1. INTRODUCTION

1.1 OBJECTIVE:

A linear program maximizes, or minimizes, a linear objective function subject to a set of linear inequalities and equalities. If the linear objective $c^T x$ (where $x \in \mathbb{R}^n$), is replaced by a ratio of two linear functions, $(c^T x + c_0) / (d^T x + d_0)$, a linear fractional program is obtained. Such program occurs in various operational research models.

The various applications of fractional programming include the following. The cutting stock problem is that of cutting a roll of paper (or a billet of steel etc) into narrower rolls of specified widths, while minimizing the amount of wastage. This can be computed as a linear program, combined with a column generator to construct cutting patterns. However, a Linear Fractional Program arises if the ratio of wastage to useful output is to be minimized. Similar Linear Fractional Programming problems arise in ship scheduling where the ratio of profit per
journey to total journey time is to be maximized, and in arc blending where the cost per unit of output is to be minimized.

It is aimed in developing an algorithm which will reduce the number of iterations and computation time for large scale problems so that the Linear Fractional Programming can be easily applied for decision making problems in industrial and business organisations.

1.2 FORMULATION:

Let n different types of raw material of weights $x_1, x_2, \ldots, x_n$ be supplied to an iron smelter. Each kilogram of raw material $j$ costs $C_j$ rupees. The ingredients in each raw material are silicon, manganese, phosphorous and sulphur. The weights of these ingredients in 1 kilogram of the $j$th raw material are $a_{1j}, a_{2j}, a_{3j}, a_{4j}$ respectively. The cost of the raw material per unit of output is to be minimized subject to upper and lower bounds on the amounts of the four ingredients in the output of cast iron. This leads to a linear fractional program of the following type.

Minimize \[
\frac{\sum_{j=1}^{n} C_j x_j}{\sum_{j=1}^{n} x_j}
\]
subject to $x_j \geq 0$

$$\sum_{j=1}^{n} x_j = 1$$

$$b \leq \sum_{i=1}^{n} a_{ij} x_j \leq B \quad (i = 1, 2, 3, 4)$$

Portfolio optimization model of Ziemba and others [55] and agricultural planning model of Bersanu are some examples for nonlinear fractional programming problem [4].

1.2.1 ALGORITHMS FOR FRACTIONAL PROGRAMS:

A linear fractional programming problem can be solved by the charnes-cooper method [10] by resolving it into two linear programming problems. However, if the problem has an unbounded feasible region, it is possible for some iterate to have $t = 0$, which corresponds to a point at infinity for the given Linear Fractional Program, while still having convergence to a maximum of the linear fractional program. Some variants of the charnes-cooper method may behave differently [19]. For bounded feasible regions, some computational evidence suggests that the Isbell-Marlow method [24] which maximizes a sequence of linear programs may reach an optimum in fewer iterations. But it may fail when the feasible region is unbounded.
Hartmut Wolf has suggested a parametric method to solve linear fractional programming problems [22]. This method not only gives the solution to the problem but also does sensitivity analysis on the problem [23].

Kantiswarup's Fractional Algorithm finds the solution of programming problems with linear fractional functionals without reducing it to linear programming problems. In all these methods it is assumed that the denominator takes only positive values.

1.3 MOTIVATION:

While there exists a variety of solution procedures for Linear Fractional Programming problems these methods and models had not been applied to actual data. The aim of this work is to find a method for the solution of Linear Fractional Programming problems in lesser number of iterations and at the same time to develop a software package so that it can be applied to actual data.

1.4 CONCLUSION:

In this chapter the use of Linear Fractional Programming Problems and the various solution techniques are generally outlined and the reasons for seeking improvement are discussed. The motivation for this work is outlined.