CHAPTER 4

VARIANTS OF REVISED COMPLEX ALGORITHM

4.1 Introduction

The Revised Complex Algorithm discussed in the previous chapter is an efficient computational procedure to solve Linear Programming Problems. As the size of the basis matrix is a dynamic one and does not exceed the number of constraints in the problem contributes to the computational efficiency of the algorithm. Here, the redundant constraints are identified and eliminated, and hence the size of the basis matrix is reduced and the computational effort is reduced. Based on the number of variables selected to enter the basis corresponding leaving variables can also be found out. This can lead to fixed reduced size of basis matrix. Using this concept the different approaches are discussed in this chapter.

4.2 Revised Complex Algorithm using variants

The concept of multiple column selection procedure [43] is used to select a set of entering and leaving variables at start and subsequent passes becomes the basis for the few variants of the Revised Complex Algorithm. In these methods the decision variables are initially arranged to enter into the basis, based on the maximum contribution of the variables to the objective function and the variables competing for the same resource and the corresponding leaving variables are identified. The product form of Inverse method is used in these methods to move from a basic feasible solution to another as well as to find the optimal solution. A two step procedure is adopted namely 1) Selection of entering and leaving variables and
2) Performing the iterations based on the selected variables. The procedure for selection of entering and leaving variables is as given below:

4.2.1 Selection of entering and leaving variables

The various steps involved in the selection of entering and leaving variables based on the maximum contribution of the variables to the objective function and type of constraints are listed below:

Step 1 to 6: as given in 3.4.1

Step 7: Arrange the variables for which the minimum occurs in the $mn^{th}$ column, and find the variable for which the $c_jx_j$ value is maximum. Increment $p_1$ by 1 and store the promising variable subscripts, column variable subscript (represents the subscript of leaving variable) as well as the product $c_jx_j$ as $p_1^{th}$ row of array G. Eliminate all other variables for which the minimum occurs in the $mn^{th}$ column except the one for which $c_jx_j$ is maximum for further consideration.

Step 8: Increment $mn$ by 1. If $mn$ is less than or equal to the number of columns of $\theta_1$ matrix go to step 6.

Step 9: Sort the array G according to the descending order of $c_jx_j$. Transfer the sorted variables to the set $J_1$ and corresponding column numbers to the set $J_2$ as the first $p_1$ elements.

ie. $J_1$ = [Ordered set of subscripts of promising variables to enter the basis ]

$J_2$ = [ Ordered set of subscripts of the variables to leave the basis ]
in the sets J1 and corresponding column numbers where the variables presented stores in J2.

**Step 11:** Repeat steps 6 to 8 for θ3 matrix. Let \( r_1 \) be the number of variables selected for arrangement and is stored as \((p_1+q_1+1)\)th element to \((p_1+q_1+r_1)\)th elements in the set J1 and corresponding column numbers stored in the set J2.

Hence the sets J1 and J2 hold the subscripts of the entering variables and leaving variables respectively. \( NJ = (p_1+q_1+r_1) \) be the number of elements in the sets J1 and J2 respectively.

### 4.2.2 Variants of Revised Complex Algorithm :

The variants to Revised Complex algorithm bring into the basis a set of variables throughout the computation. Each such group of computation is defined as a pass. Three variants are to be discussed in the following paragraphs. A set of variables is brought into the basis to obtain the basic solution. For which, the problem with \( m \) constraints is reduced to a problem with \( NJ \) (\( NJ < m \)) constraints by identifying and eliminating redundant constraints [64]. So, the size of the basis matrix and its inverse is reduced to \((NJ \times NJ)\) and based on the entering and leaving variables selected as per 4.2.1.

#### 4.2.2.1 Variant 1 :

A set of variables is brought into the basis to obtain the basic solution. In each phase, the basic solution obtained is tested for optimality and feasibility. If the solution satisfies the optimality and feasibility criteria then it is optimal. If it fails to satisfy, the next set of promising variables is brought in and this is continued until optimality is satisfied. Infeasibility, if any occurs when optimality is satisfied, is removed using the Dual Revised
Complex Algorithm. The product form of inverse is used to update the $M^{-1}$ matrix.

In the variant to the Revised Complex Algorithm, since the feasibility of the solution is not maintained in each iteration, one the following situations may arise.

i. All $\alpha_{ij}$ where $\alpha_{ij} = (B^{-1}R_j)_i$, $i=1,2...,NJ$ may be $\leq 0$ for the most promising variable. $R_j$ is the column vector of the $j^{th}$ promising variable in the redefined problem.

ii. There may be situations where all $\alpha_{ij}$s may be negative or zero where $\alpha_{ij} = (B^{-1}R_j)_i$, $i=1,2...,NJ$ except a few for which the vectors $(\alpha_0)$ where $(\alpha_0) = (B^{-1}R_0)_i$ is negative,

In this case, the $\theta$ matrix is modified as follows:

a) If the promising variable is a decision variable, then

$$\theta_{pi} = \left[ \frac{(B^{-1}R_0)_i}{(\alpha_{ij})}; (\alpha_{ij}) > 0 \right]$$

where $i = 1,2,...,NJ$ and $p = 1,2,..,NJ$

$j$ is the subscript of the promising variable

b) If the entering variable is a slack variable, then the relationship

$$\theta_{pi} = \left[ \frac{(B^{-1}R_0)_i}{(b_{ik})}; (b_{ik}) > 0 \right]$$

is used where $i = 1,2,...,NJ$ and

$b_{ik}$ is the $i^{th}$ row $k^{th}$ column element of $B^{-1}$. 

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4.2.2.2 Variant 2:

In the second method, only the promising variables with positive values alone are considered to enter into the basis. If all such variables are entered, then check for feasibility. Infeasibility if any exists, then it is removed and check for optimality. This procedure is iterated until optimal solution is obtained. In the updating of \( M^{-1} \) matrix at each stage, every variable in the set is allowed to enter the basis only with a positive value by ensuring that \( a_{ij} \) and \( b_k \) are both positive. If it fails, the current solution is checked for feasibility. Infeasibility, if any occurs, can be removed by using Dual Revised Complex Algorithm.

4.2.2.3 Variant 3:

In this method, the feasibility of the solution is tested at the end of each pass. Whenever the infeasibility occurs in the basis, it is removed then and there. The formation of \( B \) matrix and updating of \( M^{-1} \) are same as in the previous methods.

The step by step procedure of the variant algorithm is given below:

4.3 Step by Step procedure

The optimality condition to be satisfied based on the \( z_j - c_j \) value and type of the objective is given below:

In the case of maximization, all the \( z_j - c_j \) values should be positive or zero.
In the case of minimization, all the \( z_j - c_j \) values should be negative or zero.
The \( z_j - c_j \) values for all the variables are computed using the formula

\[
 z_j - c_j = (1 \ C_B \ B^{-1}) \begin{pmatrix} -c_j \\ R_j \end{pmatrix}
\]

4.3.1

where
$C_B$ is the contribution coefficient of the variables, which had already entered the basis and placed in the order in which it had entered the basis.

$R_j$ is the vector consisting of $j^{th}$ variable coefficients in the constraints in which decision variables had already entered.

**Step 1: Reduction of the problem size.**

Select the entering and leaving variables as described in 4.2.1. Hence the size of the problem is reduced to ($NJ \times NJ$) where $NJ$ – number of entering or leaving variables. $J_1$ and $J_2$ are the arrays which hold the subscripts of the entering and leaving variables respectively.

**Step 2: Updation of $M^{-1}$.**

Formulate an $M$-matrix using selected variables columns and then find $M^{-1}$. As per 2.4.6, $M^{-1} = \begin{bmatrix} 1 & C_B B^{-1} \\ 0 & B^{-1} \end{bmatrix}$ and since $B^{-1}$ is a submatrix of $M^{-1}$, it is enough to compute $M^{-1}$. In each phase, the inversion of the matrix $M$ may lead to any one of the following conditions.

i. The matrix $M$ may be singular because of the selection of dependent column.

ii. If the matrix $M$ is nonsingular, some of the variables, which have entered in a phase, may leave in the subsequent phases from the basis leading to variables popping it and out of basis.
To avoid the issues given above, the product form of inverse is used to find the basis inverse matrix.

**Step 3**: A column corresponding to an entering variable is introduced in the previously inverted matrix and the new inverted matrix is found. Let $j$ an element of $\mathbf{J}_1$ be the subscript of the entering variable and $k$ an element of $\mathbf{J}_2$ be the corresponding column into which the $j^{th}$ variable enters. The column vector $\eta$ corresponding to the $j^{th}$ variable is formed using $(z_j-c_j)$ and the vector $B^{-1}R_j$.

\[
\eta_1 = z_j - c_j \quad \text{4.3.2}
\]

\[
\eta_{i+1} = (B^{-1}R_j)_i, \text{ where } i = 1,2,...,NJ \quad \text{4.3.3}
\]

The new $\eta$ vector is determined using the above $\eta$ vector with the following convention.

\[
(\eta_i)_{\text{new}} = \frac{(\eta_i)_{\text{old}}}{(\eta_{k+1})_{\text{old}}}, i \neq k + 1 \quad \text{4.3.4}
\]

\[
(\eta_i)_{\text{new}} = \frac{1}{(\eta_{k+1})_{\text{old}}}, i = k + 1 \quad \text{4.3.5}
\]

where $i = 1,2,...,(NJ+1)$.

Before determining modified $\eta$ vector, check the optimality condition. That is whether the $j^{th}$ variable is promising or not (i.e. $\eta_1$ has promising value or not). If it is not promising, that variable is not considered to enter into the basis and the next variable is considered for entry into the basis.

Once $\eta_1$ is promising, check whether $\eta_{k+1}$ is positive. If $\eta_{k+1}$ is positive, the new $\eta$ vector is computed using the relationships 4.3.4.
and 4.3.5 If $\eta_{k+1}$ is zero or negative, then the corresponding column is ignored. Using the new $\eta$ vector, $E_0$ matrix is constructed and then update $M^{-1}$ using the relationship

$$M^{-1}_{new} = E_0 M^{-1}_{old}$$  \hspace{1cm} 4.3.6

**Step 4:** Steps 2 and 3 are repeated until the new $M^{-1}$ is computed using the list of selected variables. If the list is exhausted, go to step 5.

**Step 5 a)** The promising variables are identified using the relationship

$$\begin{pmatrix} z_j - c_j \end{pmatrix} = \begin{pmatrix} 1 & C B^{-1} \end{pmatrix} \begin{pmatrix} -c_j \\ R_j \end{pmatrix}$$

**b)** The optimality condition of the solution is checked. Under non-optimality conditions the promising variables are identified and the entering and leaving variables are selected as per the procedure given in Section 4.2.

c) Once the optimality condition is satisfied, the feasibility of the solution is verified. If the solution is feasible go to step 6.

d) Infeasibility if any exists, the dual Revised Complex procedure is used to remove the infeasibility and go to step 5a) after obtaining a feasible solution.

**Step 6:** Substitute the solutions in all the constraints. If all the constraints are satisfied then stop else go to step 5d) by including infeasible constraints..
The steps 1 through 6 are adopted for the variant 1.

For variant 2, some additional checks are to be made in step 3. That is, in addition to check whether \( \eta_i \) is still promising and \( \eta_{k+1} \) is positive, it is also to be checked whether \( B^{-1}R_0 \) is also positive. If it is positive, then that variable is made to enter into the basis otherwise it is ignored.

For variant 3, step 4 is being modified as given below:

**Step 4**: Steps 2 and 3 are repeated until the \( M^{-1}_\text{new} \) is found using the selected variables in that pass. Once the list of selected variables is exhausted the solution is checked for its feasibility. If it is feasible then go to step 5 else go to step 5d.

### 4.4 Example

Maximize  \( Z = 8x_1 + 5x_2 + 6x_3 + 9x_4 + 7x_5 + 9x_6 + 6x_7 + 5x_8 \)

subject to

\[
\begin{bmatrix}
0 & 0 & 0 & 2 & 4 & 3 & 0 & 0 \\
7 & 3 & 6 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 3 & 1 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 8 & 5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 \\
2 & 4 & 3 & 0 & 0 & 0 & 0 & 0 \\
5 & 3 & 0 & 2 & 0 & 3 & 1 & 5 \\
2 & 0 & 4 & 3 & 7 & 0 & 1 & 0 \\
5 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 6 & 4 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7 \\
x_8
\end{bmatrix}
\leq
\begin{bmatrix}
9 \\
15 \\
7 \\
25 \\
30 \\
10 \\
30 \\
20 \\
12 \\
20
\end{bmatrix}
\]

\( x_i \geq 0, \ i = 1, \ldots, 8 \)
Construct $\theta$ matrix

<table>
<thead>
<tr>
<th></th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
<th>$S_5$</th>
<th>$S_6$</th>
<th>$S_7$</th>
<th>$S_8$</th>
<th>$S_9$</th>
<th>$S_{10}$</th>
<th>$c_jx_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>-</td>
<td>15/7</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>6</td>
<td>10</td>
<td>12/5</td>
<td>19.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_2$</td>
<td>-</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>10/4</td>
<td>10</td>
<td>-</td>
<td>15</td>
</tr>
<tr>
<td>$x_3$</td>
<td>-</td>
<td>15/6</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>10/3</td>
<td>5</td>
<td>4</td>
<td>-</td>
<td>22.5</td>
<td></td>
</tr>
<tr>
<td>$x_4$</td>
<td>9/2</td>
<td>-</td>
<td>7/3</td>
<td>-</td>
<td>-</td>
<td>15</td>
<td>20/3</td>
<td>-</td>
<td>-</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>$x_5$</td>
<td>9/4</td>
<td>-</td>
<td>7</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>20/7</td>
<td>-</td>
<td>-</td>
<td>15.4</td>
<td></td>
</tr>
<tr>
<td>$x_6$</td>
<td>3</td>
<td>-</td>
<td>7/2</td>
<td>-</td>
<td>-</td>
<td>10</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>$x_7$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>25/8</td>
<td>30/7</td>
<td>30</td>
<td>20</td>
<td>-</td>
<td>20/6</td>
<td>18.75</td>
<td></td>
</tr>
<tr>
<td>$x_8$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>30/9</td>
<td>15</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>16.6</td>
</tr>
</tbody>
</table>

Arrangement of variables entering into the basis based on the maximum contribution to the objective function (i.e. $c_jx_j$). From the above table

i) $x_6$ enters $S_1$ leaves

ii) $x_3$ enters $S_2$ leaves

iii) $x_4$ enters $S_3$ leaves

iv) $x_7$ enters $S_4$ leaves

v) $x_8$ enters $S_5$ leaves

vi) $x_2$ enters $S_6$ leaves

$J_1$ = set of arranged variables entering into the basis = \{ $x_6$, $x_3$, $x_4$, $x_7$, $x_8$, $x_2$ \}

$J_2$ = set of corresponding variables leaving the basis = \{ $S_1$, $S_2$, $S_3$, $S_4$, $S_5$, $S_6$ \}

The redundant constraints are 7, 8, 9 and 10
The redundant variables are $x_1$ and $x_5$

Then the problem becomes (excluding the redundant constraints and variables)
Maximize \( Z = 5x_2 + 6x_3 + 9x_4 + 9x_6 + 6x_7 + 5x_8 \)

subject to

\[
\begin{bmatrix}
0 & 0 & 2 & 3 & 0 & 0 & x_2 \\
3 & 6 & 0 & 0 & 0 & 0 & x_3 \\
0 & 0 & 4 & 2 & 0 & 0 & x_4 \\
0 & 0 & 0 & 0 & 8 & 5 & x_6 \\
0 & 0 & 0 & 0 & 7 & 9 & x_7 \\
4 & 3 & 0 & 0 & 0 & 0 & x_8 \\
\end{bmatrix}
\begin{bmatrix}
x_2 \\
x_3 \\
x_4 \\
x_6 \\
x_7 \\
x_8 \\
\end{bmatrix}
\begin{bmatrix}
9 \\
15 \\
7 \\
25 \\
30 \\
10 \\
\end{bmatrix}
\]

\( x_i \geq 0, \ i = 2,3,4,6,7,8 \)

The problem is redefined as follows

Maximize \( Z = 5y_1 + 6y_2 + 9y_3 + 9y_4 + 6y_5 + 5y_6 \)

subject to

\[
\begin{bmatrix}
0 & 0 & 2 & 3 & 0 & 0 & y_1 \\
3 & 6 & 0 & 0 & 0 & 0 & y_2 \\
0 & 0 & 4 & 2 & 0 & 0 & y_3 \\
0 & 0 & 0 & 0 & 8 & 5 & y_4 \\
0 & 0 & 0 & 0 & 7 & 9 & y_5 \\
4 & 3 & 0 & 0 & 0 & 0 & y_6 \\
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4 \\
y_5 \\
y_6 \\
\end{bmatrix}
\begin{bmatrix}
9 \\
15 \\
7 \\
25 \\
30 \\
10 \\
\end{bmatrix}
\]

\( y_i \geq 0, \ i = 1,...,6 \)

where \( y_1 = x_2, \ y_2 = x_3, \ y_3 = x_4, \ y_4 = x_6, \ y_5 = x_7, \ y_6 = x_8 \)

\( J_1 = \) set of ordered entering variables = \( \{ y_4,y_2,y_3,y_5,y_6,y_1 \} \)

\( J_2 = \) set of corresponding leaving variables = \( \{ S_1,S_2,S_3,S_4,S_5,S_6 \} \)
Pass 1: Select variables one by one from the set J1 and J2

Entering variable \( y_4 \)
Leaving variable \( S_1 \)

\[ \eta \text{ vector for } y_4 \quad \eta = \begin{bmatrix} 3 \\ 1/3 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad M^{-1} = \begin{bmatrix} 1 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \]

Entering variable \( y_2 \)
Leaving variable \( S_2 \)

Check whether \( y_2 \) is promising or not

\[ z_2 - c_2 = \begin{bmatrix} -6 \\ 0 \\ 6 \\ 0 \\ 0 \\ 3 \end{bmatrix} = -6 \text{ (} -\text{ve) } \]

Since \( z_2 - c_2 < 0 \), \( y_2 \) is promising

\[ \eta \text{ vector for } y_2 \]

\[ \eta = \begin{bmatrix} 1 \\ 0 \\ 1/6 \\ 0 \\ 0 \\ -1/2 \end{bmatrix} \quad \text{and} \quad M^{-1} = \begin{bmatrix} 1 & 3 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/6 & 0 & 0 & 0 & 0 \end{bmatrix} \]
Entering variable $y_3$
Leaving variable $S_3$

Check whether $y_3$ is promising or not

$$ z_3 - c_3 = \begin{bmatrix} 1 & 3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 3 & 3 & 3 & 3 & 3 \end{bmatrix} = -3 \text{ (-ve)} $$

Since $z_3 - c_3 < 0$, $y_3$ is promising

$\eta$ vector for $y_3$

$$ \eta = \begin{bmatrix} 9/5 \\ -2/5 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad M^{-1} = \begin{bmatrix} 1 & 9/5 & 1 & 9/5 & 0 & 0 & 0 \\ 0 & 3/5 & 0 & -2/5 & 0 & 0 & 0 \\ 0 & 0 & 1/6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1/2 & 0 & 0 & 0 & 1 \end{bmatrix} $$

Entering variable $y_5$
Leaving variable $S_4$

Check whether $y_5$ is promising or not

$$ z_5 - c_5 = \begin{bmatrix} 1 & 9/5 & 1 & 9/5 & 0 & 0 & 0 \end{bmatrix} = -6 \text{ (-ve)} $$

Since $z_5 - c_5 < 0$, $y_5$ is promising

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\[ \eta \text{ vector for } y_5 \]
\[ \begin{bmatrix} 6/8 \\ 0 \\ 0 \\ -7/8 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 9/5 & 1 & 9/5 & 6/8 & 0 & 0 \\ 0 & 3/5 & 0 & -2/5 & 0 & 0 & 0 \\ 0 & 0 & 1/6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -7/8 & 1 & 0 \\ 0 & 0 & -1/2 & 0 & 0 & 0 & 1 \end{bmatrix} \]

Entering variable \( y_6 \)
Leaving variable \( S_5 \)
Check whether \( y_6 \) is promising or not
\[ z_6 - c_6 = \begin{bmatrix} 1 & 9/5 & 1 & 9/5 & 6/8 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = -10/8 \text{ (-ve)} \]

Since \( z_6 - c_6 < 0 \), \( y_6 \) is promising
\[ \eta \text{ vector for } y_6 \]
\[ \begin{bmatrix} 10/37 \\ 0 \\ 0 \\ -5/37 \\ 8/37 \end{bmatrix} = \begin{bmatrix} 1 & 9/5 & 1 & 9/5 & 19/37 & 10/37 & 0 \\ 0 & 3/5 & 0 & -2/5 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9/37 & -5/37 & 0 \\ 0 & 0 & 0 & 0 & -7/37 & 8/37 & 0 \end{bmatrix} \]
Entering variable $y_1$
Leaving variable $S_6$
Check whether $y_1$ is promising or not

$$z_1 - c_1 = \begin{pmatrix} 1 & 9/5 & 1 & 9/5 & 19/37 & 10/37 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 \end{pmatrix} = -2 \quad (-ve)$$

Since $z_1 - c_1 < 0$, $y_1$ is promising

$\eta$ vector for $y_1$

$$\eta = \begin{pmatrix} 2/5 \\ 0 \\ 6/5 \end{pmatrix} \quad \text{and} \quad M^{-1} = \begin{pmatrix} 1 & 9/5 & 17/15 & 9/5 & 19/37 & 10/37 & 2/5 \\ 0 & 0 & 3/5 & 0 & -2/5 & 0 & 0 \\ 0 & 0 & -8/5 & 0 & 0 & 0 & 6/5 \end{pmatrix}$$

No other elements in the ordered list
Pass 2:
Check for other variables \( y_7 \) and \( y_8 \) where \( y_7 = x_1 \) and \( y_8 = x_5 \) in the problem are promising or not.

\[
z_7 - c_7 = \begin{bmatrix} 1 & 9/5 & 17/15 & 9/5 & 19/37 & 10/37 & 2/5 \end{bmatrix} \begin{bmatrix} 0 \\ 8 \\ 0 \\ 7 \\ 0 \\ 0 \\ 2 \end{bmatrix} = 11/15 \text{ (+ve)}
\]

\[
z_8 - c_8 = \begin{bmatrix} 1 & 9/5 & 17/15 & 9/5 & 19/37 & 10/37 & 2/5 \end{bmatrix} \begin{bmatrix} 1 \\ -7 \\ 4 \\ 0 \\ 0 \\ 0 \end{bmatrix} = +2 \text{ (+ve)}
\]

Since \( z_7 - c_7 \) and \( z_8 - c_8 \) are positive, \( y_7 \) and \( y_8 \) are not promising.

The solution vector

\[
B^{-1}R_o = \begin{bmatrix} 3/5 & 0 & -2/5 & 0 & 0 & 0 \\ 0 & -8/5 & 0 & 0 & 0 & 6/5 \\ -2/5 & 0 & 3/5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9/37 & -5/37 & 0 \\ 0 & 0 & 0 & -7/37 & 8/37 & 0 \\ 0 & 1/15 & 0 & 0 & 0 & 1/5 \end{bmatrix} \begin{bmatrix} 9 \\ 15 \\ 7 \\ 25 \\ 30 \\ 10 \end{bmatrix} = \begin{bmatrix} 13/5 \\ -12 \\ 3/5 \\ 75/37 \\ 65/37 \\ 3 \end{bmatrix} y_4 \\
\text{y}_2 \\
\text{y}_3 \\
\text{y}_5 \\
\text{y}_6 \\
\text{y}_1
\]

The solution is infeasible. Dual Revised Complex Algorithm is applied to remove infeasibility.

Is there any vector has -ve value in \( B^{-1}R_7 \) and \( B^{-1}R_8 \)? Check.
Find $B^{-1}R_7$ and $B^{-1}R_8$

$B^{-1}R_7 = \begin{bmatrix} 3/5 & 0 & -2/5 & 0 & 0 & 0 & 0 \\ 0 & -8/5 & 0 & 0 & 0 & 6/5 & 7 \\ -2/5 & 0 & 3/5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9/37 & -5/37 & 0 & 0 \\ 0 & 0 & 0 & -7/37 & 8/37 & 0 & 0 \\ 0 & 1/15 & 0 & 0 & 0 & 1/5 & 2 \end{bmatrix} = \begin{bmatrix} 0 \\ -44/5 \\ 0 \\ 0 \\ 0 \\ 13/45 \end{bmatrix}$

$B^{-1}R_8 = \begin{bmatrix} 3/5 & 0 & -2/5 & 0 & 0 & 0 & 4 \\ 0 & -8/5 & 0 & 0 & 0 & 6/5 & 0 \\ -2/5 & 0 & 3/5 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 9/37 & -5/37 & 0 & 0 \\ 0 & 0 & 0 & -7/37 & 8/37 & 0 & 0 \\ 0 & 1/15 & 0 & 0 & 0 & 1/5 & 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$B^{-1}R_7$ and $B^{-1}R_8$ have negative values in II and III rows respectively.

As per the Dual Revised Complex algorithm

$$\theta_1 = \min \left\{ \frac{z_j - c_j}{B^{-1}R_j} \middle| B^{-1}R_j < 0 \right\} \text{ where } j = 7,8$$

$$= \min \left\{ \frac{11/15}{-44/5}, \frac{2}{-1} \right\} = 11/132$$

Hence, $y_7$ enters and $y_2$ leaves the basis.

$\eta$ vector for $y_7$

$$\begin{bmatrix} 1/12 \\ 0 \\ -5/44 \end{bmatrix} \quad \text{and} \quad M^{-1} = \begin{bmatrix} 1 & 9/5 & 1 & 9/5 & 19/37 & 10/37 & -1/2 \\ 0 & 3/5 & 0 & -2/5 & 0 & 0 & 0 \\ 0 & 0 & 2/11 & 0 & 0 & 0 & -3/22 \\ 0 & 0 & 0 & 9/37 & -5/37 & 0 & 0 \\ 0 & 0 & 0 & -7/37 & 8/37 & 0 & 0 \\ 13/132 & 0 & -1/11 & 0 & 0 & 0 & 7/22 \end{bmatrix}$$
Now, the solution vector is

\[
B^{-1}R_0 = \begin{bmatrix}
3/5 & 0 & -2/5 & 0 & 0 & 0 & 9 \\
0 & 2/11 & 0 & 0 & 0 & -3/22 & 15 \\
-2/5 & 0 & 3/5 & 0 & 0 & 0 & 7 \\
0 & 0 & 0 & 9/37 & -5/37 & 0 & 25 \\
0 & 0 & 0 & -7/37 & 8/37 & 0 & 30 \\
0 & -1/11 & 0 & 0 & 0 & 7/22 & 10 \\
\end{bmatrix}
\begin{bmatrix}
y_4 = x_6 \\
y_7 = x_1 \\
y_3 = x_4 \\
y_5 = x_7 \\
y_6 = x_8 \\
y_1 = x_2 \\
\end{bmatrix}
\]

The solution is feasible.

Check for other variables \(y_2, y_8\) promising or not

\[
z_2 - c_2 = \begin{bmatrix}
-5 \\
0 \\
6 \\
0 \\
0 \\
3 \\
\end{bmatrix} = \frac{3}{2} \text{ (+ve)}
\]

\[
z_8 - c_8 = \begin{bmatrix}
-7 \\
1 \\
0 \\
4 \\
0 \\
0 \\
0 \\
\end{bmatrix} = 2 \text{ (+ve)}
\]

Hence \(y_2\) and \(y_8\) are not promising and the solution is feasible.

Hence the optimal solution is

\[
x_1 = \frac{15}{11} \quad x_2 = \frac{20}{11} \quad x_4 = \frac{3}{5} \quad x_6 = \frac{13}{5} \quad x_7 = \frac{75}{37} \quad x_8 = \frac{65}{37}
\]

\[
Z = 69.75
\]

The solution is substituted in all the constraints. All the constraints are satisfied. Hence the solution of optimal and feasible.
4.5. Conclusion

In this chapter, a study on the variants of Revised Complex algorithm has been discussed. Three methods have been discussed and adopted. In these procedures, at start the variables are arranged to enter into basis based on the contribution to objective function and type of constraints. Depending on the conditions imposed on the entering and leaving variables, three variant methods are used to find the solution to linear programming problem. The variant algorithm reduces the number of iterations as compared with the conventional methods. In the subsequent chapter an algorithm to solve bounded variable problem is explained.