Chapter 6

Multifractal approach for seafloor characterization: Part 1. Application to multi-beam image data

6.1 Introduction

The seafloor bathymetry and the associated backscattering data of submerged objects have an extremely wide range of spatio-temporal scales necessitating application of “power law” to carry out the analyses. The power law behavior in such instances requires multifractal analyses (Mandelbrot\(^1\), 1967, 1989) in order to determine if both (seafloor backscatter and bathymetry image data) follow “fractal” statistics. It is imperative to treat such data as a scale invariant field requiring multifractal measures and exponent functions, rather than a unique scaling exponent (such as fractal dimension) (Hentschel and Procaccia, 1983). An infinite number of fractal dimensions would be needed to completely characterize the scaling, as evident in other fields like: satellite radiance data (Lovejoy et al., 2009b), medicine (Ivanov et al., 1999), and ecology (Seuront, 2010).

\(^1\) Benoit B. Mandelbrot is widely regarded as the “father of fractal geometry”. Mandelbrot was a mathematical genius who advanced the concept of power law scaling as the fundamental property of a broad range of natural processes and patterns in geophysics, mathematics, economics, and virtually all the branches of science. Mandelbrot died on 14 October 2010 in Cambridge, Mass., at the age of 85.
The two important formalisms of multifractal analyses such as: (i) “strange chaotic attractors” (Halsey et al., 1986) and (ii) “stochastic” multifractal fields (Schertzer and Lovejoy, 1987) have been used in this study. The first formalism is based on the “box counting” method. It involves analyses of multifractal distribution pattern using the correlation dimension \( D(q) \), and multifractal spectrum \( f(\alpha) \) related shape parameters [i.e., width of the spectrum \( W \), degree of asymmetry \( B \), and stability of spectrum \( \Delta f(\alpha) \)]. The properties of these functions at different statistical moments are used to characterize the spatial distribution of seafloor backscatter and bathymetry seepage\(^2\) blocks used in this study. The other formalism allows quantification of the seepage blocks with three fundamental parameters namely, degree of multifractality \( \alpha \), sparseness \( C_1 \), and degree of smootheness \( H \) (Gagnon et al., 2006).

Dandapath et al. (2010) had reported seafloor seepages in the WCMI using MBES backscatter and bathymetry data (Fig. 6.1). The investigations were related to underlying geology, pockmark occurrences, overlying sediment texture, and sediment movements due to strong influences of monsoonal bottom currents. Thereafter, utilizing the method proposed by Seuront and Spilmont (2002), Dandapath et al. (2012) had noted the possible multifractal behavior of the MBES backscatter and bathymetry image data. Therefore, the present study involves quantitative estimation of the multifractal parameters from seafloor seepage to improve the understanding of the processes in the WCMI.

The application of inversion modeling for seafloor roughness characterization impose a challenging task as most of the models presume the input data in stationary form. Therefore, the application of segmentation techniques (Malinverno, 1989) is indispensable and facilitates in achieving stationary profile data sets suitable for inversion modeling. The application of online segmentation techniques can also

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\(^2\) Seafloor seepages offer important proxy for shallow or deep water hydrocarbon accumulations. Pockmark associated seepages (Hovland and Judd, 1988) are prevalent from sub-arctic to tropical seas, continental shelf to deep ocean basins, even in shallow and deep lakes under different geological environments. Pockmarks are craters in the seabed caused by fluids (gas and liquids) erupting and streaming through the sediments. The presence of fluid escape features like seafloor pockmarks was first discovered by King and MacLean (1970) over the Nova Scotian shelf.
significantly reduce the frequency of time consuming ground-truth measurements required for the validation of model parameters. The use of soft-computing technique (Alexandrou and Pantartzis, 1993; Michalopoulou et al., 1995) including artificial neural networks (ANNs) were effectively demonstrated for hydroacoustic data classification (Chakraborty et al., 2015) to segment the data into stationary form.

The application of the multifractal techniques could substantiate the hitherto applied numerical inversion based characterization (De and Chakraborty, 2011; Haris et al., 2011), and the soft computational technique based classification (Chakraborty et al., 2001, 2004; De and Chakraborty, 2009) of the seafloor sediments employing the backscatter data.
Fig. 6.1 (a) Study blocks including some of the main structural features of the region of the WCMI. (b) Backscatter map of the study area showing 160–320 m isobaths with 20 m interval. Pockmarks are indicated by crossed circles. Black, blue and red color mark represents circular, elliptical and elongated pockmarks respectively. The dashed lines indicate location of the identified faults. The black arrows show bottom current directions. Solid colored squares represent the sediment types (modified after Dandapath et al. 2010).
6.2 Data sets

The backscatter and bathymetry data used in the present study were acquired from the central part of WCMI, where the water depth varies between 145–330m (Fig. 6.1). Simrad EM 1002 MBES operating at 95 kHz was used to acquire the data. The important morphological aspects of the study area have been investigated in detail, and 112 pockmarks related to the seepages were identified from the seafloor maps generated using ArcGIS (Dandapath et al., 2010). The backscatter and bathymetry image blocks having 400 x 400 pixels, were classified according to the degree of seepage based on the backscatter strength as well as fractal dimension (determined using box-dimension technique) (Dandapath et al., 2012). The analyses carried over each gray tone image blocks (with digital numbers ranging 0 to 255) suggest that the area with very high seepage has higher fractal dimension and lower dimension with very low seepage. In this study, six representative seepage blocks i.e., F20, J19, F07, Q19, S23 and N25 having very high, high, moderate, low, very low, and no evidence of seafloor seepages respectively (Fig. 6.2) have been considered for analyses.

<table>
<thead>
<tr>
<th>Sample blocks</th>
<th>F20</th>
<th>J19</th>
<th>F07</th>
<th>Q19</th>
<th>S23</th>
<th>N25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seepage type</td>
<td>Very high</td>
<td>High</td>
<td>Moderate</td>
<td>Low</td>
<td>Very low</td>
<td>No evidence</td>
</tr>
<tr>
<td>Mean backscatter (dB)</td>
<td>-51.58 (-35.71)</td>
<td>-34.40 (-42.62)</td>
<td>-18.99 (-27.50)</td>
<td>-38.96 (-33.84)</td>
<td>-43.28 (-37.24)</td>
<td>-42.62 (-41.36)</td>
</tr>
</tbody>
</table>

**Fig. 6.2** Backscatter (in digital number: 0–255) and bathymetry seafloor seepage blocks selected from the Figure 6.1 for the present investigation.
6.3 Strange attractors

Self-similar fractals are scale invariant, i.e. possessing a structure with a basic characteristic of nonscaling. They can be divided into two categories. The first one is the monofractal, having strict geometric self-similarity that can be described with a single fractal dimension. The other is the multifractal that requires a series of fractal spectrum rather than a unique fractal dimension. Highly intermittent multifractal fields common in nature are the generic outcome of multiplicative cascade processes dominated by scaling non-linear interactions.

The multifractal formalism based on the strange chaotic attractors followed here identifies a set of parameters derived from the shape of such a fractal spectrum. As a part of the image analyses, the variation of these shape parameters among the seafloor seepage blocks is examined to measure the “complexity” of the field. In view of this, a probability distribution is estimated using the box counting method (Chhabra and Jensen, 1989). The partition function \( \chi(q, \varepsilon) \) that describes the probability of “containing the object” (i.e., the values of backscatter and bathymetry in this application), within each box \( i \), can be calculated for different moments of \( q \) using:

\[
\chi(q, \varepsilon) = \sum_{i=1}^{n(\varepsilon)} m_i^q
\]

where \( m \) is the mass of the measure, \( \varepsilon \) is the length of the box and \( n(\varepsilon) \) is the number of boxes. Based on this, the mass exponent function \( \tau(q) \) shows how the moments of the measure scales with the box size and is given as:

\[
\tau(q) = \lim_{\varepsilon \to 0} \frac{\log \chi(q, \varepsilon)}{\log(\varepsilon)} = \lim_{\varepsilon \to 0} \frac{\log < \sum_{i=1}^{n(\varepsilon)} m_i^q >}{\log(\varepsilon)}.
\]

The generalized fractal dimension function \( D(q) \) can be calculated from \( \tau(q) \) as

\[
D(q) = \frac{\tau(q)}{q-1}
\]

where \( q \neq 1 \). The singularity index \( (\alpha) \) is subsequently determined to calculate the singularity spectrum \( f(\alpha) \) by Legendre transformation of the \( \tau(q) \) curve as \( \alpha(q) = d \tau(q)/dq \). Finally the \( f(\alpha) \), which represents the fractal dimension of the subset with the same singularity strength \( (\alpha) \) is determined to
describe the characteristic of the different hierarchy of fractal as
\[ f(\alpha) = q\alpha(q) - \tau(q). \]

An image can be realized as multifractal when the graph of \( \alpha \) vs. \( f(\alpha) \) (i.e., multifractal spectrum), exists and has the shape of an inverted parabola. If the curve \( f(\alpha) \) converges to a single point, it can be termed as monofractal wherein \( D(q) \) is constant for all values of \( q \). The width of the generalized dimension i.e., \( \Delta D(q) = D(q_{\text{max}}) - D(q_{\text{min}}) \), is a measure of multifractality and indicates the deviation from monofractal behavior. The particulars of the shape parameters used to describe the multifractality (based on the said formalism) are shown in Figure 6.3.

![Figure 6.3](image)

**Fig. 6.3** Schematic representation of (a) generalized correlation dimension function \( D(q) \) for estimation of the parameter \( \Delta D(q) \) and (b) multifractal spectrum \( f(\alpha) \) for assessment of the three multifractal parameters i.e., width of the spectrum \( W \), degree of asymmetry \( B \) and stability of the spectrum \( \Delta f(\alpha) \) based on strange attractor formalism.

In order to distinguish the multifractal spectrum \( f(\alpha) \) quantitatively, it is convenient to calculate the width of the spectrum \( W \) so as to measure the overall variability (Fig. 6.3). A wider \( f(\alpha) \) spectrum is indicative of larger \( W \), denoting multifractality. Such a situation reveals a “heterogeneous” seafloor. In the case of a monofractal set, \( W \) would be small and tending to “zero”. The spectrum will converge to a single point signifying a “homogeneous” seafloor. The other parameter \( B \) measures the asymmetry of the curve and shows the dominance of low or high fractal exponents (Szczepaniak and Macek, 2008). The value of \( B \) is zero for...
symmetric shapes and positive or negative for right or left-skewed shapes respectively. A left-skewed spectrum denotes low fractal exponents dominating the distribution, while a right-skewed spectrum implies dominance of high fractal exponents (Telesca et al., 2003). Thereafter, the values of \( \alpha_{\text{min}} \) and \( \alpha_{\text{max}} \) are estimated to obtain the parameter \( \Delta f(\alpha) \) as \( \Delta f(\alpha) = f(\alpha_{\text{min}}) - f(\alpha_{\text{max}}) \). \( |\Delta f(\alpha)| \) defines the undulation or instability of the system under study. The degree of undulation or instability is minimum for the smallest \( \Delta f(\alpha) \) (≈0).

### 6.4 Stochastic multifractal formalism

#### 6.4.1 Moment scaling function and universal multifractals

The seafloor bathymetry and the related backscatter data can be modeled taking into consideration a small number of (generally deterministic) or many (stochastic) degrees of freedom. In order to incorporate high degree of freedom and variability over a wide range of scales, stochastic approaches are preferred as they have infinite dimensional probability space\(^3\) (Lovejoy et al., 2009a). One way to characterize the statistics of stochastic processes is to use its statistical moments. When a multifractal cascade has proceeded over a scale ratio \( \lambda = L/l \) (\( L \) and \( l \) representing largest and smallest scale\(^4\) in the data), the statistical moments of the conserved multifractal flux (the field values of the MBES bathymetry\(\backslash\)backscatter image and the pressure values of the SBES echo envelope\(^5\)) measured at scale \( \lambda \), follow a power law that can be expressed as (Schertzer and Lovejoy, 1987, 1991):

\[
< \phi_{\lambda}^q > \sim \lambda^{K(q)}
\]  

(6.3)

where \( \phi_{\lambda} \) is the scale by scale conserved multifractal flux, \( q \) is the order of the moment, and \( K(q) \) is a nonlinear convex function. \( K(q) \) characterizes the scaling of

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\(^3\) See Figure 1.2 in Chapter 1.

\(^4\) Two types of data are utilized in this chapter. In Part 1, the MBES bathymetry and backscatter image data have been analyzed in the frame work of stochastic multifractal formalism. In Part 2, the same formalism has been applied to the time dependent SBES echo envelope data. Note that the term “scale” mentioned here is different for MBES and SBES data. The spatial scale in “meter” is applicable to MBES image data and the temporal scale in “millisecond” is ascribed to SBES echo envelope.

\(^5\) See Part 2.
the moments of the $\phi$, hence it is called the “moment scaling function”. With reference to the existence of stable attractive multifractal processes called universal multifractals (Schertzer and Lovejoy, 1987, 1991, 1997; Lovejoy and Schertzer, 1990), $K(q)$ can be expressed as:

$$K(q) = \frac{C_1}{\alpha - 1} (q^\alpha - q)$$

(6.4)

where $\alpha$ and $C_1$ are the basic parameters characterizing the scaling properties of the multifractal flux $\phi$. The parameter $\alpha$ is the degree of multifractality and varies from 0 to 2, where $\alpha=0$ is the monofractal case and $\alpha=2$ is the log normal case. This parameter describes how rapidly the fractal dimensions of the sets at different thresholds vary as they leave the mean singularity. $C_1$ is the codimension parameter of the set. Low value of $C_1$ ($\approx 0$), implies that the field values are close to the mean. $C_1 (> 0)$ indicates that the region making the dominant contribution to the mean is a sparse fractal set such that the vast majority of the field doesn’t contribute (Gagnon et al., 2006). The function $K(q)$ is related to the generalized dimension $D(q)$ as:

$$D(q) = d - \frac{K(q)}{q - 1}$$

(6.5)

where $d$ is the dimension of the space ($=2$ here) (Seuront, 2010).

### 6.4.2 Fractionally Integrated Flux model

The multiplicative process (the cascade) discussed above generates a scale by scale conserved multifractal flux $\phi$ characterized by a moment scaling function $K(q)$. The spectrum of such a conserved flux has an exponent $\beta = 1 - K(2) < 1$. In order to discriminate the seafloor echo-envelopes (having $\beta \approx 2$), FIF model (Gagnon et al., 2006) has been utilized. The FIF model of the multifractal flux provides the following statistics in relation to the intensity field$^7$ $I_\lambda$ at scale ratio $\lambda$ as (Schertzer and Lovejoy, 1987, 1991):

$$I_\lambda = \phi_\lambda \lambda^{-H}.$$  

(6.6)

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$^6$ The symbol $\alpha$ used in the strange attractor formalism represents the singularity index.

$^7$ For MBES image data the intensity field represents the values of bathymetry and backscatter within each bock. Whereas, in SBES echo envelope data the intensity field signifies the pressure values.
Here the linear scaling $\lambda^{-H}$ corresponds to a fractional integral of order $H$. The parameter $H$ can be designated as a degree of smoothness where higher $H$ signifies smoother fields. Characterization of seafloor backscattering using FIF model is difficult to distinguish the underlying cascade dynamics as it involves a convolution due to the exponent $H$. Therefore resorting to the use of “trace moments” (that directly characterizes the conserved multifractal flux $\phi_\lambda$), is necessary so that the differentiation is possible $^8$ (Gagnon et al., 2006; Chakraborty et al., 2014).

The first step to obtain $\phi_\lambda$ from the intensity field involved the removal of $\lambda^{-H}$ in the Eq. (6.6). This is equivalent to a filtering as in Fourier space with “power law”, which is a scale invariant smoothing. On elimination of $\lambda^{-H}$, only the underlying conserved multifractal flux $\phi_\lambda$ is retained. The next step was to examine the scaling of the statistical moments of $\phi_\lambda$ and compare them with Eq. (6.3). To this end, we normalized $\phi_\lambda$ so that the ensemble average of all the samples is $<\phi_{\lambda}> = 1$. Thereafter, the $q^{th}$ power of the samples (bathymetry\backscatter and pressure values) over the sets of size (or time interval) $l = L/\lambda$ was determined. It gives the moments of the normalized multifractal flux for a given value of $q$. This procedure was performed with different values of $q$ and $K(q)$ was evaluated from the logarithmic slopes (Fig. 6.4$^9$). The multifractality of the intensity field has been validated with nonlinear $K(q)$. Using the values of $K(q)$ the parameters $C_1$ and $\alpha$ were estimated as $C_1=K'(1)$ and $\alpha=K''(1)/C_1$ (Stolle et al., 2009; Gires et al., 2013). The values of $\alpha$ and $C_1$ combined with spectral slope $\beta$ were utilized to estimate values of $H$, using the relationship $\beta=1+2H-K(2)$. The three universal multifractal parameters ($\alpha$, $C_1$, and $H$) computed here, determine the statistics of the data at all scales and moments.

$^8$ The trace moment algorithm accessible on the website: http://www.physics.mcgill.ca/~eliasl/ has been used in this study.

$^9$ The corresponding figure for SBES data is illustrated in Part 2.
Fig. 6.4 The scaling behavior of the statistical moments of the two representative backscatter and bathymetry image data is illustrated here by the straightness of the Log/log curves of the normalized trace moment ($M$) as functions of the scale ratio $\lambda=L/l$. The values of the exponent $q$ of each trace moments are varied between 0-2. The linear deviation of dashed curves for $q \geq q_c > 1.5$ is indicative of a multifractal phase transition.
6.5 Results and discussion

6.5.1 Strange attractors based technique

As mentioned earlier, the degree of multifractality can be easily related to the width of the generalized dimension $\Delta D(q)$ and the $f(\alpha)$ spectrum. The computed $\Delta D(q)$ values get successively reduced from maximum to minimum, in the case of the backscatter blocks: Q19, J19, S23, F07, N25 and F20, associated with low, high, very low, moderate, nil, and very high seepages respectively (Fig. 6.5). Such reductions in $\Delta D(q)$ values indicate decrease in the degree of multifractality. Generally, low $\Delta D(q)$ values of bathymetry data blocks indicate comparatively reduced multifractality than the corresponding backscatter block (Fig. 6.5). However, gradual reduction of $\Delta D(q)$ values among the bathymetry data blocks show successive reduction in the degree of multifractality or monofractality (particularly in Q19 and S23 blocks) as $D(q)$ vs. $q$ curves are unvarying. Interestingly, the overall observation of the $D(q)$ vs. $q$ plots of backscatter as well as bathymetry data blocks imply similar construal for $f(\alpha)$ spectrum. The Q19 and S23 bathymetry blocks show single data point in the $f(\alpha)$ spectrum i.e., monofractality (Fig. 6.5).

It is further observed that the shape parameters estimated from the $f(\alpha)$ spectrum also provide information about the multifractality (Telesca et al., 2003). The $W$ values of the four backscatter blocks, Q19, S23, J19 and F07, show gradual decrease in the degree of multifractality (heterogeneity) in a decreasing order. Though, Q19 and S23 blocks are located at a relatively shallower depth ($\approx$180m), they have low to very low backscatter strength indicating dominant multifractality (heterogeneity) (Fig. 6.6). Intriguingly, bathymetry blocks with negligible $W$ values display relatively reduced degree of multifractality as compared to the corresponding backscatter blocks. This may be due to the presence of shell materials along with coarse sediments and the changes in the seafloor roughness at the textural level caused by bottom currents (Fig. 6.1) (Dandapath et al., 2010, 2012). Such changes at the textural level can only be notably detected in the backscatter data as compared to the bathymetry data.
Fig. 6.5 Block wise generalized correlation dimension $D(q)$ and multifractal spectrum $f(\alpha)$ plots for (a) backscatter strength, and (b) bathymetry data of the study blocks.
Fig. 6.6 Results obtained using the two multifractal formalisms. (a) Block wise multifractal spectrum $f(\alpha)$ related shape parameters estimated using strange attractor formalism. (b) Scatter plots of the three multifractal parameters [$W$, $B$, and $\Delta f(\alpha)$]. The bathymetry blocks are encircled. (c) Block wise multifractal parameters estimated using stochastic multifractal formalism. (d) Scatter plots of the three multifractal parameters ($\alpha$, $C_1$ and $H$). The solid and hollow shapes represent different study blocks as shown in the legend.
The estimated $B$ values using backscatter image blocks reveal that all the blocks are positive or right skewed except S23. Among them, the blocks J19 and F20, located away from fault regime (Fig. 6.1), possess higher $B$ values indicating dominance of higher fractal exponents (Fig. 6.6). Whereas S23 block possess a negative value of $B$ showing left skewed spectrum i.e., the dominance of lower fractal exponents. The estimated $B$ parameters of all the bathymetry blocks are significantly low and negative (left skewed), indicating that the distributions are dominated with lower fractal exponent.

The positive values of the parameter $\Delta f(\alpha)$, are seen successively diminishing in the case of backscatter image blocks J19, F07, F20, Q19, N25 and S23, suggesting reduction in the undulations or instability at the textural level (Fig. 6.6). However in the case of bathymetry image blocks, low negative values of $\Delta f(\alpha)$ are observed in all the six blocks. The variability of the three parameters [$W$, $B$, and $\Delta f(\alpha)$] of the backscatter blocks is more conspicuous than their corresponding parameters of the bathymetry blocks, indicating dominant fine scale undulations in the backscatter as compared to the depth data. The scatter plots (Fig. 6.6) of the three parameters affirm the location wise study results using the strange attractor technique.

### 6.5.2 Stochastic multifractal field based technique

In our analyses, the universal form [determined from Eq. (6.4) based on the estimated $\alpha$ and $C_1$] fits the empirical $K(q)$ (determined from the logarithmic slopes of trace moments) quite well. A “multifractal phase transition” (Schertzer and Lovejoy, 1992) is observable in the plot (Fig. 6.4) of empirical and theoretical $K(q)$ curves, indicating that the measured moments will only have the theoretical $K(q)$ for $q$ below a critical moment $q_c$. Beyond $q_c$ there is a multifractal phase transition where $K(q)$ becomes asymptotically linear for $q>q_c>1.5$ (a sample size-dependent effect corresponding to the domination of the statistics by the largest flux values present). Indeed, for $q<1.5$, the deviations from the universal form are negligible.

The $\alpha$ values of the bathymetry and backscatter blocks show identical trend ($\approx 2$) expressing similar degree of multifractality excluding Q19 and S23 of the bathymetry blocks (Fig. 6.6). The low $\alpha$ values of the two blocks are well
corroborated with the results of the other formalism signifying monofractality. The $C_1$ values of the backscatter and bathymetry blocks are found to be varying between 0.058 and 0.091 and between 0.038 and 0.690 respectively. Excluding the S23 bathymetry block ($C_1 = 0.690$), the lower values of $C_1$ attributed to the remaining blocks, indicate that the field values are close to the mean values. Fluctuations in higher values of $H$ between 0.636 and 0.706 are observed in the backscatter image blocks, except in the N25 block (0.412) having no evidence of seepages. Whereas, lower $H$ values (0.255–0.480) are observed in the bathymetry as compared to the backscatter blocks at the same location. The estimated $H$ value of N25 bathymetry block is the lowest.

Cluster analyses output (Fig. 6.6) of $H$ vs. $\alpha$ reveals a uniform $\alpha$ in the backscatter and bathymetry blocks except in the Q19 bathymetry block. However, the $H$ value of the bathymetry block S23, does not subsist owing to the isotropic field condition as mentioned in p. 549 of Gagnon et al. (2006). Remarkably the clustering tendency around the high $\alpha$ (≈ 2) and low $C_1$ of the bathymetry and backscatter data, show extremely close relationship among, the six backscatter and four bathymetry blocks (Fig. 6.6). However both Q19 and S23 bathymetry blocks possess comparable $\alpha$ values and relatively higher $C_1$ values. The $C_1$ (0.690) value of the S23 bathymetry block espouse the setting wherein the depths at specific locations (i.e., center of the pockmark) are higher as compared to the rest of the locations within the block. In this study, the stochastic multifractal based technique show no significant variation in $\alpha$ and $C_1$ of the backscatter field data, except parameter $H$.

### 6.6 Concluding remarks

Two multifractal formalisms (strange attractor and stochastic) were applied to the backscatter and corresponding bathymetry blocks to characterize the pockmark seepage associated seafloor in the WCMI. The outcome of the application of the strange attractor technique, $\Delta D(q)$ and the $f(\alpha)$ spectrum related shape parameters [$W$, $B$ and $\Delta f(\alpha)$] reveal multifractal character of the six backscatter blocks. The variability of the estimated shape parameters is more apparent in the backscatter as
compared to the corresponding bathymetry blocks. This can be related to the interpretation of $\Delta D(q)$, and $f(\alpha)$ parameters of the ECG signal of a healthy and a diseased heart (Stanley et al., 1999). The above referred multifractality aspects support the fact that greater the data heterogeneity, higher would be the system stability. Characteristically, higher stability can be realized when $|\Delta f(\alpha)|$ is low. Appropriately Q19 and S23 backscatter blocks reveal low $|\Delta f(\alpha)|$ values. The stability attributed to the blocks could be ascribed to their location in shallow depth ($\approx 180$ m) as compared to the rest of the blocks. Moreover, the above blocks possess coarse seafloor sediments having dominant shell materials, and are influenced by the monsoonal bottom currents, resulting in greater heterogeneity (Dandapath et al., 2010).

The three computed parameters ($\alpha$, $C_1$, and $H$) from bathymetry and backscatter blocks using stochastic multifractal formalism show almost similar degree of multifractality with the exception of Q19 and S23 bathymetry blocks. The low values of the two blocks are in sound corroboration as they show monofractality. Generally the level of the $C_1$ values of backscatter data blocks are found to be higher (0.058–0.091) than the bathymetry (0.038–0.690). In the case of backscatter image blocks, higher $H$ values (0.636–0.706) are observed except in N25 block (0.412) having no seepages. On the other hand, lower $H$ values (0.255–0.480) are observed in the bathymetry blocks in relation to the backscatter. The advantage of the stochastic multifractal technique is that it provides three distinct parameters, with which it is easier to comprehend the multifractality aspects of the seafloor than the strange attractor based $f(\alpha)$ spectrum related shape parameters. Cheng and Agterberg (1996) had made assessment between the interrelationships of the two methods (akin to the ones we have used here) and had suggested the aptness of the multifractal spectrum $f(\alpha)$ over the codimension function $C_1$. The two multifractal techniques utilized in our work is a first-time attempt to analyze the high resolution MBES backscatter and bathymetry data. The present investigation employing both the methods ascertain an important finding, however further interest is required to expound the techniques.