Chapter III

Converged Welch’s Method for Energy Detection Spectrum Sensing in Cognitive Radio
3.1 Introduction
CR working is based on the dynamic spectrum access model are one of the best available solutions for efficient use of spectrum. The secondary or the cognitive user can access the unused spectrum band which has been allotted to the PU or the licensed user in their absence. The phenomenon of working with CR users deals with the non-interference with the PU [169, 170]. It simply means the spectrum sensing at the CR receiver should be with maximum accuracy as well as computationally fast and simple. As mentioned in Eq (3.1), the hypothesis testing represents, where the null hypothesis indicates the absence of PUs and presence of AWGN and alternate hypotheses as [171]:

\[ H_0: \text{PU is absent}; \\
H_1: \text{PU is in operation.} \]

\[ x_i(t) = \begin{cases} 
 w_i(t), & H_0, \\
 h_i(t)s(t) + w_i(t), & H_1, 
\end{cases} \]  

(3.1)

In this case, \( x_i(t) \) is the received complex baseband signal at \( i^{th} \) CR, whereas \( s(t), h_i(t) \) and \( w_i(t) \) are PU signals, the complex channel gain of the sensing channel between the PU and the \( i^{th} \) and AWGN respectively.

In this chapter, we consider the problem of BER based on the spectrum sensing signals for CRs. It is shown that there is an improved BER performance of the receiving signal of interest from NI-USRP 2920 using the modified CWPS. \( P_d \) and \( P_f \) have been found using Fast Fourier Transform (FFT) or N-Point FFT (NFFT) conventional Periodogram and then the BER are compared with the Welch’s Periodogram which is modified. The BER performance analysis has been evaluated and compared with standard error function for binary FSK. Simulation results show that substantial improvement of more than 5% has been obtained over standard bit error rate.

The signal is fed through a low pass filter and the audio signal is sampled at 15 KHz, considering the interpolation factor, decimation factor and gain is taken as 3, 2 and 20 dB, respectively.
3.1.1 Effect of Noise Uncertainty

In most communication systems, noise is a collection of various independent sources, including thermal noise as well as interferences due to nearby unintended emissions. So the assumption that the noise is a Gaussian random process is always appropriate. Further, the variance of the noise could change over time and it cannot be estimated exactly. The below section shows that the detection of threshold, \( \lambda \) is proportional to \( \sigma_w^2 \). In most practical situations, \( \sigma_w^2 \) (and \( \lambda \) consequently) would need to be estimated at the receiver [175, 187, 188].

Denoting the estimates by \( \hat{\sigma}_w^2 \) and \( \hat{\lambda} \). Assume the error in estimating \( \sigma_w^2 \) is:

\[
(1 - \varepsilon_1)\sigma_w^2 \leq \hat{\sigma}_w^2 \leq (1 + \varepsilon_2)\sigma_w^2, \quad 0 \leq \varepsilon_1 < 1; \varepsilon_2 \geq 0 
\]

(3.2)

Let, \( U = \frac{1 + \varepsilon_2}{1 - \varepsilon_1} \geq 1 \), this \( U \) is taken for the detection requirement of \( (P_d, P_f) \).

Observation window \( N \) and SNR can be related as:

\[
SNR = \frac{(U - 1)\sqrt{N} + \sqrt{2UQ^{-1}(P_f)} - \sqrt{2Q^{-1}(P_d)}}{\sqrt{2Q^{-1}(P_d)} + \sqrt{N}} 
\]

(3.3)

In the large observation window the relation between these two can be shown as:

\[
SNR = U - 1 + \frac{\sqrt{2UQ^{-1}(P_f)} - \sqrt{2Q^{-1}(P_d)}}{\sqrt{2Q^{-1}(P_d)} + \sqrt{N}} 
\]

\[
= U - 1 + Q\left(\sqrt{\frac{1}{N}}\right) \geq (SNR)_{\text{min}} 
\]

(3.4)

One of the common and general methods for spectrum sensing is an energy detection method [172]. It calculates the power, in terms of energy contained in the signal and compared with a threshold to detect the presence or absence of PU [171, 191]. Signal and noise variance is one of the important parameters while calculating the energy from the band of interest.

In the next section, implementing the test statistics of the conventional energy detection technique of the spectrum sensed from USRP and then the Welch’s Power Spectral is modified from the conventional Welch’s Cross Power Spectral [190, 182] by squaring the \( M \)th segments of
the Periodogram. CWPS method is proposed to be applied to a signal after sensing through NI USRP-2920. Center frequency has been set at Frequency Modulation (FM) band (93.5 MHz) and Ultra High Frequency (UHF) band (845 MHz) and the signal is passed through demodulator and De-emphasis filter. Known modulation technique makes our demodulation technique fixed. The experimental setup of NI USRP-2920 for signal detection between 93.5 MHz and 98.5 MHz has been shown in Fig 3.1. The results of the USRP are then studied and analyzed in the later sections of the chapter.

Fig 3.1 NI USRP-2920 experimental setup

After analyzing the PSD, a threshold has been set at -80 dBm and received signal is compared to this threshold to find out the ED performance. After implementing ED, the behavior of $P_d$ and $P_f$ is shown in the Fig 3.2 and Fig 3.3 at two different frequencies of 93.5 MHz (FM band) and 845 MHz (UHF band) respectively.
The $P_d$ is calculated using the comparison of the signal PSD and the given threshold (non-cooperative). $P_d$ increases if a signal more than the given threshold exists and vice versa for $P_f$ [175]. We can clearly see that there occurs some blank $P_f$ in between and that’s the target areas [164] for the CR transmitter to work based on the $P_d$ vs. $P_f$ performance.
The proposed method is implemented and tested for high data rate CR transmitter using error correction methods. In the proposed case, we have considered SNR as 10 dB and Rayleigh channel and noise variance in AWGN as 0.5. Here $P_f$, as shown in the Fig 3.3 is having some notches which explain the vacant spectrum and thus we can accommodate a CR user or SU in that place. But not only the power level, but also the BER needs to be considered for accommodating CR user in the vacant band. Fig 3.4 shows the basic block structure of an ED to be implemented in cognitive transmitter [179].

$$P_d = P(Y > \lambda / H_i) = Q_m(\sqrt{2y}, \sqrt{\lambda})$$  \hspace{1cm} (3.5)  \\
$$P_f = P(Y > \lambda / H_0) = \Gamma(m, \lambda / 2) / \Gamma(m)$$  \hspace{1cm} (3.6)  \\
$$P_m = 1 - P_d$$  \hspace{1cm} (3.7)

Here, $P_m$ is the probability of miss detection, $Y$ is the SNR, $m = TW$ is the (observation/sensing) time bandwidth product, $Q_m(.)$ is the generalized Marcum Q-function, whereas $\Gamma(.)$ and $\Gamma(\ldots)$ are denoting complete and incomplete gamma functions.

The sampling frequency is changed to 200 MHz and number of samples of the filtered output as 2048. Fig 3.5 shows the Periodogram illustrating the frequency elements from 93.5 MHz to 98.5 MHz.
3.1.2 $P_d$ vs $P_f$ (Theoretical)
In theoretical $P_d$ vs $P_f$ as shown in Fig 3.5, inverse $Q$ function is used to determine the threshold value and it is given by:

$$\text{thresh} = \left( Q^{-1}(P_f - t) / \sqrt{L} \right) + 1$$

(3.8)

Here, $P_f - t$ is $P_f$ at a particular threshold value, $L$ is the number of samples.

Inverse $Q$ function is denoted as:

$$Q^{-1}(x) = \sqrt{2} \text{erf}^{-1}(1 - 2x)$$

(3.9)

The $P_d$ at particular threshold value can be written as:

$$P_d \_\text{the} = Q((\text{thresh} - (\text{snr} \_t + 1) \times (\text{snr} \_t + 1)))$$

(3.10)

Here, it is calculated using the $Q$ function.

$Q$ function is denoted as:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{u^2}{2}\right) du$$

(3.11)
Fig 3.6 $P_d$ vs. $P_f$ (theoretical)

$P_d$ vs $P_f$ is plotted in Periodogram in Fig 3.6. The signal acquired by USRP (NI-2920) has been passed through low pass filter and re-sampled at 15 KHz. The interpolation factor, decimation factor and antenna gain have been kept at 3, 2 and 20dB, respectively.

The output graph is shown which shows the theoretical comparison of $P_d$ and $P_f$. Comparing the simulated and theoretical graphs it can be seen that there is a smooth increase in $P_d$ as $P_f$ increases due to averaging of observations (cooperative sensing).

In section 3.3, CWPS is estimated from the conventional Welch’s Cross Power Spectral Density [183] using the conventional energy detection technique of the sensed spectrum from NI USRP-2920 by squaring the $M^{th}$ segments of the Periodogram. In this case, BER is calculated taking data from CWPS using conventional error function from an input signal with center frequencies at 93.5 MHz and 845 MHz, respectively. Demodulation technique is fixed since the modulation technique is known (Frequency Shift Keying as used in the FM band and Gaussian Minimum Shift Keying (GMSK) for GSM band).
3.3 BER Calculation Based on Conventional Periodogram

3.3.1 Conventional Periodogram
One of the most important applications of communication is a PSD estimation of random or periodic signals. A general analysis of PSD can be done using Periodogram. This method performs the Fourier transform of the autocorrelation estimate to find the power spectrum. The method uses DFT for estimating the frequency components in terms of power.

3.3.1.1 BER Calculation
Varying threshold from -75 dBm to -80 dBm has been used for detection of signal. Theoretical and real-time signal BER has been computed using Eq. (3.14) and Eq. (3.29). It can be observed that as the threshold changes, there is a change in BER. This is because of averaging of threshold in cooperative sensing results in an increase in BER. For calculation of BER, considering cooperative sensing for $K$ continuous channels.

\[
Q_c = \Pr(\bar{H}_1|H_0) = \sum_{l=0}^{K} P_f^l (1 - P_f)^{K-l} \tag{3.12}
\]

\[
Q_m = \Pr(\bar{H}_0|H_1) = \sum_{l=0}^{K} P_d^l (1 - P_d)^{K-l} \tag{3.13}
\]

Considering here the Binary FSK modulation technique, the BER can be written as:

\[
P_{BER} = Q\left(\sqrt{\frac{E_b}{N_0}}\right) = \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right) \tag{3.14}
\]

where, $N_0$ is the noise variance and symbol or total energy $E_s=E_b\log_2M$. $E_b$ and $M$ are per bit energy and $M$ constellation respectively. As shown in the Fig 3.7 below, the error rates are getting changed as we are increasing the value of threshold for multiple channels. Thus averaging of threshold is increasing the error rates consistently. The threshold scale has been taken along x-axis as 10 is -75 dBm and scaling the other values accordingly on the graph.

In this case, the performance of detection is analyzed by taking 4000 elements of the PSD obtained. In the Eq. (3.29), it has been simulated with change of noise variance and keeping input signal variance constant at 0.5.
Thus, by adding a random variable to the SNR, makes the noise variance normalized. A random variable is taken to be uniformly distributed between [-1, 1] which is being added to the SNR input. The noise uncertainty becomes ±1 dB (maximum) with respect to the signal variance.

3.4 Analysis of BER Based on CWPS with Hamming Window

The Welch method of finding power spectral density is finding out the power of a signal at different frequencies. The Welch’s method is an improved version of the Periodogram and Bartlett’s method. The method reduces noise in the estimated power spectral density by reducing the frequency resolution. This is the most common method used for PSD estimation on noisy environment in real time applications. The general method of Welch’s PSD estimation includes the splitting of overlapping segments and these overlapping segments are windowed using different windowing technique(s). This improves the lack of ensemble averaging as compared with Periodogram. Thus the method is also known as modified Periodogram.

The idea of CWPS comes from the uniqueness of WPSD. In the proposed method the normalized variance is kept constant to maintain and improve the variability product of resolution. In this method, the data segments are being varied continuously based on the number
of channels being sensed. The number of samples has been kept constant, while the coefficient of convergence is getting increased for each channel. The implementation is applied therefore for high data rates with error correcting codes.

The Welch’s PSD is applied by splitting the signal into 2048 overlapping segments having length of 3999 samples. These overlapping segments are then windowed after the data is split up into overlapping segments, all the 2048 data segments have a window applied to them in the time domain.

As per Welch’s method for calculation of cross power spectral density based on Bartlett’s method and Hamming window:

\[
x(n) = \begin{cases} 
x(n + iD), & n = 0,1,2,...,M-1 \\
i = 0,1,2,...,L-1 &
\end{cases}
\]  

(3.15)

Where, \( iD \) is the starting point for the \( i^{th} \) sequence. If \( D = M \), the segment does not overlap and the \( L \) of data sequence is identical to the data segment of Bartlett method.

To prior the computing the Periodogram data segments are windowed in Welch’s method

\[
P_{xx}^i(f) = \frac{1}{MU} \sum_{k=0}^{M-1} x(n)w(n)e^{-j2\pi kl} \quad l = 0,1,...,L-1
\]  

(3.16)

Where, \( U \) is a normalization factor for power, and can be written as:

\[
U = \frac{1}{M} \sum_{n=0}^{M-1} w^2(n)
\]  

(3.17)

The Welch power spectrum estimate

\[
P_{xx} = \frac{1}{L} \sum_{i=0}^{L-1} P_{xx}^i(f)
\]  

(3.18)

is the average of modified Periodogram mean value of Welch estimate
\[ E[P^w_{xx}(f)] = \frac{1}{L} \sum_{i=0}^{L-1} E[P^i_{xx}(f)] \] (3.19)

The improved BER is calculated using the modified Welch’s equation which is derived in the Appendix I. The comparison between the earlier BER in Fig 3.7 and Fig 3.10 shows an improvement in the error rates.
Fig 3.9 Converged Welch’s PSD plot with a hamming window at 845 MHz

Fig 3.10 BER of Converged Welch’s PSD

Thus, by applying CWPS, the signal resolution is converging with respect to the constant phase from 0 to 90° as taken from Eq. (3.29).
The modified Hamming window considers that windows are triangular in nature which helps in the proper convergence of the resolution of the PSD. Here the hamming window is designed with variable widths of the sample as per the real time sensing output. The variation of the width of the sample with modified Hamming window analyzes the real-time sensing outputs for further proposed operations. As shown in Fig 3.10, the plot of CWPS analysis, this shows a drastic improvement in PSD as compared with the earlier Periodogram technique. The corresponding BER shows an improved error rate as when found out during the conventional method.

3.5 Conclusion and Discussion
The CWPS illustrates an improved performance in BER while assuming a constant SNR and varying threshold. Moreover the SUs can be implemented based on the $P_d$ and $P_f$ with improved BER. A novel approach has been implemented by considering the fixed effective radius of the network and by assuming the symbol energy as total energy while calculating the BER. A sample of FM and UHF band signal has been tested using NI-USRP (2920) and analyzed the $P_d$ and $P_f$ by implementing ED. Further, it has been compared with the Welch’s conventional Periodogram which was modified during the analysis of BER. The nature of the CWPS output is observed and it is found to be an improved version of conventional Welch’s Power Spectral density.

This work assumes SU’s activity is based on constant power level and constant SNR with an implementation of the ED technique on their receiver’s side. This chapter also considers the AWGN channel for sensing and other simulation operations. This work is extended by performing the source detection and applying the same for cyclostationarity based sensing for comparison in Chapter 6.