CHAPTER II

REACTIVE POWER MARGINS WITH RESPECT TO VOLTAGE COLLAPSE

2.1 Introduction:

As a result of the major voltage collapse incidents experienced in the recent past, there has been a strong incentive to detect potential voltage collapse problems through appropriate indices.

Conceptually, most (but not all) steady-state indices: (i) consider some perturbation in the load; (ii) rely on well-known Q-V curves; (iii) monitor some quantity whose variation when approaching the collapse point can be predicted.

Sensitivity-type indices consider infinitesimal load perturbation and use system linearisation through the Jacobian matrix. Methods based on Jacobian eigen value or determinant analysis conceptually belong to the same category.

As pointed out by several researchers, the weakness of these indices result from the major non-linearities imposed by generator reactive power limitations. The latter strongly affect the shape of the voltage versus load curve, where each discontinuity in the derivative corresponds to the loss of voltage control by one generator (and hence to a significant change in the Jacobian). Sensitivity based indices always characterise a configuration whose collapse point is beyond the real one (except very close to collapse). In practice,
this may lead to poor prediction capabilities or, equivalently, it can make the choice of a security threshold difficult.

Methods exploiting properties related to multiple load flow solutions use, in some sense, the distance between the upper and lower parts of the Q-V curves as an indicator of the critical point proximity. Although these methods allow a deeper understanding of the nature of the load flows solutions, the effect of generator reactive limits still deserves further investigation.

Margin-type indices, as used in [30], aim at directly locating the operating point with respect to a critical one, corresponding to some way of increasing the load. Generator limits are taken into account. The resulting margin \( (Q^* - Q^0) \) is an explicit measure of the distance to collapse in terms of meaningful quantities from an operator's viewpoint. In addition, unlike some other indices, this margin is still defined beyond collapse, where it merely takes on a negative value.

The problem is obviously to determine the corresponding collapse point. It is well known that repeated load flow calculations are not adequate for this purpose; indeed, besides being heavy, usual load flow algorithms have no solution beyond collapse and unreliable convergence properties around it (owing to the singularity of the Jacobian). To circumvent this problem, several methods have been proposed, of which two promising ones are the successive linearisation and the approach using optimisation [38].
Simply stated, the principle of an optimisation-based method is to obtain the collapse point as the maximum of the load expressed as a function of network voltages, while taking into account the multi-dimensional aspect and the generator limits.

2.2 Reactive power margin:

There are (at least) two basic viewpoints as regards the way to define and use the above margins:

(i) Either an exact network loadability limit is sought, i.e., the value of the maximum load, interface flow, etc., that can be tolerated without causing voltage instability. This viewpoint probably prevails in systems where the dread source of voltage collapse is the load increase itself. In such a case, as an accurate limit has to be obtained, the load variation should be as realistic as possible (therefore involving both the active & reactive powers).

(ii) Or one seeks a measure of the system robustness with respect to incidents likely to cause voltage collapse. Within the context of static indices, this measure should faithfully reflect the distance between the current operating point and the limit of existence of a load flow solution.

The reactive power margins considered in [30] falls within the second category. It is defined as the difference between the maximum reactive load that can be consumed in a
given area and the corresponding load at the operating point where the robustness is being analysed.

What is the meaning of increasing reactive power only? First, one should avoid the confusion resulting from the fact that this margin involves a load increase – as in (i) above – while it does not intend to give a network loadability limit. Rather, the reactive load increase should be considered as a large perturbation applied to the system to appraise its robustness. Second, it is clear that reactive power plays an important role, owning to the reactive nature of the network, to the generator limitations and its effect on the voltage profile, already under normal conditions. Furthermore, the reactive power margin implicitly reflects the system stress imposed by the active power transfers, since the margin is a reserve after imposing the active load to be satisfied. As an example, if one would (fictitiously) approach collapse through purely active load increases, the reactive margin would tend to zero as well.

Finally, using reactive power only: (a) makes the margin easier to compute and (b) does not require active power generation rescheduling, hence avoiding possibly questionable assumptions. For instance, meeting the active demand increase might require to (fictitiously) commit new units and hence to modify the system configuration being analysed.
2.3 **Electrical Decoupling:**

The sought margin being of pure reactive nature, some form of electrical decoupling is advisable to make the procedure more efficient. The decoupling allows the active power flows to be taken into account, while avoiding to treat active power – phase angle relationships explicitly.

A classical decoupled scheme would result in solving alternate sequences of active / reactive sub-problems. The active one would merely amount to adjusting phase angles and therefore appear to be a computational burden. On the other hand, considering constant phase angles would amount to assuming a strong electrical decoupling; this is questionable in view of the following two facts: (a) unlike to many network calculations, the collapse point determination generally involves voltages far below 1.0 per unit; (b) in some systems, taking into account the full electrical distance between loads & generators requires to include sub-transmission networks whose lines can exhibit low X/R ratios.

In view of these findings, the CRIC transformations proposed in [39] appear to be an attractive compromise between accuracy and efficiency. This transformation is roughly detailed in Appendix-1, in brief, it consists of:

1. eliminating from the load flow equations, the individual branch phase angle differences. For this purpose, the active power flow relationship, symbolically written as:
\[ P_{ij} = f_P (V_i, V_j, \theta_i - \theta_j) \]  \hspace{1cm} (2.1)

(where \( V \) denotes voltage magnitude & \( \theta \) denotes phase angle)

is used to obtain:

\[ \theta_i - \theta_j = \phi (V_i, V_j, P_{ij}) \]  \hspace{1cm} (2.2)

which is substituted into the corresponding reactive power flow equation:

\[ Q_{ij} = f_Q (V_i, V_j, \theta_i - \theta_j) = f_Q \{ V_i, V_j, \phi (V_i, V_j, P_{ij}) \} \]  \hspace{1cm} (2.3)

to yield:

\[ Q_{ij} = F_Q (V_i, V_j, P_{ij}) \]  \hspace{1cm} (2.4)

(2) assuming constant active power flows in all network branches, the \( P_{ij} \)'s becoming fixed parameters, reactive power flows involve voltage magnitudes only. The same holds for the reactive injection at any bus – say the \( i^{th} \) one – which can be denoted by:

\[ Q_i(V) = \sum_j F_Q (V_i, V_j, P_{ij}) \]  \hspace{1cm} (2.5)

where \( V \) is the vector of voltage magnitudes.

Fixing individual active power flows is an approximation in the sense that when voltages change, the active power flows are somewhat redistributed (their nodal sums remaining unchanged); the active power losses also change. However neglecting these variations only marginally influences the collapse point.
It should also be noted that equation (2.5) fully retains the sparse structure of standard load flow equations. For instance, the Jacobian matrix associated with equation (2.5) offers the same sparsity pattern as the $B''$ matrix of the fast decoupled load flow [40].

### 2.4 The optimisation zone:

Generally, the maximum reactive load will be determined in a given zone $z$ of the network. Let $j$ be any node in $z$. The reactive power consumption at this bus is denoted by $Q^0_j$ in the base case and by $Q'_j$ at the collapse point. The reactive power margin ($Q$) is defined as:

$$Q = \sum_{j \in z} Q'_j - \sum_{j \in z} Q^0_j$$

--- (2.6)

Moreover, each bus in $z$ is required to participate at the rate of an $\alpha_j$ coefficient, to the global load increase. This is expressed by:

$$Q'_j - Q^0_j = \alpha_j Q ; \; j \in z$$

--- (2.7)

The $\alpha_j$'s are called participation coefficients of the buses (or the loads) of $z$. By adding all the equations of the type (2.7), we get:

$$\sum_{j \in z} \alpha_j = 1$$

--- (2.8)
The choice of the $\alpha_j$'s and hence the $z$ zone obviously influences the value of the margin.

However, even when applied to a large zone, the proposed margin is said to have the ability to detect local weaknesses.

2.5 Problem formulation:

Let $N$ be the total number of buses in the system, $N'$ be the number of load buses ($N' < N$) and $(N-N')$ be the number of generator buses.

Let $\mathbf{V}$ be the $N'$- dimensional vector of voltage magnitudes at load buses.

Each load bus is associated with one reactive power balance relationship of the type:

$$Q_j(\mathbf{V}) = q_j(v_j) - \alpha_j Q; \quad j = 1,2, ..., N'$$  \hspace{2cm} (2.9)

where:

$\alpha_j$, $Q$ & $Q_j$ are already defined symbols; and $q_j(v_j)$ is reactive power injection at load bus $j$ as a function of its bus voltage magnitude $v_j$.

Examples of load buses are as follows:

Case A: Non-participating load buses: correspond to $\alpha_j = 0$ and $q_j(v_j) = Q^o_j$ (base case value), $j \notin z$

Case B: Participating load buses: correspond to $\alpha_j \neq 0$ and $q_j(v_j) = Q^o_j$ (base case value), $j \in z$
Case C: Buses fed by generators under rotor current limit: correspond to $a_j = 0$ and $q_j(v_j)$ obtained from the machine equivalent scheme, under constant excitation.

Case D: Buses fed by generators under reactive power limit: correspond to $a_j = 0$ and $q_j(v_j) = Q_{\text{max}}$ (constant value)

The reactive power limitations of generators play an important role in voltage collapse and any method which does not take them into account will yield optimistic and unacceptable results. Cases C & D correspond to generators switched under limit in the course of maximising the load. Case C corresponds to the action of a rotor current limitation device while Case D represent the operator’s action on the voltage set point to decrease the reactive power production.

These limitations can be formally expressed by inequality constraints of the type:

$$g_k [Q_k (v), v_k] \leq 0$$

(2.10)

where $k$ is the number of generator bus of concern. In particular case of a reactive power limit (Case D), the above inequality reduces to:

$$Q_k (v) - Q_{k, \text{max}} \leq 0$$

(2.11)

All the needed relationships are now available to formulate the optimisation problem. The sought reactive power margin is the solution of the $Q$ variable of the following problem:
Maximise $Q$ 

subject to: $Q_j (V_j) - q_j (v_j) + \alpha_j Q = 0; \ j = 1, 2, ..., N' \quad (2.13)$

and $g_k [Q_k (V_k), v_k] \leq 0; \ k \in \text{generator buses} \quad (2.14)$

The design variables of this problem are the components of $V$ and $Q$ variable itself.

2.6 Solution Approach:

The solution approach proposed in [30] appears to be little cumbersome for the problem formulation described by the set of equations (2.12 to 2.14). The inequality constraints were considered separately and a two-stage approach was presented. It is felt that the total problem with both the equality and inequality constraints should be dealt together within the optimisation region to achieve effective solution. Motivated by this, solution approach using Interior Penalty Function (IPF) method is proposed in [31].

2.7 Solution process by IPF method [41]:

The general formulation of the Interior Penalty Function (IPF) method and the solution process as described in [41] is indicated in Appendix-2.

The objective function $Q$ is to be maximised. Converting the maximisation problem into minimisation by multiplying each term with (-1), we have:

Minimise: $-Q$

subject to: $Q_j (V_j) - q_j (v_j) + \alpha_j Q = 0; \ j = 1, 2, ..., N' \quad (2.15)$

and $g_k [Q_k (V_k), v_k] \leq 0; \ k \in \text{generator buses}$
By applying IPF method, the unconstrained minimisation problem takes the form:

$$\Phi_k = -Q - \tau \sum_k \left( \frac{1}{\varnothing_k} [Q_k(\mathbf{V}), \nu_k] + \frac{1}{\sqrt{\tau}} \sum_{j=1}^{N'} [Q_j(\mathbf{V}) - q_j(\nu_j) + \alpha_j Q] \right)^2$$

The equation (2.16) is solved using the IPF algorithm to obtain the optimal reactive power margin Q. Here both the equality and inequality constraints are taken unlike in [30] where equality constraints are solved first and then the inequality constraints.

2.8 **Program development:**

A computer program for IPF method employing Powell's pattern search method [42] as a major sub-routine, is utilised by suitably customising the problem under consideration for obtaining solution of the un-constrained non-linear optimisation problem.

2.9 **Sample case study:**

2.9.1 **Base case sample system:**

The overall method is illustrated on a sample 4-bus system shown in Figure-2.1(a). Corresponding one-line diagram, per unit values and load flow data are given in Figure-2.1(b). Set \( V_B = V_C = 1.0 \) p.u. (flat start). A maximum reactive power production for generator D \( (Q_b^{\text{max}}) \) is fixed as 2.2 p.u.
FIGURE 2.1(a) SAMPLE FOUR NODE SYSTEM

FIGURE 2.1(b) SYSTEM ONE LINE DIAGRAM
The load to be optimised is the one at bus C, with a base case $Q^\text{BC} = 0.23$ p.u. and a participation co-efficient $\alpha = 1$. Using the active-reactive power decoupling criteria, we can derive the following equations [2-1, 2-3]:

\[
Q_C = 16.1667 V_C^2 - \sqrt{(277.7778 V_B^2 V_C^2 - 9)} \\
Q_{BC} = 16.1667 V_B^2 - \sqrt{(277.7778 V_B^2 V_C^2 - 9)} \\
Q_{BD} = 28.5714 V_D^2 - \sqrt{(816.3265 V_B^2 V_D^2 - 1)} \\
Q_{BA} = 4 V_B^2 - \sqrt{(16 V_B^2 V_A^2 - 4)} \\
Q_D = 28.5714 V_D^2 - \sqrt{(816.3265 V_B^2 V_D^2 - 1)}
\]

According to the procedure described in Section 2.7, the Interior Penalty Function (IPF) method is applied by keeping the voltages constant for those generators which do not have any reactive power limitation. In the sample problem, generator–A does not have any constraint on it and hence controlled generator voltage $V_A$ does not participate in the optimisation problem. $V_A$ is simply set to its base case value.

The corresponding optimisation problem (equation 2.15) amounts to:

Minimise : $-Q$

Subject to :

\[
16.1667 V_C^2 - \sqrt{(277.7778 V_B^2 V_C^2 - 9)} + 0.23 + Q = 0 \\
49.2381 V_B^2 - \sqrt{(816.3265 V_B^2 V_D^2 - 1)} - \sqrt{(18.6624 V_B^2 - 4)} - \sqrt{(277.7778 V_B^2 V_C^2 - 9)} = 0 \\
28.5714 V_D^2 - \sqrt{(816.3265 V_B^2 V_D^2 - 1)} - 2.2 \leq 0
\]
The unconstrained $\Phi$ function can be written as:

$$
\Phi_k = -Q - n_k \left[ 1/\{28.5714 V^2_B - \sqrt{(816.3265 V^2_B V^2_D - 1) - 2.2} \} 
+ (1/n_k) \left[ 16.1667 V^2_C - \sqrt{(277.7778 V^2_B V^2_C - 9) + 0.23 + Q} \right]^2 
+ (1/n_k) \left[ 49.2381 V^2_B - \sqrt{(816.3265 V^2_B V^2_D - 1) - J(8.6624 V_B - d(277.7778 V^2_B - 9))} \right]^2 \right] 
- \sqrt{(18.6624 V^2_B - 4) - \sqrt{(277.7778 V^2_B V^2_C - 9)}^2} 
\right] \right] 
$$

(2.23)

The initial point $x_i = [V_B, V_C, V_D, Q]$ is taken as $[1.0, 1.0, 1.05, 0.0]$ and the first value of $n_k$ is taken as 1000 and is successively reduced by a factor of $c = 0.1$. A tolerance of 0.01 p.u., between successive values of each variable of the design vector $x$ is enforced. The convergence is obtained for $n_k = 0.00001$. The value of the reactive power margin $Q$ and the voltage at the load bus $C (V_C)$ are tabulated below in Table-2.1.

<table>
<thead>
<tr>
<th>Serial No.</th>
<th>$Q$</th>
<th>$V_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.3033</td>
<td>1.0516</td>
</tr>
</tbody>
</table>

**2.9.2 Case where load is simulated to be far away:**

The effect of load being far away on the reactive power margin is studied here. The following three (3) different cases are considered:

(a) with $X_{BC} = 0.07$ (with $X_{AB} = 0.25$ and $X_{BD} = 0.035$)

(b) with $X_{BC} = 0.08$ (with $X_{AB} = 0.25$ and $X_{BD} = 0.035$)

(c) with $X_{BC} = 0.09$ (with $X_{AB} = 0.25$ and $X_{BD} = 0.035$)
For the above three cases, system equations similar to equations (2.17) to (2.21) are derived and corresponding optimisation problems are solved. The results are tabulated below in Table-2.2.

<table>
<thead>
<tr>
<th>Serial No.</th>
<th>$X_{BC}$</th>
<th>$Q$</th>
<th>$V_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.06</td>
<td>1.3033</td>
<td>1.0516</td>
</tr>
<tr>
<td>2</td>
<td>0.07</td>
<td>1.2219</td>
<td>1.0771</td>
</tr>
<tr>
<td>3</td>
<td>0.08</td>
<td>1.1034</td>
<td>1.1042</td>
</tr>
<tr>
<td>4</td>
<td>0.09</td>
<td>1.0304</td>
<td>1.1354</td>
</tr>
</tbody>
</table>

From the Table-2.2, we observe that the reactive power margin $Q$ decreases as the distance between the load and the generation is increased. The voltages at the load bus are found to increase.

2.9.3 Case where generation is simulated to be far away:

The effect of generation being far away on the reactive power margin is studied here. The following three (3) different cases are considered:

(a) with $X_{AB} = 0.30$ and $X_{BD} = 0.040$ (with $X_{BC} = 0.06$)
(b) with $X_{AB} = 0.35$ and $X_{BD} = 0.045$ (with $X_{BC} = 0.06$)
(c) with $X_{AB} = 0.40$ and $X_{BD} = 0.050$ (with $X_{BC} = 0.06$)

For the above three cases, system equations similar to equations (2.17) to (2.21) are derived and corresponding optimisation problems are solved. The results are tabulated below in Table-2.3.
Table-2.3: Variation of $Q$ and $V_C$ when generation is far away

<table>
<thead>
<tr>
<th>Serial No.</th>
<th>$X_{AB}$</th>
<th>$X_{BD}$</th>
<th>$Q$</th>
<th>$V_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25</td>
<td>0.035</td>
<td>1.3033</td>
<td>1.0516</td>
</tr>
<tr>
<td>2</td>
<td>0.30</td>
<td>0.040</td>
<td>1.2332</td>
<td>1.0580</td>
</tr>
<tr>
<td>3</td>
<td>0.35</td>
<td>0.045</td>
<td>1.1165</td>
<td>1.1811</td>
</tr>
<tr>
<td>4</td>
<td>0.40</td>
<td>0.050</td>
<td>0.9997</td>
<td>1.2448</td>
</tr>
</tbody>
</table>

From the Table-2.3, we observe that the reactive power margin $Q$ decreases as the generation is far away. The voltages at the load bus are found to increase.

2.9.4 Case where generation and load are simulated to be far away:

The effect of generation and load being far away on the reactive power margin is studied here. The following three (3) different cases are considered:

(a) with $X_{AB} = 0.30$ and $X_{BC} = 0.07$ and $X_{BD} = 0.040$

(b) with $X_{AB} = 0.35$ and $X_{BC} = 0.08$ and $X_{BD} = 0.045$

(c) with $X_{AB} = 0.40$ and $X_{BC} = 0.09$ and $X_{BD} = 0.050$

For the above three cases, system equations similar to equations (2.17) to (2.21) are derived and corresponding optimisation problems are solved. The results are tabulated below in Table-2.4.

Table-2.4: Variation of $Q$ and $V_C$ when generation and loads are far away

<table>
<thead>
<tr>
<th>Serial No.</th>
<th>$X_{AB}$</th>
<th>$X_{BC}$</th>
<th>$X_{BD}$</th>
<th>$Q$</th>
<th>$V_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25</td>
<td>0.06</td>
<td>0.035</td>
<td>1.3033</td>
<td>1.0516</td>
</tr>
<tr>
<td>2</td>
<td>0.30</td>
<td>0.07</td>
<td>0.040</td>
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<td>1.1458</td>
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<tr>
<td>3</td>
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<td>0.08</td>
<td>0.045</td>
<td>1.0299</td>
<td>1.2285</td>
</tr>
<tr>
<td>4</td>
<td>0.40</td>
<td>0.09</td>
<td>0.050</td>
<td>0.9281</td>
<td>1.3578</td>
</tr>
</tbody>
</table>
From the Table-2.4, we observe that the reactive power margin Q decreases as the generation and load are far away. The voltages at the load bus are found to increase.

Based on the results of the case study presented, the following observations can be mentioned:

(1) For a particular power system, as the length of transmission line increases, the reactive power margin Q decreases.

(2) Motivated by the above result, the case of parallel transmission lines at the load end was considered and significant improvement in the reactive power margin Q was observed over the base case result.

2.10 Conclusions:

In the analysis of voltage collapse problem, there is a need for the formulation of appropriate models for reactive power margins and for development of effective tools for their evaluation. These aspects are addressed in [31]. An attempt is made towards this objective and the proposed methodology is expected to be useful to the utility engineers and researchers working in this area.