Chapter 3

Electromagnetic interaction of radiation with nuclei

Study of electromagnetic properties have provided a wealth of information about the structure of nuclear physics. One reason for studying the electromagnetic properties is that unlike nuclear residual interaction which is introduced phenomenologically the electromagnetic interaction of radiation with nuclei is completely understood. A $\gamma$ radiation emitted during a transition between nuclear states carries information in its angular momentum, energy and associated transition probabilities. These allow direct conclusion about the angular momentum, parity, excitation energy and the transition matrix elements of the stationary states of the nucleus. In some cases the information may be summarized in terms of electric multipole moments such as electric quadrupole moment, which can be more directly related to the shape of the nuclear charge distribution.

3.1 Coupling of radiation and matter

A starting point in the description of electromagnetic interaction of radiation with a quantum system like nucleus is the Hamiltonian of the system. A general form of the Hamiltonian describing such a system can be written as

$$H = H_{\text{nuclear}} + H_{\text{em}} + H_{\text{int}}.$$  \hfill (3.1a)

The nuclear part of the Hamiltonian $H_{\text{nuclear}}$ is provided by the nuclear model specifically adopted for the problem in question. The Hamiltonian of the electromagnetic field $H_{\text{em}}$
is given by the expression

\[ H_{em} = \sum_{\kappa \mu} \hbar \omega_{k} (\beta_{\kappa \mu}^\dagger \beta_{\kappa \mu} + \frac{1}{2}) , \]  

(3.1b)

where \( H_{em} \) is written in second quantization form and \( \beta_{\kappa \mu}^\dagger \) (\( \beta_{\kappa \mu} \)) are the creation (destruction) operators creating (destroying) a photon in state \( |\kappa \mu\rangle \). The interaction part \( H_{int} \) describes the coupling of electromagnetic field with nucleus and is given by the expression

\[ H_{int} = -\frac{1}{c} \int A(r,t) j(r) d^3r , \]  

(3.1c)

where \( A(r,t) \) stands for the operator corresponding to vector potential describing the radiation field and \( j(r) \) stands for current operator for the nucleons. Thus the problem is to find the solution of the equation

\[ H(t)\psi(t) = E\psi(t) . \]  

(3.2)

Since the coupling \( H_{int} \) is weak, it is treated as perturbation and the perturbation theory is applied to find the solution of above equation. The Hamiltonian for the above equation then can be written as

\[ H = H_{o} + H_{int} \]  

(3.3)

with the first order states as tensor product of nuclear state \( |\psi\rangle \) and electromagnetic field state \( |\kappa \mu\rangle \), i.e, \( |\psi; \kappa \mu\rangle \). The electromagnetic interaction then induces a transition between these first order states.

### 3.2 Properties of electromagnetic transitions and selection rules

There are certain selection rules for a \( \gamma \) transition to occur. These selection rules follow from the conservation rules for energy, angular momentum and parity of the system.

a) Conservation of energy and angular momentum: This law is a restatement of the fact that energy and momentum of a closed system cannot be created or destroyed. If \( |\psi_i\rangle \) is an initial nuclear state undergoing transition to a final state \( |\psi_f\rangle \), then from the conservation rules we have

\[ E_i - E_f = \hbar \omega \]  

(3.4a)
and
\[ \vec{T}_i = \vec{T}_f + \vec{\lambda}, \]  
(3.4b)
where \( E_i \) and \( E_f \) are the excitation energies and \( \vec{I}_i \) and \( \vec{I}_f \) are the corresponding angular momentum of initial and final states respectively. \( \hbar \omega \) and \( \vec{\lambda} \) are the energy and the angular momentum of the radiation emitted during the process. If the excitation energy and angular momentum of the final state are known, then from the experimentally observed properties of radiation, the information about the excitation energy and angular momentum of the initial state can be determined using the above equations. In Section 3.4, methods to determine the multipolarity of \( \gamma \) radiations are described. From Eq. (3.4b) it follows that
\[ |I_i - I_f| \leq \lambda \leq I_i + I_f, \]  
(3.5a)
\[ m_i - m_f = m. \]  
(3.5b)
Above equation imposes a restriction on the values of angular momentum quantum number and is one of the selection rule which have to be satisfied in these processes. It follows from above equation that \( 0^+ \rightarrow 0^+ \) radiative transitions are not possible, since a transition can not have \( \lambda=0 \).

b) Conservation of parity : the rule states that the parity of a system before \( (\pi_i) \) and after \( (\pi_f) \) the transition is conserved. Expressing it another way, the law states the parity of the integrand in Eq. 3.1c is positive. Since the parity of current operator is -1, it follows that for electric and magnetic transitions following relations hold
\[ \pi_i \pi_f = (-1)^\lambda \text{ Electric transition} \]  
(3.6a)
and
\[ \pi_i \pi_f = -(-1)^\lambda \text{ Magnetic transition} \]  
(3.6b)
respectively. The information about the parity of the transition can be obtained from polarization experiments and is described in Section 3.5.
3.3 Reduced transition probability

As mentioned in Section 3.1, the electromagnetic interaction induces the transition between the stationary nuclear states. The rate at which the transition is induced by electromagnetic field is given by the Fermi Golden rule

\[ T_{if} = \frac{2\pi}{\hbar} |\langle f | H_{int} | i \rangle|^2 \rho_f = \frac{2\pi}{\hbar} |M_{if}| \rho_f , \]

(3.7)

where the \( |i\rangle \) is the initial state with no photon, i.e. \( |i\rangle \equiv |\psi_{i,m_i}; 0\rangle \) and \( |f\rangle \) is the final state with a photon of momentum \( \kappa \), i.e. \( |f\rangle \equiv |\psi_{f,m_f}; \kappa \mu\rangle \). Treating the outgoing photon as plane wave, the multipole expansion of the matrix element \( M_{if} \) in terms of electric and magnetic multipole fields can be made. Often in experiments, one is interested in average transition rates therefore an average over 4\(\pi\) direction of the outgoing photon is carried out. With these considerations the transition rate for a particular mode is then given by

\[ T_{if} = \frac{8\pi(\lambda + 1)}{\lambda ((2\lambda + 1)!!)^2} \frac{k^{2\lambda + 1}}{\hbar} |\langle \psi_{f,m_f} | \hat{\Omega}_{\lambda \mu}(R) | \psi_{i,m_i} \rangle|^2 , \]

(3.8)

where \( R \) stands for either \( E \) (electric transition) or \( M \) (magnetic transition). The matrix elements for electric and magnetic case respectively are

\[ \langle \psi_{f,m_f} | \hat{\Omega}_{\lambda \mu}(E) | \psi_{i,m_i} \rangle = \int \langle \psi_{f,m_f} | \hat{j}(r) | \psi_{i,m_i} \rangle r^{\lambda} Y_{\lambda \mu}(\Omega) d^{3}r \]

(3.9a)

and

\[ \langle \psi_{f,m_f} | \hat{\Omega}_{\lambda \mu}(M) | \psi_{i,m_i} \rangle = \frac{-1}{c(\lambda + 1)} \int \langle \psi_{f,m_f} | \hat{j}(r) | \psi_{i,m_i} \rangle L(r^{\lambda} Y_{\lambda \mu}(\Omega)) d^{3}r . \]

(3.9b)

In above equation, \( j(r) \) contains contribution from orbital motion of protons and spin of both neutrons and protons. For experiments with unpolarized target and projectile, the angular momentum projections of the initial states are averaged and of the final states are summed over in the expression of transition rates

\[ T_{if} = \frac{8\pi(\lambda + 1)}{\lambda ((2\lambda + 1)!!)^2} \frac{k^{2\lambda + 1}}{\hbar} B(R \lambda, I_i \rightarrow I_f) , \]

(3.10)

where \( B(R \lambda, I_i \rightarrow I_f) \) is the reduced transition probability of the transition containing information about the structure of nucleus and is independent of the orientation of state.
Table 3.1: Relation between the transition probability $T(E\lambda)$ or $T(M\lambda)$ and the reduced transition strengths $B(E\lambda)$ or $B(M\lambda)$. $T(E\lambda)$'s and $T(M\lambda)$'s are in $sec^{-1}$, $B(E\lambda)$'s in $e^2 fm^{2\lambda}$, $B(M\lambda)$'s in $\mu^2 fm^{2\lambda-2}$ and $E$'s in $MeV$.

<table>
<thead>
<tr>
<th>Transition Probability</th>
<th>Reduced Transition Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T(E1)$</td>
<td>$1.59 \times 10^{15} E^3 B(E1)$</td>
</tr>
<tr>
<td>$T(E2)$</td>
<td>$1.22 \times 10^9 E^5 B(E2)$</td>
</tr>
<tr>
<td>$T(E3)$</td>
<td>$5.67 \times 10^2 E^7 B(E3)$</td>
</tr>
<tr>
<td>$T(E4)$</td>
<td>$1.69 \times 10^{-4} E^9 B(E4)$</td>
</tr>
<tr>
<td>$T(M1)$</td>
<td>$1.76 \times 10^{13} E^3 B(M1)$</td>
</tr>
<tr>
<td>$T(M2)$</td>
<td>$1.35 \times 10^7 E^5 B(M2)$</td>
</tr>
<tr>
<td>$T(M3)$</td>
<td>$6.28 \times 10^0 E^7 B(M3)$</td>
</tr>
<tr>
<td>$T(M4)$</td>
<td>$1.87 \times 10^{-6} E^9 B(M4)$</td>
</tr>
</tbody>
</table>

It is given by

$$B(R\lambda, I_i \rightarrow I_f) = \frac{1}{2I_i + 1} |\langle \psi_f | \hat{\Omega}_{\omega \mu}(R) | \psi_i \rangle|^2 .$$

(3.11)

The relation between the transition probability $T(R\lambda)$ and the reduced transition strengths $B(R\lambda)$ for first four multipole radiative transitions are listed in Table 3.1.

Since the matrix elements in Eqs. (3.11) depends on the dimension of a nucleus as $B(R\lambda) \sim r^{\lambda}$, the transition probability is proportional to $(kr)^{2\lambda}$. As the dimension of a radiating nucleus is smaller then the wavelength of radiation emitted, the factor $kr \ll 1$ and $(kr)^{2\lambda}$ is a rapidly decreasing function of $\lambda$. Hence the higher multipole transitions have lesser probability of emission than the transitions of lower multipole. In fact the probability is so much hindered that finding a transition of multipolarity greater than $\lambda=3$ is a rare event. From above equation the relative probability of electric and magnetic transition of same multipolarity can also be deduced. The ratio of reduced matrix element of magnetic transition and reduced matrix element of electric transition $\frac{B(M\lambda)}{B(E\lambda)}$ is proportional to $\frac{\mu_n}{e}$. Since the velocity of nucleons inside the nucleus is quite less than the velocity of light this implies that electric transitions are faster than magnetic transition of same multipolarity.

Since the transition probability depends on the matrix element of the interaction between the initial and final states, it provides information about the configuration of nucleons participating in the transition. The transition probabilities $T_{if}$ are obtained
Figure 3.1: A schematic diagram illustrating the decay of states through a cascade of transitions of mixed multipole.

from lifetime measurements. They are related by the expression

\[ \frac{1}{\tau_{if}} = T_{if}, \] (3.12)

where \( \tau_{if} \) is the transition lifetime. Depending on the range of lifetime of a nuclear state, there are several techniques for employed. For lifetime in picosecond and subpicosecond range the techniques used are Recoil Distance Doppler Shift method (RDDS) and Doppler Shift Attenuation Method (DSAM) respectively. The DSAM is described in Section 5.10, Chapter 5.

3.4 Angular Distribution and Correlation from Oriented nuclei

Angular distribution and correlation measurements have been one of the most important and useful method of assigning spins of the oriented nuclear states\(^1\). These measurements correspond to the determination of probability distribution in space of transitions emitted

\(^1\)A nuclear state is said to be oriented if the relative populations \( P(m) \) of the angular momentum substates are unequal, i.e. \( P(m) \neq P(m') \).

38
from the oriented nuclear states. In particular, they give information about the relative
correlation of various multipole components involved. A general expression for angular
distribution of mixed transition from an oriented state, \( I_i \), without the observation of
polarization is given by [2]

\[
W(\theta) = \sum_k A_k(\lambda, I_i I_f) P_k(\cos \theta),
\]

(3.13a)

where \( A_k(\lambda, I_i I_f) = \rho_k(I_i) B_k(\lambda, I_i I_f) \) and \( \theta \) is the angle with respect to orientation
axis. The coefficients \( B_k(\lambda, I_i I_f) \) are related to \( F_k(\lambda, I_i I_f) \) coefficients [2] and mixing
ratio \( \delta \) by the relation

\[
B_k(\lambda, I_i I_f) = \frac{1}{1 + \delta^2} \left[ F_k(\lambda, I_i I_f) + \delta^2 F_k(\lambda, I_i I_f) + 2\delta F_k(\lambda, I_i I_f) \right].
\]

(3.13b)

The statistical tensor \( \rho_k(I_i) \) describes the orientation of the initial state and are defined
by [3]

\[
\rho_k(I_i) = \sqrt{2I_i + 1} \sum_m (-1)^{I_i - m} \langle I_i m | I_i - m | 0 \rangle P(m),
\]

(3.13c)

where in above equation \( P(m) \)'s are the relative population of the substates and are
normalized such that \( \sum_m P(m) = 1 \). In above equations, the mixing ratio \( \delta \) is defined
as the ratio of transition amplitudes i.e., \( \delta = \langle I_f | \hat{\Omega}_\lambda(\mathbf{R}) | I_i \rangle / \langle I_i | \hat{\Omega}_\lambda(\mathbf{R}) | I_f \rangle \). The \( F_k \)
coefficients are related to the Racah coefficients \( W(\lambda, I_i I_f; k, I_f) \) as

\[
F_k(\lambda, I_i I_f) = (-1)^{(I_i - I_f + 1)} (2\lambda + 1)(2\lambda' + 1)(2I_i + 1)^{1/2} \times \langle \lambda, I_i I_f \rangle \langle 0 | I_f, k, I_f \rangle.
\]

(3.13d)

In case, where the initial state \( I_i \) of the \( \gamma \) transition is populated via preceding
transitions, as shown in Fig. 3.1, its spin orientation is attenuated. This attenuation is
due to the change in the spin orientation due to the preceding transition. For the case
shown in Fig. 3.1, the change in orientation due to the \((i - 1)\)th preceding transition is
given by the attenuation factor \( U_k(\lambda_{i-1} I_{i-1}, I_{i-1} I_i) \) which is expressed in terms of Racah
coefficients as [4]

\[
U_k(\lambda_{i-1} I_{i-1}, I_{i-1} I_i) = \frac{1}{1 + \delta^2} u(\lambda_{i-1} I_{i-1} I_i) + \frac{\delta^2}{1 + \delta^2} u(\lambda_{i-1} I_{i-1} I_i)
\]

(3.14)
Figure 3.2: The angles in a directional correlation of two successive radiations emitted from an axially oriented state (DCO).

with \( u(\lambda_{i-1}I_iI_f) \) given by

\[
u(\lambda_{i-1}I_iI_f) = (2\lambda_{i-1} + 1)(2\lambda'_{i-1} + 1)^{1/2}(-1)^{(I_f-I_i-1)}W(I_iI_iI_fI_f; k\lambda_{i-1}) \quad (3.15)\]

The orientation of the \( i^{th} \) level can now be written as

\[
\rho_k(I_i) = \rho_k(I_o)U_k(\lambda_o\lambda'_o, I_zI_z)U_k(\lambda_1\lambda'_1, I_1I_2)...U_k(\lambda_{i-1}\lambda'_{i-1}, I_{i-1}I_i) \quad (3.16)
\]

If a \( \gamma \) transition is pure, then angular distribution measurement is sufficient to deduce the spin of the transition. But in the case of \( \gamma \) transition having a mixed multipolarity, the angular distribution measurements cannot resolve the ambiguity in the determination of mixing ratio. However, if the angular correlation measurements of two \( \gamma \) radiations emitted from an oriented initial state is carried out, then in many cases an unambiguous determination of mixing ratio is possible. If only the directional correlation of the two \( \gamma \) radiations is measured, the method is called as Directional Correlation from Oriented nuclear state (DCO). A general expression of DCO is given by [5]

\[
W(\theta_1, \theta_2, \Phi) = \sum_{k_1k_2} \rho_{k_1}(I_1)A_{k_1k_1}(\gamma_1)A_{k_2}(\gamma_2) \frac{4\pi}{2k_2 + 1} \sum_q \langle k_10q|k_2q \rangle Y_{k_2}(\theta_1, 0)Y_{k_2}^*(\theta_2, \Phi) \quad (3.17)
\]
where $\theta_1$ and $\theta_2$ are the angles between the directions of $\gamma_1$ and $\gamma_2$ radiations with the orientation axis, $\Phi$ is the angle between the planes formed by the direction of radiations with the orientation axis, $A_k^{h_kl}(\gamma_1)$ and $A_k(\gamma_2)$ are the radiation distribution coefficients [5] and $\rho_k(I_1)$ is the orientation parameter of initial state $I_1$ as defined in Eqs. 3.13c and 3.16. Often, the data is not available for a complete range of $\theta_1$, $\theta_2$ and $\Phi$ and hence the determination of complete distribution is not carried out. Instead, the measurement is carried out for specific values of $\theta_1$, $\theta_2$ and $\Phi$ and the ratio

$$R_{DCO} = \frac{W(\theta_1 = \alpha, \theta_2 = \beta, \Phi)}{W(\theta_1 = \beta, \theta_2 = \alpha, \Phi)}$$

(3.18)

is determined. The ratio, called as DCO ratio, is highly sensitive to mixing ratio ($\delta$) and the sensitivity is largest if one of the angle is chosen close to $\pi$ and other close to zero or $\pi$. For pure stretched transitions of same multipolarity the ratio is close to 1.0 and for different multipolarity (usually $l=1$ and $2$) it is close to either 0.5 or 2.0 depending on how the ratio is defined. If one of the transition is having a mixed multipolarity then the ratio move towards 1.0 or even beyond for positive values of $\delta$ while for negative values it moves towards 0.0. Hence the $R_{DCO}$ cannot distinguish between a mixed transition and a pure quadrupole. Further ambiguity arises from the non stretched ($I_i - I_f = 0$) pure $E1$(or $M1$) transition which have same value of $R_{DCO}$ as for stretched $E2$ transition. It is to be noted that both angular distribution and angular correlation are sensitive only to the multipolarity of the transition and not on its parity. For determining the later, polarization measurement would be required (see Section 3.5).

3.4.1 Angular distribution and correlation in heavy ion (HI) fusion evaporation reaction.

For an unoriented state, define by equal population of angular momentum substates, i.e. $P(m) = 1/(I + 1)$, the statistical tensors have the value $\rho_k(I) = 0$ for $k \neq 0$ and $\rho_0(I) = 1$ (see Eqs. (3.13c)). From Eqs. (3.13a) it is deduced that for such a state the angular distribution is isotropic in space ($W(\theta) = P_0(\cos \theta) = 1$) and thus cannot provide any information about the radiation emitted. Therefore, the orientation of a decaying state is a necessary requirement to observe an angular distribution of the transition emitted.
Alignment of a state is a type of orientation defined by $P(m) = P(-m)$ and can be produced by various means. For example, cryogenic methods, $\gamma-\gamma$ correlations and heavy ion fusion-evaporation reactions. In the cryogenic methods [6], orientation is achieved through the interaction of external fields with the static moments of the nuclei at low temperature. The interaction lifts up the degeneracy of the nuclear magnetic substates which are then populated at low temperature according to the Boltzmann distribution. In $\gamma-\gamma$ correlations method from unoriented nuclei, the direction of first $\gamma$ radiation is chosen as the quantization axis. Thus the angular correlation distribution become equivalent to the angular distribution of second $\gamma$ radiation and by the detection of first $\gamma$ radiation population of substates of intermediate state is altered from their unoriented value.

In heavy-ion reaction, since the angular momentum of the system is perpendicular\(^2\) to the beam direction\(^3\) an oblate alignment\(^4\) of a state, as shown in Fig. 3.3, is produced.

\(^2\)The angular momentum of the system for a HI reaction is mainly due to the relative orbital angular momentum.
\(^3\)In HI reaction, beam axis is chosen as the orientation axis.
\(^4\)An oblate alignment is defined by $P(m_i = 0) > P(m_i)$
The degree of alignment is higher for heavier projectile and higher beam energy. The advantage of HI fusion reaction over other methods is that they populate the states of radioactive nuclei which are not accessible via other means. For an oblate alignment, \( p_2(I) < 0 \) and \( p_4(I) \geq 0 \). For a pure dipole radiation, since \( F_2(\lambda\lambda', I_iI_f) \) is positive and \( F_4(\lambda\lambda', I_iI_f) = 0 \), the relation for angular distribution Eqs. (3.13a) reduces to

\[
W(\theta) = 1 - C_d P_2(\cos\theta)
\]

while for a pure quadrupole, since both \( F_2(\lambda\lambda', I_iI_f) \) and \( F_4(\lambda\lambda', I_iI_f) \) are negative, the relation for angular distribution is

\[
W(\theta) = 1 + C_q P_2(\cos\theta) - D_q P_4(\cos\theta).
\]

The \( C_d, C_q \) and \( D_q \) are positive constants. These constants depend on the mixing ratio of the transition and the angular momentum of decaying and feeding states and the transition. Above equation implies that the emission probability for a dipole transition is in a direction normal to the orientation axis is higher while for a quadrupole transition it is in the direction of orientation axis. An illustrative figure has been shown in Fig. 3.4 for dipole and quadrupole transition for different mixing ratios in the case of complete alignment.

For a cascade of stretched transitions, i.e. \( \lambda_{i-1} = I_{i-1} - I_i \), the attenuation factors \( U_k(I_i\lambda I_{i-1}) \) are close to one. This implies that, in this case, there is no effect on the orientation of nuclear state \( I_i \) due to preceding transitions, i.e. \( \rho_k(I_i) \approx \rho_k(I_o) \). Hence the angular distribution of all the transitions in a cascade is same.

### 3.5 Linear Polarization from oriented Nuclei

As mentioned in previous sections, the directional correlation and the distribution measurements both cannot distinguish whether the transition is of electric or magnetic character because they have the same distribution pattern for both the type of transitions of same multipolarity. To determine the parity of the radiation it is necessary to have a linear polarization measurement. The general relation of angular distribution of linearly
Figure 3.4: An illustration of the angular distribution $W(\theta)$ of (a) dipole and (b) quadrupole radiation for different values of mixing ratio. The distributions have been obtained assuming complete alignment of decaying state. In case (a) the mixed transitions have $\lambda's = 1$ and $2$ while for case (b) $\lambda's = 2$ and $3$. 
polarized gamma rays emitted in a cascade (see Fig. 3.1) from an axially oriented nuclear state \( \psi_o \) is given by \[7\]

\[
W(\theta, \psi) = \frac{d\Omega}{8\pi} \sum_{\lambda=\text{even}} B_\lambda U_\lambda [A_\lambda P_\lambda(\cos \theta) + 2A_{\lambda 2} P_{\lambda 2}^{(2)}(\cos \theta) \cos (2\psi)] ,
\]  

(3.21)

where \( \theta \) is the angle between the orientation axis and the direction of emitted \( \gamma \)-ray, \( \psi \) is the angle between electric vector \( E \) and the reaction plane\(^5\), \( B_\lambda \) are orientation tensors describing the degree of orientation of the parent nuclear state, \( U_\lambda \) are deorientation-orientation coefficients which include the effect of the preceding radiation between the oriented state and the initial state of the observed radiation and \( A_\gamma \) are angular distribution coefficients which depend on the characteristics of a \( \gamma \)-ray, namely the initial state spin \( I_i \), final state spin \( I_f \) and the radiation multipolarity.

Since for an electric transition the probability of electric vector oscillating in the reaction plane is higher while the opposite is true for a magnetic transition, the above expression of angular distribution therefore can be used to determine the character of the transition. A ratio called as the degree of polarization of a transition can be defined as \[7\]

\[
P(\theta) = \frac{W(\theta, \psi = 0^\circ) - W(\theta, \psi = 90^\circ)}{W(\theta, \psi = 0^\circ) + W(\theta, \psi = 90^\circ)} ,
\]  

(3.22)

where the normalization is such that \(-1 \leq P(\theta) \leq 1\). In above equation \( W(\theta, \psi = 0^\circ) \) \( (W(\theta, \psi = 90^\circ)) \) is the probability of photon emitted in direction \( \theta \) with electric vector oscillating parallel (perpendicular) to the reaction plane. The above definition implies a positive value of \( P(\theta) \) for radiation of electric character and negative value for magnetic character. For a mixed multipole radiation, where the mixing is of opposite character \( (\text{e.g. } E2 + M1) \), it may be close to zero depending upon the amount of mixing. Since the magnitude of \( P(\theta) \) is maximum for \( \theta \) close to 90°, this fact is often used for setting of the experiments.

Among several processes sensitive to the polarization of \( \gamma \)-radiation, Compton scattering is the most often used process \[8\]. The reason being a high differential Compton scattering cross section and polarization sensitivity over a wide range of photon energy

\(^5\)The reaction plane is defined as the plane determined by the orientation axis and the direction of emitted \( \gamma \)-ray.
range. The Compton cross section is given by Klein-Nishina formula [9] with the assumption that the scattered electron is not polarized and the final photon's polarization is not detected

\[ \frac{d\sigma}{d\Omega}(\nu, \chi) = \frac{r_o^2}{2} \left( \frac{E}{E_0} \right)^2 \left[ \frac{E_0}{E} + \frac{E}{E_0} - 2 \sin^2 \nu \cos 2\chi \right] , \]  

(3.23)

where \( r_o \) is the classical radius of the electron, \( E_0 \) and \( E \) are the energies of the incident and the scattered photons respectively, \( \nu \) is the scattering angle of the photon and \( \chi \) is the angle between the electric vector of the incident photon and the scattering plane defined by the direction of incident and scattered photon. From above equation it follows that the cross section of Compton scattering is maximum in a plane normal to the electric vector of the incident photon \((\chi = 90^\circ)\). As mentioned in last paragraph, for in-beam experiments, the probability of electric vector oscillating in reaction plane is higher for the electric transitions which implies they have more probability of getting scattered perpendicular to the reaction plane. The opposite is true for the magnetic transition. Therefore on the basis of character, a \( \gamma \) radiation can be discriminated in the Compton scattering process.

A setup employing the process of Compton scattering to determine the parity of radiation is called Compton polarimeter. The Compton polarimeter consists of two de-
tectors, one of them is a 'scatterer' placed at angle $\theta$ with respect to orientation axis and other is an 'analyzer' placed at angle $\nu$ with respect to the direction of incident photon in the scattering plane. By changing the angle between the scattering and the reaction plane, it is possible to measure the probability of emission of radiation in perpendicular and parallel plane with respect to the electric vector $E$ of radiation. This probability can be measured by calculating the experimental asymmetry of the radiation defined by [10]

$$\Delta = \frac{aN_{\perp} - N_{\parallel}}{aN_{\perp} + N_{\parallel}} \quad (3.24)$$

and using the relation $P = \Delta/Q$ between the degree of polarization $P$ of the radiation, defined in Eq. (3.22), and the polarization sensitivity $Q$ of the polarimeter. In Eq. (3.24), $N_{\perp}(N_{\parallel})$ is the number of counts of $\gamma$ transitions scattered perpendicular (parallel) to the reaction plane. The variable 'a', called as correction factor, is a measure of any instrumental asymmetry in the response of the perpendicular and the parallel detectors and is obtained from a radioactive source data ($^{152}$Eu). It is defined as

$$a = \frac{N_{\parallel}(\text{unpolarized})}{N_{\perp}(\text{unpolarized})} \quad (3.25)$$

The polarization sensitivity $Q$ of the Compton polarimeter of finite dimensions as a function of photon energy can be approximated as $Q(E_{\gamma}) = Q_o(E_{\gamma})(CE_{\gamma} + B)$ [11]. $Q_o(E_{\gamma}, \nu)$ is the polarization sensitivity of a point detector and for $\nu = \pi/2$ is given by

$$Q_o(E_{\gamma}) = \frac{\alpha + 1}{\alpha^2 + \alpha + 1} \quad (3.26)$$

Here $\alpha = E_{\gamma}/mc^2$ with $mc^2$ being the rest mass energy of the electron. The constants $B$ and $C$ are obtained from fit to the experimental data. A typical plot for a point detector and a clover detector is shown in Fig. 3.6. It can be seen from the figure that the polarization sensitivity of the clover detector is always less than that of the point detector which is due to its finite size.

The arrangement of four crystal segments inside the clover detector makes it possible to be used as a Compton polarimeter. In the clover detector any one of the segments can act as a scatterer and the other two neighboring segments as an analyzer. This makes eight pair of segments to be used as an individual polarimeter as shown in Fig. 3.7. Often
Figure 3.6: Polarization sensitivity of the clover detector measured as a function of gamma ray energy. The dotted line is the sensitivity for the point scatter and analyzer. The solid line the the fitting of the experimental data. The data is taken from Ref. [12].

Figure 3.7: A schematic diagram of a Clover as Compton polarimeter. Here the double sided arrow has been used to indicate the Compton scattering in both the direction.
the data from all the eight pairs are combined to reduce any systematic errors arising from different efficiencies of the segments. The angle between the scattering and the reaction planes are fixed to $\chi = 0^\circ$ and $\chi = 90^\circ$. 
Bibliography


