Acknowledgement

“When we do the best that we can, we never know what miracle is wrought in our life or in the life of others.”

Hellen Keller.

I think the emotions of Hellen Keller are true up to a great extent because to make a play successful on the stage, a lot of work is done behind the curtains by many people. The most pleasurable aspect of writing a thesis is that it provides the opportunity to express most heartfelt gratitude to all the persons who have been sources of encouragement and enlightenment. The existence of my research journey wouldn’t have come to an end if my near and dear ones would not have supported me during, the tough journey of its completion.

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I am grateful to my family, who always taught me “Never bend your head, hold it high, look the world straight in the eye.” never let me even thinking of putting down my foot to not leave this in between.

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The chain of my gratitude would be definitely incomplete if I forget to thank the almighty God, the premier mover.

I dedicate this thesis to my Father.

Deepti
(Research Scholar)
Banasthali Vidyapith
Preface

This thesis contains six chapters beginning with the introduction which deals with the literature review and the methods which have been used as tools in this thesis.

In Chapter 2, the $L$-dual of a Finsler space with special $(\alpha, \beta)$-metric is obtained. The importance of $L$-duality is by far not limited to computing the dual of some Finsler fundamental functions but there are so many problems which have been solved by taking the $L$-duals of Finsler spaces.

In Chapter 3, a study is done on the nonholonomic Finsler frame for a class of Generalized Lagrange spaces with $(\alpha, \beta)$-metric and then the two Finsler deformations for the afore said metrics are obtained. Consequently, the nonholonomic Finsler frame for afore said Finsler space is obtained.

In Chapter 4, the conditions for a Douglas space of second kind with $(\alpha, \beta)$-metric to be a Douglas space of second kind under conformal transformation are discussed. Further, it is shown that Douglas space of second kind with Matsumoto and generalized Kropina metric is a Douglas space of second kind under conformal transformation.

In Chapter 5, the conditions for a Finsler space $F^n$ with an $(\alpha, \beta)$-metric to be a weakly-Berwald space are discussed. In particular, the conditions for $F^n$ with second approximate Matsumoto metric to be weakly-Berwald space are obtained.

In Chapter 6, a two-dimensional Landsberg space with certain $(\alpha, \beta)$-metrics satisfying some conditions is discussed. First the conditions for a
\( F^n \) with a special \((\alpha, \beta)\)-metric to be a Berwald space are obtained and then the difference vector and the main scalar of \( F^2 \) with the aforesaid metrics are obtained.
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