO algorithm has been proved to have the enhanced features in terms of achieve better optimal solutions for reducing the fuel cost of the generating units and improving the convergence characteristics. Non-linear characteristics of the generators such as prohibited operating zones and ramp-rate limit constraints are considered for the selected test systems. The results obtained by the proposed HPSO algorithm are compared with EP, PSO and other methods reported in recent literatures. The comparative study is done based on the optimum fuel cost and has been established that the proposed HPSO algorithm can be an alternative approach for finding a better solution for the non-linear economic dispatch problems.