CHAPTER VI

GROUNDWATER MODELING

6.1 INTRODUCTION

Groundwater model is a simplified representation of the actual hydrogeological system. In order to simulate groundwater flow in an aquifer, analytical or numerical modeling techniques are used to approximate aquifer conditions. It is a methodology for the analysis of mechanisms and controls of groundwater systems and for the evaluation of models. It is powerful tools for understanding the mechanisms of compositions. Models can help in orderly interpretation of the data describing a system and provide a quantitative indicator for precise evaluation where financial resources are limited for additional field data collection. Finally models can also been used in a predictive way for assessing the input of an impaired stress on the system.

Flow modeling studies are usually under taken to determine response of an aquifer to pumping, injection or recharge stresses. Models try to circumvent these difficulties by either (i) Simulating the behaviour of the aquifer systems on a small scale; (ii) Using simple assumptions or numerical approximations to the governing equations.

An attempt has been made in this study to (i) evaluate groundwater system of lower Gundar basin by constructing and operating a mathematical model, which assumes the appearance of the actual behaviour, and (ii) to assess the groundwater available in storage under existing conditions.
6.2 Major Features in Groundwater Modeling

Formulation, Approximation, Computation and Application are the four basic steps involved in mathematical modeling.

1. Formulation refers to the process of deriving or selecting the basic equation(s) governing the flow of groundwater in the system, with the domain specifications and initial boundary conditions.

2. Approximation refers to the selection of numerical method, which can be used to solve the system of algebraic equations. Finite difference (FD), Finite Element (FE) and Integrated Finite difference (IFD) methods are some of the widely used concepts in modeling the groundwater systems.

3. The most important and troublesome of these steps is the computational part, which refers to the process of obtaining a solution to the equations. This usually requires the use of a digital computer and a method for coding the steps.

4. The application part of the groundwater modeling includes calibration on history matching of the observed and simulated heads, sensitivity analysis and prediction. Sensitivity tests are used to show how the model reacts to various extreme values of transmissivity, storage coefficient of recharge/discharge volumes.

Walton (1962) presented the analytical methods of aquifer evaluation, which formed the basis for all the later orientations towards the numerical approaches. Prickett (1975) gave a comprehensive outlook on the modeling
techniques for groundwater evaluation by properly explaining the equations of flow, given an overview of the types of analog and numerical models used prior to 1975. Balasubramanian (2001) provided an overview of groundwater models.

6.3 GROUNDWATER MODEL OF THE STUDY AREA

The groundwater model of lower Gundar basin aquifer has been developed using standard finite difference techniques and has been simulated to make a fair assessment of the groundwater in storage. The detailed mathematical description of the techniques employed, model designed, program used and the quantification obtained are discussed in the following sections.

6.4 MATHEMATICAL DESCRIPTIONS

The digital simulation model used in this study computes the head changes in the aquifer system at any specified time as a function of the various aquifer characteristics. These changes are obtained by solving the equation of flow through porous media. Although numerous techniques exist, a digital model is necessary to treat the complex boundary conditions, aquifer heterogeneity and scale of the aquifer systems. The governing equation of groundwater flow and its solution strategies for the aquifer of lower Gundar basin under study are given in this section.
6.5 Governing Equation of Groundwater Flow

The Partial Differential Equation (PDE) governing the flow of groundwater in non-heterogeneous, anisotropic aquifer, where the vertical components of flow are small to be neglected (Pricket et al., 1976, Rushton et al., 1983) is

\[
\frac{\partial}{\partial x} (T_x \cdot \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y} (T_y \cdot \frac{\partial h}{\partial y}) = S \cdot \frac{\partial h}{\partial t} + \omega (x, y, t) \tag{6.1}
\]

where,

- \( T_x \) and \( T_y \) are the Transmissivity in \( x \) and \( y \) co-ordinates (\( L^2/T \)).
- \( S \) is the approximate storage coefficient (dimensionless).
- \( h \) is the hydraulic head (\( L \)).
- \( t \) is the time increment (\( T \)) and
- \( \omega \) is the fraction of recharge or discharge (\( L/T \)) with reference to space and time.

The general solution of equation 6.1 does not exist. The approximate solution of the equation is obtained by numerical method like finite difference or finite element approaches. Finite difference method has been adopted in this study.

6.6 Finite Difference Method

Finite difference method is the process of replacing the partial derivatives at a point by the ratio of the changes in variables. Both time and space variables are treated as discrete parameters. The values of all variables that best represent the conditions of the grids are assigned to nodes, which are located at the center of the sub regions. The distance between the nodes is represented by \( \Delta x \) and \( \Delta y \) in \( x \) and \( y \) directions. The transient flow conditions depicting the head
distribution at (k) and (k+1) time intervals. Subscripts J and I are used to index the rows and columns of nodes and k is the index of time.

6.7 SOLUTION STRATEGIES

The Finite difference form of the general flow equation has been derived by many works (Remson et al., 1971). The following formulations are popular:

1. Formed difference or explicit method
2. Backward difference or implicit method
3. Alternating direction Explicit (ADE) and
4. Iterative alternating direction implicit (IADI) method.

In the backward difference method form the known values of heads at the previous time level (k-1) are considered for approximating time derivatives; while in the forward — difference scheme, the same is done by taking into amount the difference between the known head values of the present time (k) and the unknown values of head at the next time level (k+1).

6.8 FORWARD DIFFERENCE (EXPLICIT) FORMULATION

This is one of the earliest methods used for regional groundwater problems. The finite difference form of the partial differential equation 1 is

\[
\frac{2}{\Delta x} \cdot T_x(I,J) / \Delta x \left[ H(I, J+1, k) - H(I,J,k) \right] \\
+ T_x(I,J-1) / \Delta x \left[ H(I, J-1, k) - H(I,J,k) \right] \\
+ \frac{2}{\Delta y} \cdot T_y(I,J) / \Delta y \left[ H(I+1, J,k) - H(I,J,k) \right] \\
+ T_y(I-1, J) / \Delta x \left[ H(I-1, J,k) - H(I,J,k) \right] \\
S(I,J) \left[ H(I,J,k+1) - H(I,J,k) / \Delta t \right] \pm \omega(I,J) \quad 6.2
\]
where,

$T_x$ and $T_y$ are average transmissivities in x and y direction between
the nodes (I,J) and (I, J+1) and nodes (I, J) and (I+1, J)
respectively ($L^2/T$),

$H(I,J,k)$ is the head (L) at (k) time level.

$H(I,J,k+1)$ is the head (L) at (k+1) time level;

$H(I,J)$ is the storage coefficient of node (I,J);

$\omega (I,J)$ is the fraction of recharge or discharge ($L/T$) and
$t$ is the time increment (T).

It is convenient to write Eq. 6.2 as

$$AH(I,J+1,k) + BH(I-1,J,k) + CH(IJ-1,k)$$
$$+ DH(I+1,J,K) - (A+B+C+D) H(IJ,K)$$
$$= F [H(IJ,K+1) - H(IJ,k)] + \omega (I,J) \quad \dots \quad 6.3$$

where

$$A = 2T_x(I,J)/2X \Delta x X \Delta x \quad \dots \quad 6.4$$
$$B = 2T_y(I-1,J)/2X \Delta x X \Delta x \quad \dots \quad 6.5$$
$$C = 2T_x(IJ-1)/2X \Delta x X \Delta x \quad \dots \quad 6.6$$
$$D = 2T_y(I,J)/2X \Delta y X \Delta y \quad \dots \quad 6.7$$
$$F = S(I,J)/\Delta t \quad \dots \quad 6.8$$

In order to find out the potential at (k+1) time level, Eq.6.3 can be assumed as,

$$H(I,J,K+1) = [AH(I,J+1,k) + BH(I-1,J,k) + CH(I,J-1,K)$$
$$+ DH(I+1,J,K) (A+B+C+D-F) H(I,J,K) \pm \omega (I,J)/F \quad \dots \quad 6.9$$

As the unknown potential $H(I,J,K+1)$ is written explicitly in terms of the
known heads at the time step (k) and flows $\omega (I,J)$, this is termed as an explicit
method. This method is very straightforward solving only the Eq. 6.9, which
needs only a less comp coding.
6.9 

**BACKWARD DIFFERENCE (IMPLICIT) FORMULATION**

The partial difference expression (PDE) of backward difference (Implicit) formulation of Eq. 1 becomes as (After Rushton et al., 1979).

\[
\frac{\partial}{\partial x} \cdot \frac{\partial h}{\partial x} (T_x (K+1)) + \frac{\partial}{\partial y} \cdot \frac{\partial h}{\partial y} (T_y . K+1 / \partial y) \\
= S \frac{\partial h}{\partial t} \cdot \frac{\partial h}{\partial t} \pm \omega \-------------------------- 6.10
\]

However, writing this infinite difference form using the notations of Eq. 6.1 and Eq. 6.2 gives,

\[
AH (I,J+1, K+1) + BH (I-1, J,K+1) + CH (I,J-1, K+1) + DH (I+1, J, K+1) \\
- (A+B+C+D) H (I,J, K+1) = F | H (I,J,K+1) - H (I,J,K) + \\
\omega (I,J) ------- 6.11
\]

It is not possible to write this in an explicit way to find out head distribution, since this and the equivalent equation of the other node form a set of simulation equation. Solving all the equations at each time step is a time consuming process, which can become manageable only by adopting iterative techniques. The method that has proved to be convenient for groundwater flow problems is the techniques of Successive Over Relationship (SOR).

In describing the SOR method it is helpful to denote a further coefficient E which reads as

\[
E = A + B + C + D + S (I,J) / \Delta t = A + B + C + D + F --------------------- 6.12
\]
Eq. 6.11 can be written in a form similar to that explicit procedure as

\[
H(I,J,k+1) = \left[ AH(I,J+1, K+1) + BH(I+1, J, K+1) + CH(I,J-1, K+1) \\
+ DH(I+1, J, K+1) + FH(I,J,K) \omega(I,J) \right] / E \quad 6.13
\]

This equation allows the direct computation of head at \((K+1)\) time level but it depends on the other heads of the same time step, which in the early stages of the computation, might not have reached their correct values. Therefore, the head distribution given by the use of Eq. 6.13 will be an over relaxation factor \((\omega)\). Remson et al., (1971) and Rushton et al., (1983) introduced to Eq. 6.13 so that the new approximation to the potential is given by

\[
H(I,J,M+1) = H(I,J,K+1, M) \pm \omega \Delta H(I,J,K+1,M+1; M) \quad 6.14
\]

where, \(\Delta H(I,J,K+1,M+1; M)\) is the change in potential between \(M\) and \(M+1\) iterations.

Eq. 6.14 is used at each node in term. As the iterations proceed, convergence to a steady value is achieved. The over relaxation factor normally ranges from 1.2 to 2.0 (Remson et al., 1971). After several trial runs with the computer model, a factor of 1.0 has been taken and used in this study.

In order to test the convergence the error in satisfying the finite difference form of the differential Eq. 6.10 is calculated from equation 6.13 as

\[
\text{Error} = AH(I, J+1, K+1) - BH(I-1, J, K+1) + CH(I,J-1, K+1) + DH(I+1, J, K+1) - EH(I, J, K+1) \\
+ FH(I, J, K) \pm \omega(I,J) \quad 6.15
\]
An error criterion of 0.01 has been used in the present analysis. Iteration should continue in the process till the computed error value becomes less than this amount.

6.10 Flow Program Adopted for the Study Area

A computer program – FLOW that can perform the computations of the backward (Implicit) difference method with built in provision to compute nodal yields, flows permeability has been used. The detailed program listing in BASIC with simplified steps and definition of variables has been given in Appendix - B. The program can take the inputs only in metric units.

Each step of the computation has 61 days time steps and gives the result of water level distribution at the first day of every alternate month. The nodal transmissivity values are being updated at all the steps to adopt transient flow conditions.

In order to avoid reading voluminous flow data from month to month and year to year, multiplication factor has been included in the program, which can increase / decrease the flow data according to user’s stress.

The difference between predicted and observed head values have been taken as output at all the steps of the model, along with nodal total flow.

6.11 Model Design

6.11.1 Finite Difference Network

A finite difference grid has been superposed over the base map of the area of study. The aquifer is divided into values having dimension in $\Delta x$, $\Delta y$, where $\Delta h$
is the thickness of aquifer. The position of nodes can be identified by their column and row integer subscripts, i.e. J and I starting from the left hand top corner. The network of this model contains 38 rows and 30 columns.

6.11.2 Nodal Spacing

The study area covers an aerial extent of 970 Km² and considering the limitations of computer time and data processing.

6.11.3 Basic Assumptions

It is very essential in these techniques that the head computed for a node containing a pumping well is not the head of the well, but rather it represents an average hydraulic head for entire node block. The basic assumptions of the present model for calculation of the head in a node are as follows:

- The aquifer is unconfined in major part of the model is bounded with an impermeable basement at depths.
- Groundwater obeys Darcy's law and flow is two-dimensional
- Transmissivity in the system is not constant with space and time.
- The interval of normal with the flow has a fairly narrow range of hydraulic conductivity is very small and have permeability in the vertical sector is assumed to be constant.
6.12 **INPUT DATA**

6.12.1 **Water Level**

The water level above mean sea level (amsl) data of 1992 forms the initial data used as input to the node. Additional water level arrays have been prepared in order to calculate the discharge and recharge volume. Figure 6.1 shows the initial water level data used for the simulation.

6.12.2 **Transmissivity**

Initial nodal Transmissivity value for the model has been taken from the results of the dug well pump tests. Figure 6.2 shows the data matrix used for modeling.

6.12.3 **Aquifer Basement**

Using the results of geoelectrical prospecting the aquifer basement above mean sea level (msl) has been determined and figure 6.3 shows the data used for this purpose.

6.13 **INFLOW AND OUTFLOWS**

The nodal inflow and outflow volumes have been controlled through an external constant and the amount of recharge is expected only during the last (Time-step) phase of the year. And all other time-steps have only nodal discharges. The nodal flows are computed and taken as output at every simulation.
FIG. 6.1 INITIAL WATER TABLE DATA FOR WATERSHED I - B - V, BOLIVIA - SEE APPENDIX B
FIG. 6.2 INITIAL TRANSMISSIVITY DATA AND MATRIX USED FOR MODELING IN LOWER KULIFAH BASIN
6.14 RESULTS

The results of the simulation runs (61 days cycle) for 10 years of calibration and two years of prediction have been shown in the calibration graphs given in figures 6.4 to 6.7 and output is given in Appendix – C.

Any model can be used for predictive simulations when there is a good match between observed and computed heads during the calibration periods. There is a remarkable match among these variables and the computed heads show the long-term trend approximately. This model could be used for practical simulations for analyzing any future stress to be imposed or anticipated. The model is suitable for all practical applications.
Fig. 6.4 Observed Water Levels Vs Predicted Water Levels - Sirampur

Fig. 6.5 Observed Water Level Vs Predicted Water Level - Muthuramalingapuram
Fig. 6.6  Observed Water Level Vs Predicted Water Level - Pudukottai

Fig. 6.7  Observed Water Level Vs Predicted Water Level, Chatram