APPENDIX A

Derivation of the analytical model of SEIG operating with constant speed prime mover [34]

The per-phase steady-state equivalent circuit of a three phase SEIG supplying a balanced R-L load is shown in fig.2.1 circuit symbols. Here a new symbol of per unit frequency (F) is introduced. The loop equations for the current Iₙ can be written as:

\[ Zₙ Iₙ = 0 \]  \hspace{1cm} (A.1)

Where \( Zₙ = Zₐ + Zₖ + Zₜ \) and

\[ Zₐ = \left[ \frac{Rₙ + jXₙ}{F} \right]_{[Fₙ Xₙ]} \]

\[ Zₖ = \left[ \frac{Rₖ + jXₖ}{F} \right]_{[Fₖ Xₖ]} \]

\[ Zₜ = \left[ \frac{Rₜ + jXₜ}{F} \right]_{[Fₜ Xₜ]} \]

\[ Zₕ = Rₕ + jXₕ; \quad Zₜ = \frac{(Rₜ + jXₜ)}{F} \]

Under Steady-State Self Excitation, \( Iₙ \neq 0 \); Therefore form equation (A.1), \( Zₙ = 0 \), which implies that both the real and the imaginary parts of \( Zₙ \) should be independently zero. This leads to the following two simultaneously nonlinear equations with \( Xₖ \) and \( F \) as unknown variables.

\[ P(Mₙ) = a₁ Xₖ F^2 + a₂ Xₖ F^₆ + (a₃ Xₖ + a₄) F^3 + (a₅ Xₖ + a₆) \]

\[ + (a₇ Xₖ + a₈) Xₖ + a₉ = 0 \]  \hspace{1cm} (A.2)

And

\[ Q(Xₖ F) = (b₁ Xₖ + b₂) F^4 + (b₃ Xₖ + b₄) F^₆ + (b₅ Xₖ + b₆) F^3 \]

\[ + (b₇ Xₖ + b₈) F + b₉ = 0 \]  \hspace{1cm} (A.3)

The value of coefficients \( a₁ \) to \( a₁₀ \) and \( b₁ \) to \( b₉ \) are given below:

\[ a₁ = Xₕ Xₙ Xₖ \]  \hspace{1cm} (A.4)

\[ a₂ = -a₁ \]  \hspace{1cm} (A.5)

\[ a₃ = -Xₖ [Xₕ Xₙ + Rₕ (Rₖ + Rₜ) + Rₘ Rₙ] - Rₙ [Xₕ Xₙ + Rₙ] \]

\[ + Xₙ (Rₕ + Rₜ) - Xₙ Xₙ Xₖ \]  \hspace{1cm} (A.6)

\[ a₄ = -Rₙ Xₙ [Xₕ (Rₙ Xₙ + Rₕ Xₙ) + (Rₙ Xₙ Xₖ)] \]  \hspace{1cm} (A.7)

\[ a₅ = Ω [Rₙ (Rₙ Xₙ + Rₕ Xₙ) + Rₕ (Rₙ Xₙ + Rₕ Xₙ)] + Xₙ (Xₙ Xₙ + Rₙ Rₙ) + Xₙ (Xₙ + Xₙ) \]  \hspace{1cm} (A.8)

\[ a₆ = Rₙ Xₙ Xₙ Ω (Rₙ Xₙ + Rₙ Xₙ) \]  \hspace{1cm} (A.9)

\[ a₇ = Xₕ Xₙ Xₙ (Rₙ + Rₙ) \]  \hspace{1cm} (A.10)

\[ a₈ = Rₙ Xₙ Xₙ Xₙ (Rₙ + Rₙ) \]  \hspace{1cm} (A.11)

\[ a₉ = Rₙ Xₙ Xₙ Ω (Rₙ + Rₙ) \]  \hspace{1cm} (A.12)

\[ a₁₀ = a₉ Xₙ \]  \hspace{1cm} (A.13)

and

\[ b₁ = -Xₕ (Xₙ (Rₙ + Rₙ) + Xₙ (Rₙ + Rₙ)) - Rₙ Xₙ Xₙ \]  \hspace{1cm} (A.14)

\[ b₂ = -Xₙ Xₙ Xₙ Rₙ \]  \hspace{1cm} (A.15)
For special cases, the above, coefficients should be simplified as follows:-

(a) For No Load Condition: We substitute $X_L = \infty$ and set limit as $R_1 \to 0$ in the equations (A.4) to (A.22)

(b) For Resistive Load: We put $X_1 = 0$ in equations A.4 to A.22. For a set of value of generator speed, load impedance, terminal capacitor and other machine electrical parameters equations A.2 and A.3 are used to determined $X_m$ and $F$. The air gap voltage $V_m$ subsequently is determined from the plot between $X_m$ and $V_m / F$. The $V_m / F-X_m$ plot is obtained by driving the induction motor as synchronous speed ($\Omega = 1$) and measuring $X_m$ at different input voltages of input frequency $F = 1$. The other system variables such as stator, rotor and currents $I_1$, $I_2$ Terminal voltages, machines losses and input and output power are obtained by applying ac current theory to the equivalent circuit. These expressions are given below:-

\[
I_s = V_m / F / [R_s / F + jX_{1s} + Z_c] \quad (A.23)
\]

\[
I_r = -V_m / F / [R_s / (F - \Omega) + jX_{1r}] \quad (A.24)
\]

\[
I_L = -jX_c I_s / [R_1 F + j(X_1 F^2 - X_c)] \quad (A.25)
\]

\[
V_1 = (R_1 + jX_1 F) I_L \quad (A.26)
\]

Total Input Power = $3[I_s^2 R_{1s} / (F - \Omega)]$ \quad (A.27)

Total Output Power = $3[I_L^2 R_1]$ \quad (A.28)

Total Losses = $3 [I_{r1}^2 R_{r1} + I_d^2 R_d + |V_m|^2 / R_d]$ \quad (A.29)
APPENDIX B

Base Values and Machine Parameters:
(1) Base Voltage = 110 Volts/Phase
(2) Base Current = 4.5 Amp
(3) Base Frequency = 50 Hz
(4) Base Angular Frequency = 314.15 sec
(5) Base Impedance = 24.44 \( \Omega \)
(6) Base Power = 495 Watt/Phase

For 6 Pole Generator
(7) Base Speed = 1000 rpm
(8) Base Torque = 4.729 N-m

Machine Parameters
(9) \( R_s = 0.0681 \) pu
(10) \( R_r = 0.0548 \) p.u.
(11) \( L_{rs} = L_s = 0.072 \) p.u.
(12) \( R_c = 10 \) p.u.

For 4-Pole Generator
(13) Base Speed = 1500 rpm
(14) Base Torque = 3.151 N-m

Machine Parameters
(15) \( R_s = 0.08 \) pu
(16) \( R_r = 0.07 \) p.u.
(17) \( L_{rs} = L_s = 0.085 \) p.u.
(18) \( R_c = 15 \) p.u.

The constants for Torque-Speed characteristics of the prime mover
For 4-Pole Generation operation:
\( T_o = 1.02 \) p.u, \( \Omega = 1.06 \) p.u, \( m = 31.4 \)
For Six Pole Generation Operations:
\( T_o = 1.6 \) p.u, \( \Omega_0 = 1.04 \) p.u, \( m = 12.3 \)
APPENDIX C

Representation of magnetizing current of the model of the model of SEIG operating with unregulated prime movers:

Stator flux linkage versus magnetizing current characteristic of the generator is approximated by

\[ \Psi_1 = K_1 I_m \text{ for } 0 \leq I_m < I_0 \]  \hspace{1cm} (C.1)

\[ \Psi_2 = K_1 I_0 + [K_1 / b] \tan^{-1} b (I_m - I_0) \text{ for } I_0 \leq I_m \]  \hspace{1cm} (C.2)

The constants \( K_1, b, I_0 \) are determined such that the derivative \( d\Psi / dt \) is continuous at \( I_m = I_0 \) and the measured curve is adequately approximated substituting \( \Psi_s = V_m / \omega \) in equation (C.2) we get

\[ V_m / \omega = K_1 I_0 + [K_1 / b] \tan^{-1} b (I_m - I_0) \]

Or

\[ I_m = I_0 + [1 / b] \tan b (V_m / \omega K_1 - I_0) \]  \hspace{1cm} (C.3)

The \( (V_m / F - X_m) \) characteristic for 4-Pole Generator is approximated as

\[ V_m / F = 3.09 - 1.14 X_m \]  \hspace{1cm} (C.4)

For 4-Pole Generator,
\( b = 1.15, K_1 = 1.49, I_0 = 0.46 \text{ pu} \)

For 6-Pole Generator,
\( b = 1.03, K_1 = .80, I_0 = 0.70 \text{ pu} \)
APPENDIX – D

Details of Induction Machine:
- k.W : 2 kW
- Phase : 3
- Pole : 4
- Frequency : 50
- Phase to Phase Voltage : 380 V
- Full Load Current : 5.4 A
- Connection : Y (Star)
- Type of Rotor : Squirrel Cage Rotor

Per Phase Equivalent Circuit Constant in per-unit
- \( R_1 = 0.098, R_2 = 0.062, X_1 = 0.112, X_2 = 0.095 \)
- \( X_m = 2.58 \)
- Load Impedance: \((0.8 + j0.6)\) pu.
(considered here)
APPENDIX E

To compute the coefficients $P_0$ to $P_6$ in equation (3.91), the following are defined as:

\[ X_3 = X_m + X_l \]  \hspace{1cm} (E.1)

\[ \text{DENOM} = R_2^2 + (a-b)^2X_3^2 \]  \hspace{1cm} (E.2)

\[ \text{NUM1} = R_1 \text{DENOM} + a \ (a-b) \ R_2 X_m^2 \]  \hspace{1cm} (E.3)

\[ \text{NUM2} = a [X_1 \text{DENOM} + X_m \ (R_2^2 + (a-b)^2 X_3^2)] \]  \hspace{1cm} (E.4)

\[ Z_{ad} = R_{ad} + jX_{ad} = \text{NUM1} + j\text{NUM2} / \text{a.DENOM} \]  \hspace{1cm} (E.5)

Equation (3.89), upon cross-multiplication, becomes,

\[ R_L (\text{NUM1}^2 + \text{NUM2}^2) + (R_L^2 + a^2 X_l^2).\text{DENOM}.\text{NUM1} = 0 \]  \hspace{1cm} (E.6)

\[ \text{DENOM}, \text{NUM1} \text{ and } \text{NUM2} \text{ can be reduced to the following forms:} \]

\[ \text{DENOM} = f_2 a^2 + f_1 a + f_0 \]  \hspace{1cm} (E.7)

\[ \text{NUM1} = g_2 a^2 + g_1 a + g_0 \]  \hspace{1cm} (E.8)

\[ \text{NUM2} = h_3 a^3 + h_2 a^2 + h_1 a \]  \hspace{1cm} (E.9)

Where,

\[ \begin{aligned} f_0 &= R_2^2 + b^2 X_3^2 \\ f_1 &= -2bX_3^2 \\ f_2 &= X_3^2 \end{aligned} \]  \hspace{1cm} (E.10)

\[ \begin{aligned} g_0 &= R_1 (R_2^2 + b^2 X_3^2) \\ g_1 &= -b (R_2 X_m^2 + 2R_1 X_3^2) \\ g_2 &= R_2 X_m^2 + R_1 X_3^2 \end{aligned} \]  \hspace{1cm} (E.11)

\[ \begin{aligned} h_1 &= (X_1 R_2^2 + X_m R_2^2) + b^4 (X_1 X_3^2 + X_2 X_3 X_m) \\ h_2 &= -2b (X_1 X_3^2 + X_2 X_3 X_m) \\ h_3 &= X_1 R_2^2 + X_m R_2^2 \end{aligned} \]  \hspace{1cm} (E.12)

Each of the terms in equation (E.6), after expansion, reduces to a 6th degree polynomial, hence the coefficients $P_0$ to $P_6$ are given by:

\[ P_i = m_i + n_i \quad i = 0, 1, 2 \ldots 6 \]

$m_i$ and $n_i$ can be systematically expressed in terms of $R_L$, $X_L$ and the constants defined in equation (E.10) and (E.12)  \hspace{1cm} (E.13)
Assuming: $X_e = X_s = X_1$

\[ C_1 = -2X_1R_L, \ C_1 = -X_1^2R_e, \ C_3 = -C_1, \ C_4 = -C_2 \]

\[ C_5 = X_{ch}R_L - (X_{ce} - X_{ch})(R_s + R_t) \]

\[ C_6 = (X_sX_{ch} + R_sR_t)R_L - (X_{ce} - X_{ch})(R_s + R_t)X_1 \]

\[ C_7 = -bX_{ch}R_L + b(X_{ce} - X_{ch})R_s \]

\[ C_8 = X_1C_7, \ C_9 = 0.0, \ C_{10} = X_{ce}X_{ch}R_t \]

And,

\[ D_1 = -2(X_{ce} - X_{ch})X_1 + (R_s + R_t)R_L \]

\[ D_2 = (R_s + R_t)X_1R_L - (X_{ce} - X_{ch})X_1^2 \]

\[ D_3 = 2b(X_{ce} - X_{ch})X_1 - bR_2R_4 \]

\[ D_4 = b(X_{ce} - X_{ch})X_{12} - bX_1R_1R_4 \]

\[ D_5 = X_{ch}X_{ce} \]

\[ D_6 = (X_sX_{ce} - R_sR_t)X_{ch} + (X_{ce} - X_{ch})R_tR_s \]

\[ D_7 = -bX_{ce}X_{ch}, \ D_8 = D_2 . X_1 \]
APPENDIX G

Base Quantities,
Power, 3.7 kW, Voltage / Phase - 415 Volt
Current / Phase, 4.42 Amp, Frequency - 50 Hz
Speed, 1430 rev/min
Equivalent Circuit Parameters,
R_s = 0.050 p.u., R_r = 0.060 p.u.
K_1 = 1.623, K_2 = 0.342
X_s = X_r = 0.090 p.u.
Appendix H

The Fourier Coefficients value is given as follows:

\[ a_n = -\frac{2I_{cm}}{(1+n)\pi} \sin((1+n)(\alpha + \pi/2)) \cos((1+n)4\pi/3 - \pi/6) + \frac{2I_{pm}}{(1-n)\pi} \sin((1-n)(\alpha + \pi/2)) \cos((1-n)4\pi/3 - \pi/6) + \frac{2I_{pm}}{(1-n)\pi} \cos((1+n)\pi/2) \sin((1+n)(\alpha - \pi/3)) + \cos((1+n)5\pi/6) \sin((1+n)(\alpha - \pi/3)) + \cos((1-n)5\pi/6) \sin((1-n)(\alpha - \pi/3)) + \frac{4I_d}{nn} \sin(n\pi/3) \sin(n\alpha) \]

\[ b_n = \left[ \frac{2I_{cm}}{(n + 1)\pi} \sin((n + 1)2\pi/3 - \pi/6) \sin((n + 1)(\alpha - \pi/2)) + \frac{2I_{pm}}{(n - 1)\pi} \sin((n - 1)2\pi/3 + \pi/6) \sin((n - 1)(\alpha - \pi/2)) + \frac{2I_{pm}}{(n + 1)\pi} \sin((n + 1)\pi/2) \sin((n + 1)(\alpha - \pi/3)) + \frac{2I_{pm}}{(n + 1)\pi} \sin((n + 1)5\pi/6) \sin((n + 1)(\alpha - \pi/3)) + \frac{2I_{pm}}{(n - 1)\pi} \sin((n - 1)\pi/6) \sin((n - 1)(\alpha - \pi/3)) + \frac{2I_{pm}}{(n + 1)\pi} \sin((n + 1)7\pi/6) \sin((n + 1)(\alpha - \pi/3)) + \frac{2I_{pm}}{(n - 1)\pi} \sin((n - 1)7\pi/6) \sin((n - 1)(\alpha - \pi/3)) + \frac{2I_{pm}}{(n + 1)\pi} \sin((n + 1)7\pi/6) \sin((n + 1)(\alpha - \pi/3)) + \frac{4I_d}{nn} \sin(n(\alpha + \pi/2) \sin(n\pi/3 - \pi/6)) \right] - \frac{2I_{cm}}{(n + 1)\pi} \sin((n + 1)5\pi/6) \sin((n + 1)(2\pi/3 - \alpha)) - \frac{2I_{cm}}{(n - 1)\pi} \sin((n - 1)5\pi/6 + 5\pi/6) \sin((n - 1)(2\pi/3 - \alpha)) - \frac{I_{cm}}{(n + 1)\pi} \sin((n + 1)(\alpha + 2\pi/3) + 5\pi/6) \sin((n + 1)\pi/6) - \frac{I_{cm}}{(n - 1)\pi} \sin((n - 1)(\alpha + 2\pi/3) - 5\pi/6) \sin((n + 1)\pi/6) \quad \text{(H.1), And} \]

\[ b_n = \left[ \frac{2I_{cm}}{(n + 1)\pi} \sin((n + 1)2\pi/3 - \pi/6) \sin((n + 1)(\alpha - \pi/2)) + \frac{2I_{pm}}{(n - 1)\pi} \sin((n - 1)2\pi/3 + \pi/6) \sin((n - 1)(\alpha - \pi/2)) + \frac{2I_{pm}}{(n + 1)\pi} \sin((n + 1)\pi/2) \sin((n + 1)(\alpha - \pi/3)) + \frac{2I_{pm}}{(n + 1)\pi} \sin((n + 1)5\pi/6) \sin((n + 1)(\alpha - \pi/3)) + \frac{2I_{pm}}{(n - 1)\pi} \sin((n - 1)\pi/6) \sin((n - 1)(\alpha - \pi/3)) + \frac{2I_{pm}}{(n + 1)\pi} \sin((n + 1)7\pi/6) \sin((n + 1)(\alpha - \pi/3)) + \frac{2I_{pm}}{(n - 1)\pi} \sin((n - 1)7\pi/6) \sin((n - 1)(\alpha - \pi/3)) + \frac{2I_{pm}}{(n + 1)\pi} \sin((n + 1)7\pi/6) \sin((n + 1)(\alpha - \pi/3)) + \frac{4I_d}{nn} \sin(n(\alpha + \pi/2) \sin(n\pi/3 - \pi/6)) \right] - \frac{2I_{cm}}{(n + 1)\pi} \sin((n + 1)5\pi/6) \sin((n + 1)(2\pi/3 - \alpha)) - \frac{2I_{cm}}{(n - 1)\pi} \sin((n - 1)5\pi/6 + 5\pi/6) \sin((n - 1)(2\pi/3 - \alpha)) - \frac{I_{cm}}{(n + 1)\pi} \sin((n + 1)(\alpha + 2\pi/3) + 5\pi/6) \sin((n + 1)\pi/6) - \frac{I_{cm}}{(n - 1)\pi} \sin((n - 1)(\alpha + 2\pi/3) - 5\pi/6) \sin((n + 1)\pi/6) \quad \text{(H.2)} \]
APPENDIX I

Fourier Coefficients of Fundamental Components $a_n$ and $b_n$ (odd harmonics of current $i$):

$$a_n = \begin{align*}
&\left[4Id / \pi n\right] \sin(n\pi / 3) \cos(n\beta / 2) \cos(2n + n\gamma + n\beta / 2) \\
&-\left[Id / \pi (k + n)\right] \sin((k + n) \beta / 2) \cos((k + n) (5\pi / 3 + \gamma + \beta / 2)) \\
&-\left[Id / \pi (k - n)\right] \sin((k - n) \beta / 2) \cos((k - n) (5\pi / 3 + \gamma + \beta / 2)) \\
&+\left[Id / \pi (k + n)\right] \sin((k + n) \beta / 2) \cos((k - n) (7\pi / 3 + \gamma + \beta / 2)) \\
&+\left[Id / \pi (k - n)\right] \sin((k - n) \beta / 2) \cos((k - n) (7\pi / 3 + \gamma + \beta / 2))
\end{align*}$$  \hspace{1cm} (I.1)

And,

$$b_n = \begin{align*}
&\left[4Id / n\pi\right] \cos(n\pi / 3) \cos(n\beta / 2) \sin(2n\pi + n\gamma + n\beta / 2) \\
&-\left[Id / \pi (n + k)\right] \sin((n + k) \beta / 2) \sin((n + k) (5\pi / 3 + \gamma + \beta / 2)) \\
&-\left[Id / \pi (n - k)\right] \sin((n - k) \beta / 2) \sin((n - k) (5\pi / 3 + \gamma + \beta / 2)) \\
&+\left[Id / (n + k)\pi\right] \sin((n + k) \beta / 2) \sin((n + k) (7\pi / 3 + \gamma + \beta / 2)) \\
&+\left[Id / \pi (n - k)\right] \sin((n + k) \beta / 2) \sin((n - k) (7\pi / 3 + \gamma + \beta / 2))
\end{align*}$$  \hspace{1cm} (I.2)
The equations that model the induction generator, referred to a frame rotating at rotor speed, are as follows:

\[ V_a = -[R_a][I_a] \cdot d(\lambda_a) / dt + \omega_a[N]. \lambda_a \]  
\[ 0 = -[R_e][I_e] \cdot d(\lambda_e) / dt \]  
With \[ N = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \]

\[ W = n_p \omega_m; \quad \omega_m = \text{rotor angular speed}, \; n_p = \text{number of pole pairs} \]
\[ W = d \theta / dt; \quad \theta = n_p \theta_m; \quad \theta_m = \text{rotor angle} \]

\[ V_a = \text{Col} (V_{oa}, V_{da}, V_{qa}): \text{The armature voltage vector}, \]
\[ I_a = \text{Col} (I_{oa}, I_{da}, I_{qa}): \text{The armature current vector}, \]
\[ \lambda_a = \text{Col} (\lambda_{oa}, \lambda_{da}, \lambda_{qa}): \text{The armature linkage flux}, \]
\[ I_e = \text{Col} (I_{oe}, I_{de}, I_{qe}): \text{The rotor current vector}, \]
\[ \lambda_e = \text{Col} (\lambda_{oe}, \lambda_{de}, \lambda_{qe}): \text{The rotor linkage flux}, \]

\[ [B_a] = \text{diag} (R_a, R_a, R_a): \text{The armature resistance matrix} \]
\[ [R_e] = \text{diag} (R_e, R_e, R_e): \text{The rotor resistance matrix}, \]

The currents are related to the flux linkages by:

\[ \lambda_a = [L_{ia}] I_a + \lambda_{ma} \]  
\[ \lambda_e = [L_{ie}] I_e + \lambda_{me} \]  
Where, \[ [L_{ia}] = \text{diag} [L_{oa}, L_{da}, L_{qa}] \]
\[ [L_{ie}] = \text{diag} [L_{oe}, L_{de}, L_{qe}] \] are the leakage reactances
\[ \lambda_{ma} = \text{Col} (0, \lambda_{md}, \lambda_{mq}) \]
\[ \lambda_{me} = \text{Col} (0, \lambda_{md}, \lambda_{mq}) \] are the magnetizing fluxes

The main flux relates to the magnetizing current by:

\[ \lambda_{md} = L_{md} (I_m) \cdot I_{md} + \lambda_{res}.\cos \theta \]  
\[ \lambda_{mq} = L_{mq} (I_m) \cdot I_{mq} + \lambda_{res}.\sin \theta \]  
Where, \[ L_{md} (I_m) \] and \[ L_{mq} (I_m) \] are the main inductance along the d and q axes, and are residual flux which corresponds to each segment of the saturation curve, \( \theta \) is the angle of the total magnetizing current \( I_{md} \), w.r.t. the rotating reference frame. \( I_{md} \) and \( I_{mq} \) are the magnetizing current components and calculated as each time step from their partial derivatives w.r.t. the magnetizing fluxes and from the values of the saturated inductance.

Here \[ L_{md} = L_{mq} = \text{assumed}. \]

\[ T_e = (\lambda_{md} I_q - \lambda_{md} I_d) n_p; \quad T_m = J \cdot d\omega_m / dt \]  
\[ T_m = \text{Torque of shaft and } J = M \cdot I_c, \text{ here } V_a = \begin{pmatrix} Z_m \end{pmatrix} I_a + V_m \]  
This leads to Thevenin – Equivalent circuit.
APPENDIX K

Ratings of IG

a. As a motor
   3φ, 220 / 380V, 50 Hz, 2.5 kW, 6A, 0.83 pt, 2865 rpm
b. As Induction Generator
   3φ, 220 / 380V, 50 Hz, 1.7 kW, 6A, 0.43 pt, 3040 rpm
c. $R_s = 2.8 \Omega$, $R_e = 0.8 \Omega$, $L_{ie} = 20.37 \text{ mH}$, $L_e = 20.37 \text{ mH}$,
d. The magnetization curve at 50 Hz.

![Magnetization Curve](image)

fig. 1 - K1
Shaft torque harmonics obtained through Fourier analysis of the $t_0$ curve fig. 7.1 are given below:

Fourier analysis of harmonics:

Average $t_0 = 2757.5$ Nm
Magnitude of fundamental $t_1 = 496.4$ Nm
Magnitude of II harmonic $t_2 = 2907.2$ Nm
Magnitude of III harmonic $t_3 = 543.8$ Nm
Magnitude of IV harmonic $t_4 = 114.7$ Nm
Phase angle of fundamental $a_1 = 0.537$ mech. rad.
Phase angle of second harmonic $a_2 = 2.839$ mech. rad.
Phase angle of third harmonic $a_3 = 1.983$ mech. rad.
Phase angle of fourth harmonic $a_4 = 0.0175$ mech. rad.

The induction motor parameters are as follows:

Number of poles: $p = 14$
Stator resistance per phase $r_s = 0.054$ ohm
Rotor resistance per phase wrt stator $r_r = 0.031$ ohm
Stator leakage inductance $l_s = 0.0051$ H
Rotor leakage inductance wrt stator $l_r = 0.0057$ H
Mutual inductance $m = 0.005$ H
Rated line current $i_{rated} = 258$ A
Rated phase voltage $v_{rated} = 265.6$ V

The operating conditions are:

Supply phase voltage $V = 265.6$ V
Supply frequency $f = 50$ Hz,
MI of the system $j = 22.67 \text{ K}\text{m}^2$

The base values used for conversion to and from the pu, system are calculated from expressions are given in table L-1 (Choice of Base Values).

### Table: L-1 (Choice of Base Values)

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Base Value</th>
<th>Symbol</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Power</td>
<td>Rated 3 Phase VA</td>
<td>PB</td>
<td>W</td>
</tr>
<tr>
<td>Reactive Power</td>
<td>Rated 3 Phase VA</td>
<td>PB</td>
<td>VA</td>
</tr>
<tr>
<td>Voltage</td>
<td>Rated Phase Voltage</td>
<td>VB</td>
<td>V</td>
</tr>
<tr>
<td>Current</td>
<td>$PB / (3VB)$</td>
<td>IB</td>
<td>A</td>
</tr>
<tr>
<td>Impedance</td>
<td>$VB / IB$</td>
<td>ZB</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>Mech. cmf. Velocity</td>
<td>$L\pi f / P$</td>
<td>WB</td>
<td>rad / s</td>
</tr>
<tr>
<td>Torque</td>
<td>$\beta B / WB$</td>
<td>TB</td>
<td>Nm</td>
</tr>
<tr>
<td>Moment of inertia</td>
<td>$(2 / P) TB / (WB)^2$</td>
<td>JB</td>
<td>kgm$^2$</td>
</tr>
</tbody>
</table>
APPENDIX – M

SIMULINK BLOCKS FOR COMPLEX ARITHMETIC

In the titles of the blocks 'c' refers to quantities in Cartesian co-ordinates and 'p' refers to quantities in polar co-ordinates. Thus c/c = p means two numbers in Cartesian co-ordinates are divided to produce result in polar coordinates. p₁ to p₂ implies a complex polar number is split into Cartesian real and imaginary quantities. p₂ to p₁ implies the Cartesian number given in terms of real and imaginary quantities is converted to polar form and split into magnitude and phase.

The figures of SIMULINK diagram for analysis of the compressor drive through the steady state equivalent circuit method has been given below between fig. M-1 and fig. M-6. Fig. M-7 to Fig. M-15 shows the SIMULINK blocks for complex arithmetic.

The blocks are given below:
Fig. M1. SIMULINK diagram for analysis of the compressor drive through the Steady State Equivalent Circuit method.

Fig. M2. Internal details of "Induction Machine Steady State Model". Block.

Fig. M3. Internal details of block No.1.
Fig. M4. Internal details of block No.2

Fig. M5. Internal details of block No.3

Fig. M6. Internal Details of block No.4
Fig. M-7 to M-15: SIMULINK Blocks for complex arithmetic
APPENDIX – N

M – File for loading initial parameters of compressor drive into MATLAB workspace before SIMULATION through either steady state method or D-Q method.

\[ f = 50; \quad p = 14; \quad w = 2\pi f; \quad ws = 2 / P w; \quad j = 22.67; \quad rs = 0.054; \quad r = 0.031; \quad is = 0.0051; \]
\[ ir = 0.0057; \quad lm = 0.005; \quad d = is*lr - lm^2; \quad xs = w*(is - lm); \quad xr = w*(ir - lm); \quad xm = w*lm; \]
\[ zs = (rs*xs) ; \quad zr = (rr*xr) ; \quad zm = [0 \quad xm]; \quad vph = 440 / sqrt(3); \quad vsd = 0; \quad vsq = sqrt(2)*vph; \]
\[ vraq = 0; \quad vrd = 0; \]

\%
Choice of base values
%

% Choice of base values
%

\[ \text{wbase} = w; \]
\[ \text{wmbase} = 2*\text{wbase} / p; \]
\[ \text{ibase} = 258; \]
\[ \text{vbase} = vph; \]
\[ \text{pbase} = 3*\text{vbase}^2 / \text{ibase}; \]
\[ \text{zbase} = \text{vbase} / \text{ibase}; \]
\[ \text{tbase} = \text{pbase} / \text{wmbase}; \]
\[ \text{jbase} = 2/p*\text{tbase} (\text{wmbase})^2; \]

% Load torque harmonics
%

% average / load torque +ve for motoring
\[ t0 = 2759.7; \]
% fundamental load torque
\[ t1 = 496.4; \]
% second harmonic load torque
\[ t2 = 2907.2; \]
% third harmonic load torque
\[ t3 = 543.8; \]
% fourth harmonic load torque
\[ t4 = 114.7; \]
% phase of t1 mech. rad
\[ a1 = 0.537; \]
% phase of t2 mech. rad
\[ a2 = -2.839; \]
% phase of t3 mech. rad
\[ a3 = -1.983; \]
% phase of t4 mech. rad
\[ a4 = -0.0175; \]
Derivation of equations for calculation of induction machine torque, power, current and slip variations using hunting network method.

Here generator convention is used in the nomenclature, and this is clear by using italics where appropriate. This was done to make it easy to applying the hunting network methodology later to generate analysis. The prime mover shaft torque can be represented by a Fourier series as:

\[ TT(\theta) = TT_0 - \sum_{n=1}^{N_{\text{MAX}}} \left[ TT_n \cos (n\theta + \angle TT_n) \right] \]  
\[ \text{(O.1)} \]

The oscillation of the rotor produced by the \( n \)th harmonic shaft torque gives rise to inertia and electrical torques which may be equated to the shaft torque to give,

\[ J \frac{d^2\delta_n}{dt^2} + T_{dn} \frac{d\delta_n}{dt} + T_{sn}\delta_n = TT(\theta) \]  
\[ \text{(O.2)} \]

The first term on the left hand side is the inertia torque and the remaining two terms adds up to the generator torque which opposes the shaft torque. The synchronizing and damping torque coefficients may be calculated using the following formulae;

\[
\begin{align*}
T_{sn} &= R_s (IR_1 l_n + E_2 n)^2 + (IR_1 l_n + IR_2 e_2 e_0)^2 \\
T_{dn} &= l_m (IR_0 (E_2 n - E_1 n) + IR_1 l_n - IR_2 e_2 e_0) / h_n
\end{align*}
\]  
\[ \text{(O.3)} \]

The steady-state solution of equation (O.2) is

\[ \delta_n(\theta) = |\delta_n| \cos(n\theta + \angle \delta_n) \]

Where,

\[ |\delta_n| = |TT_n| / \left[ T_{sn} \left( X_n^2 + Y_n^2 \right) \right] ; \angle \delta_n = \angle TT_n + \tan^{-1}(Y_n / X_n) \]  
\[ \text{(O.4)} \]

\[ X_n = [h_n^2 / T_{sn}] - 1 \text{ and } Y_n = h_n T_{dn} / T_{sn} \]  
\[ \text{(O.5)} \]

The total rotor displacement is obtained by adding all the harmonics with an average displacement (of zero) to give,

\[ \delta(\theta) = \sum_{n=1}^{N_{\text{MAX}}} \delta_n(\theta) \]  
\[ \text{(O.6)} \]

The electrical torque developed by the generator for each value of \( 'n' \) is given by:

\[ T.G_n = T_{dn} d\delta_n / dt + T_{sn} \delta_n = |TG_n| \cos(n\theta + \angle TG_n) \]

Where,

\[ |TG_n| = T_{sn} \sqrt{[1 + (\gamma_n)^2]} |\delta_n| \text{ and } \angle TG_n = \angle \delta_n - \tan^{-1} Y_n \]  
\[ \text{(O.7)} \]

The total generator torque is\( TG(\theta) = \)

\[ TT_0 - TL + \sum_{n=1}^{N_{\text{MAX}}} \sum TG_n(\theta) \]  
\[ \text{(O.8)} \]

The \( n \)th harmonic slip can be related to the \( n \)th harmonic rotor displacement by,
\[ s_n(\theta) = -\left[ \frac{1}{\omega_s} \right] \frac{d\delta_n}{dt} = |s_n| \cos (n \theta + \angle s_n) \]

where

\[ |s_n| = h_n \delta_n, \quad \angle s_n = \angle \delta_n + \pi/2 \quad \text{and} \quad h_n = \left[ -\frac{n}{\omega_s} \right] \frac{d(\omega T)}{dt} \]

\[ = n\omega T / \omega_n = n \omega f / \omega_n \]

is the \( n \)th harmonic hunting frequency.

Hence total slip is

\[ s_0 + \sum_{n=1}^{N_{MAX}} s_n(\theta) \]

\[ \text{(O.9)} \]

The \( n \)th harmonic stator current is found as follows:

\[ I_{Sn} = \delta_n \left( I_{Sn} + j I_{Sn} \right) = \delta_n \left[ \sqrt{S_{\text{SR}^n}} \cdot \text{DI}^2 \right] \cos(n \theta + \arctan \frac{\text{DI}}{\text{SR}}) + \]

\[ j \left( \sqrt{S_{\text{SR}^n}} \cdot \text{DI}^2 \right) \cos(n \theta - \arctan \frac{\text{DI}}{\text{SI}}) = \text{Re}(I_{Sn}) + j \text{Im}(I_{Sn}) \]

\[ \text{(O.10)} \]

Where,

\[ \begin{align*}
SR_n &= \text{Re}(I_{Sn}) + \text{Re}(I_{Sn}) \\
\text{DI}_n &= \text{Im}(I_{Sn}) + \text{Im}(I_{Sn}) \\
\text{SI}_n &= \text{Im}(I_{Sn}) - \text{Im}(I_{Sn}) \\
\text{DR}_n &= \text{Re}(I_{Sn}) - \text{Re}(I_{Sn})
\end{align*} \]

\[ \text{(O.12)} \]

Here the current obtained from the hunting network have been multiplied by rotor displacement since those currents are for rotor displacement of one radian. The \( n \)th harmonic real and reactive stator currents are given by,

\[ \text{Re}(I_{Sn}(\theta)) = |\text{Re}(I_{Sn})| \omega s(n \theta + \angle \text{Re}(I_{Sn})) \]

\[ \text{Im}(I_{Sn}(\theta)) = |\text{Im}(I_{Sn})| \omega s(n \theta + \angle \text{Im}(I_{Sn})) \]

\[ \text{(O.13)} \]

The real power fed to the grid and the lagging reactive power drawn from the grid is given by:

\[ P(\theta) = \text{Re}(I_{Sn}(\theta)) = \sum_{n=1}^{N_{MAX}} \text{Re}(I_{Sn}(\theta)) \]

\[ \text{(O.14)} \]

\[ Q(\theta) = \text{Im}(I_{Sn}(\theta)) = \sum_{n=1}^{N_{MAX}} \text{Im}(I_{Sn}(\theta)) \]

The total stator current magnitude and the power factor are given by

\[ |I_{S}(\theta)| = \sqrt{\left( \sum_{n=1}^{N_{MAX}} \text{Re}(I_{Sn}(\theta))^2 \right)^2 + \left( \sum_{n=1}^{N_{MAX}} \text{Im}(I_{Sn}(\theta))^2 \right)^2} \]

\[ \text{pf}(\theta) = \frac{\text{Re}(I_{S}(\theta))}{|I_{S}(\theta)|} \]

\[ \text{(O.15)} \]
APPENDIX – P

Interrelations amongst harmonics of various quantities:

The relationships among the magnitudes and phase angles of the \( n \)th harmonic components of pertinent quantities are shown in Table P-1 and Table P-2 respectively. Note that these tables can be used to obtain the harmonic components of all other quantities, given the harmonic components of any one quantity. Table P-1 shows for example that the magnitude of the \( n \)th harmonic slip can be obtained by multiplying the magnitude of the \( n \)th harmonic shaft torque by \(-j\beta_n / [T_{Sn} \sqrt{X_n^2 + Y_n^2}]\). Similarly, Table P-2 shows that the phase angle of the \( n \)th harmonic slip can be obtained by adding \( \tan^{-1} Y_n / X_n + \pi / 2 \) to the \( n \)th harmonic phase angle of the shaft torque.

Table P-1

<table>
<thead>
<tr>
<th>(TT(_n))</th>
<th>(TG(_n))</th>
<th>(S(_n))</th>
<th>(Re(IS(_n)))</th>
<th>(Im(IS(_n)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>TT(_n)</td>
<td>)</td>
<td>1</td>
<td>( \sqrt{(X_n^2 + Y_n^2)} / \sqrt{(1 + Y_n^2)} )</td>
</tr>
<tr>
<td>(TG(_n))</td>
<td>( \sqrt{(1 - Y_n^2)} / \sqrt{(X_n^2 + Y_n^2)} )</td>
<td>1</td>
<td>( T_{Sn} \sqrt{(1 + Y_n^2)} / h_n )</td>
<td>( T_{Sn} \sqrt{(1 + Y_n^2)} / \sqrt{(SR_n^2 + DL_n^2)} )</td>
</tr>
<tr>
<td>(S(_n))</td>
<td>( h_n / T_{Sn} \sqrt{(X_n^2 + Y_n^2)} )</td>
<td>( h_n / T_{Sn} \sqrt{(1 + Y_n^2)} )</td>
<td>1</td>
<td>( h_n / \sqrt{(SR_n^2 + DL_n^2)} )</td>
</tr>
<tr>
<td>(Re(IS(_n)))</td>
<td>( \sqrt{SR_n^2 + DL_n^2} / T_{Sn} \sqrt{(X_n^2 + Y_n^2)} )</td>
<td>( \sqrt{SR_n^2 + DL_n^2} / T_{Sn} \sqrt{(1 + Y_n^2)} )</td>
<td>( \sqrt{SR_n^2 + DL_n^2} / h_n )</td>
<td>1</td>
</tr>
<tr>
<td>(Im(IS(_n)))</td>
<td>( \sqrt{IS_n^2 + DR_n^2} / T_{Sn} \sqrt{(X_n^2 + Y_n^2)} )</td>
<td>( \sqrt{IS_n^2 + DR_n^2} / T_{Sn} \sqrt{(1 + Y_n^2)} )</td>
<td>( \sqrt{IS_n^2 + DR_n^2} / h_n )</td>
<td>( \sqrt{SR_n^2 + DR_n^2} / \sqrt{(SR_n^2 + IS_n^2)} )</td>
</tr>
</tbody>
</table>
Table P-2: Relationships among phases of the $n^{th}$ Harmonic components:

<table>
<thead>
<tr>
<th></th>
<th>$\angle TT_n$</th>
<th>$\angle TG_n$</th>
<th>$\angle S_n$</th>
<th>$\angle R_d(IS_n)$</th>
<th>$\angle I_m(IS_n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\angle TT_n$</td>
<td>0</td>
<td>$-\tan^{-1} Y_n / X_n - \tan^{-1} Y_n$</td>
<td>$\pi / 2$</td>
<td>$-\tan^{-1} Y_n / X_n - \tan^{-1} D_l_n / S_l_n$</td>
<td>$-\tan^{-1} Y_n / X_n - \tan^{-1} D_l_n / S_l_n$</td>
</tr>
<tr>
<td>$\angle TG_n$</td>
<td>$-\tan^{-1} Y_n / X_n + \tan^{-1} Y_n$</td>
<td>0</td>
<td>$\tan^{-1} Y_n - \pi / 2$</td>
<td>$\tan^{-1} Y_n - \tan^{-1} D_l_n / S_l_n$</td>
<td>$\tan^{-1} Y_n + \tan^{-1} D_l_n / S_l_n$</td>
</tr>
<tr>
<td>$\angle S_n$</td>
<td>$\tan^{-1} Y_n / X_n + \pi / 2$</td>
<td>$-\tan^{-1} Y_n + \pi / 2$</td>
<td>0</td>
<td>$-\tan^{-1} D_l_n / S_l_n + \pi / 2$</td>
<td>$\tan^{-1} D_l_n / S_l_n + \pi / 2$</td>
</tr>
<tr>
<td>$\angle R_d(IS_n)$</td>
<td>$\tan^{-1} Y_n / X_n - \tan^{-1} D_l_n / S_l_n$</td>
<td>$-\tan^{-1} Y_n + \tan^{-1} D_l_n / S_l_n$</td>
<td>$\tan^{-1} D_l_n / S_l_n - \pi / 2$</td>
<td>0</td>
<td>$\tan^{-1} D_l_n / S_l_n + \tan^{-1} D_l_n / S_l_n$</td>
</tr>
<tr>
<td>$\angle I_m(IS_n)$</td>
<td>$\tan^{-1} Y_n / X_n - \tan^{-1} D_l_n / S_l_n$</td>
<td>$-\tan^{-1} Y_n - \tan^{-1} D_l_n / S_l_n$</td>
<td>$-\tan^{-1} D_l_n / S_l_n + \pi / 2$</td>
<td>$-\tan^{-1} D_l_n / S_l_n - \tan^{-1} D_l_n / S_l_n$</td>
<td>0</td>
</tr>
</tbody>
</table>
MATLAB Program for hunting network analysis of the compressor drive

f = 50;  % supply frequency in Hz
p = 14;  % number of poles
rs = 0.054;  % stator resistance per phase in ohms
rr = 0.031;  % rotor resistance per phase in ohms
j = 22.67;  % moment of inertia in kgm²
ls = 0.0051;  % stator leakage inductance in H
lr = 0.0057;  % rotor leakage inductance referred to stator in H
lm = 0.0005;  % mutual inductance in H
irate = 258;  % rated line current in A
vrate = 265.6;  % rated phase voltage in V
vph = vrate;  % rated phase voltage V rms
w = 2*pi*f;  % synch. speed in elec. rad/s
t0 = 2759.7;  % av. Load torque in Nm (+ = motoring, - = generating)
t1 = 496.4;  % fundamental load torque Nm
t2 = 2907.2;  % 2nd harmonic load torque Nm
t3 = 543.8;  % 3rd harmonic load torque Nm
t4 = 114.7;  % 4th harmonic load torque Nm
a1 = 0.537;  % phase angle of t1 mech rad
a2 = -2.839;  % phase angle of t2 mech rad
a3 = -1.983;  % phase angle of t3 mech rad
a4 = -0.0175;  % phase angle of t4 mech rad.
wbase = w;  % base electrical frequency rad/sec
wmbase = 2*wbase/p;  % base mechanical frequency rad/sec
ibase = irate;  % base line current A
vbase = vrate;  % base phase voltage V
phbase = 3*vbase*ibase;  % base power watts
zbase = vbase / ibase;  % base impedance ohms
tbase = phase / wmbase  % base elec. torque Nm
jbase = 2/p*tbase(wbase*wbase);  % base inertia in kgm²

t0 = t0, tmax = [t1, t2, t3, t4]; ttang = [a1, a2, a3, a4],
% Approx slip values between which av. slip corresponding to tt0 will lie. (for iteration).
smin = 0.02; smax = 0.03;
% Conversion to per unit
xs = w*(ls - lm) / zbase;
tt0 = tt0 / tbase;
tmax = ttmax / tbase;
xs = w*(ls - lm) / zbase;
\[
x_r = w^*\left(l_r - l_m\right) / z_{base};
\]
\[
x_m = w*l_m / z_{base};
\]
\[
rs = rs / z_{base};
\]
\[
rr = rr / z_{base};
\]
\[
v0 = vph / v_{base};
\]
\[
j = j/j_{base};
\]

% Solution of steady state equivalent circuit, for funding average slip from known average torque and thence all steady state quantities:

\[
s0 = s_{min};
\]
\[
torque = d + 1;
\]
\[
while \ s0 <= smax and abs(torque - tt0) >= 0.001 \\
\begin{align*}
\ s0 &= s0 + 0.001; \\
\ zs0 &= rs + xs^*i; \\
\ zr0 &= rr/s0 + xr^*i; \\
\ zm &= xm^*i; \\
\ is0 &= -v0 / (zs0 + zr0^*zm / (zr0-zm)); \\
\ ir0 &= is0*(1+zs0/zm) + v0/zm; \\
\ torque &= rr/s0^*abs(ir0)^2;
\end{align*}
\]
\end{while}

% Display average torque and slip.

%*************U*******************************

% Calculate hunting frequency

%**************+*************************************************

% Solution of positive hunting frequency equivalent circuit

%**************+*************************************************

% Solution of negative hunting frequency equivalent circuit

u
\[ \text{im} = \text{isn} \cdot (1 + \text{zsn}/\text{zm}); \]
\[ \text{en} = \text{real} (\text{zm}) \cdot \text{im} - \text{vm}; \]

\% Synchronising and damping torques

\[ \text{ts} = \text{real} ((\text{ir}0 \cdot \text{conj} (\text{en} + \text{cn})) + ((\text{irp} + \text{im}) \cdot \text{rr}/\text{s0} \cdot \text{conj} (\text{ir}0))); \]
\[ \text{td} = \text{imag} ((\text{ir}0 \cdot \text{conj} (\text{en} - \text{ep})) + ((\text{irp} - \text{im}) \cdot \text{rr}/\text{s0} \cdot \text{conj} (\text{ir}0)))./\text{h}; \]
\[ \text{u} = \text{ts} + \text{i} \cdot \text{h} \cdot \text{td}; \]
\[ \text{v} = \text{u} - \text{h}^2 \cdot \text{j}; \]

\% Complex quantities calculated from synchronising and damping torques

\[ \text{ttt} = \text{ttmax} \cdot \exp (\text{i} \cdot \text{ttang}); \] % complex shaft torque
\[ \text{ttg} = \text{ttt} / \text{v} \cdot \text{u}; \] % complex generated torque
\[ \text{sss} = \text{ttt} / \text{v} \cdot \text{h} \cdot \text{j}; \] % complex slip
\[ \text{rii} = -\text{ttt} / \text{v} \cdot (\text{isp} + \text{conj} (\text{isn})); \] % complex real current
\[ \text{iii} = -\text{ttt} / \text{v} \cdot (\text{isp} - \text{conj} (\text{isn})) \cdot \text{i}; \] % complex reactive current

\% Solution in time domain

\% for \( \omega t = 0: \pi /180 : 2 \cdot \pi \),
\[ \text{theta} = [\text{theta} \cdot \text{cot} \cdot 180 / \pi]; \]
\[ \text{tt} = [\text{tt} \cdot \text{tt} 0 + \text{sum} (\text{abs} (\text{tt}) \cdot \text{cosn} \cdot \text{wt} + \text{angle} (\text{ttt}))); \]
\[ \text{tg} = [\text{tg} \cdot \text{tt} 0 + \text{sum} (\text{abs} (\text{ttg}) \cdot \text{cosn} \cdot \text{wt} + \text{angle} (\text{ttg}))); \]
\[ \text{s} = [\text{s} \cdot \text{s} 0 + \text{sum} (\text{abs} (\text{sss}) \cdot \text{cosn} \cdot \text{wt} + \text{angle} (\text{sss}))); \]
\[ \text{ri} = [\text{ri} \cdot \text{real} (\text{is} 0 + \text{sum} (\text{abs} (\text{rii}) \cdot \text{cos} (\text{n} \cdot \text{w} t + \text{angle} (\text{riii}))); \]
\[ \text{ij} = [\text{ij} \cdot \text{imag} (\text{is} 0 + \text{sum} (\text{abs} (\text{iii}) \cdot \text{cos} (\text{n} \cdot \text{w} t + \text{angle} (\text{iii}))); \]
end
\[ \text{it} = \text{abs} (\text{ri} - \text{j} \cdot \text{ii}); \]
\[ \text{pf} = \text{abs} (\text{ri} / \text{it}); \]

\% Output plot in per unit

\[ \text{t} = [\text{theta} \cdot \text{tt} \cdot \text{rg} \cdot \text{s} \cdot \text{ri} \cdot \text{ii} \cdot \text{it} \cdot \text{pt}]; \]
\[ \text{plot} (\text{r}(:, 1), \text{r}(:, 2:8)) \]
\[ \text{grid} \]
\[ \text{zoom on} \]
MATLAB program for hunting network analysis of the WAVE energy system:

% Load machine parameters, operating conditions and torque harmonics
f = 48.96;  % supply frequency in Hz
p = 6;  % number of poles
rs = 0.0123;  % stator resistance per phase in ohms.
\( \tau \) = 0.07436;  % rotor resistance per phase in ohms
rm = 63.787;  % core loss resistance
\( j \) = 330;  % moment of inertia in Kg m\(^2\)
xs = 0.0747;  % stator leakage reactance in ohms
\( \tau \) = 0.0747;  % rotor leakage reactance referred to stator in ohm
xm = 2.5312;  % mutual reactance in ohm
\( \text{srated} \) = 151.934;  % rated line current in A
\( \text{vrated} \) = 440/sqrt(3);  % rated phase voltage in V
\( \text{vph} \) = \( \text{vrated} \);  % rated phase voltage in V rms
w = 2*pi*f;  % synch speed in elec. rad/s
\( f_t \) = 1/200;  % period of torque-waveform (assumed)

% Selection of base values

\( \omega_{base} \) = \( w \);  % base electrical frequency rad/sec
\( \omega_{base} \) = 2*\( \omega_{base} \);  % base mechanical frequency rad/sec
\( i_{base} \) = \( \text{srated} \);  % base line current A
\( v_{base} \) = \( \text{vrated} \);  % base phase voltage V
\( p_{base} \) = 3*\( v_{base} \)*\( i_{base} \);  % base power Watts
\( z_{base} \) = \( v_{base} / i_{base} \);  % base impedance ohms
\( t_{base} \) = \( \text{phase} / \omega_{base} \);  % base elec. Torque Nm
\( j_{base} \) = 2/\( p \)*\( t_{base} \)*(\( \omega_{base} \)*\( \text{rm} \));  % base inertia in kg m\(^2\)
\( t_{0} \) = \( t_{0} \); \( t_{\text{max}} \) = [\( t_{1} \), \( t_{2} \), \( t_{3} \), \( t_{4} \)]; \( t_{\text{tang}} \) = [\( a_{1} \), \( a_{2} \), \( a_{3} \), \( a_{4} \)];
% = shaft torque values
\( t_{0} \) = -117.813000
\( t_{\text{max}} \) =

\[
\begin{array}{cccc}
44.193630 & 176.876800 & 15.206960 & 84.240910 \\
61.995390 & 105.434900 & 60.429880 & 138.933700 \\
28.182040 & 69.564170 & 25.785070 & 38.533100 \\
19.272770 & 76.277180 & 60.889590 & 17.783650 \\
65.011700 & 29.702650 & 17.753460 & 29.230950 \\
82.562410 & 12.205920 & 89.303090 & 69.668740 \\
48.227490 & 27.261360 & 57.899090 & 36.748670 \\
27.719880 & 65.993250 & 37.466110 & 34.891080 \\
48.372850 & 155.468400 & 53.610000 & 78.509960 \\
56.752720 & 34.479520 & 62.579930 & 103.409300 \\
44.726330 & 114.506100 & 39.796760 & 64.481270 \\
\end{array}
\]
Approx. slip values between which av. slip corresponding to $t_t0$ will lie.

$s_{\text{min}} = 0.02; s_{\text{max}} = 0.03;$

Conversion to per unit

$tt_0 = tt_0 / t_{\text{base}};$
$tt_{\text{max}} = tt_{\text{max}} / t_{\text{base}};$
$xs = w*(ls - lm) / z_{\text{base}};$
$xr = w*(lr - lm) / z_{\text{base}};$
$xm = w*lm / z_{\text{base}};$
$rs = rs / z_{\text{base}};$
$rr = rr / z_{\text{base}};$
$v_0 = v_{\text{ph}} / v_{\text{base}};$
$j = j / j_{\text{base}};$

Solution of steady state, equivalent circuit for finding average slip from known average torque and thence all steady state quantities:

$s_0 = s_{\text{min}};$
$\text{torque} = tt_0 + 1;$
while $s_0 <= s_{\text{max}}$ and $\text{abs(torque-} tt_0) >= 0.0001;$
\[ s_0 = s_0 + 0.00001; \]
\[ z_{s_0} = r_s + x_s i; \]
\[ z_{r_0} = r_r s_0 + x_r i; \]
\[ z_m = x_m i; \]
\[ i_{s_0} = -v_0 / (z_{s_0} + z_{r_0} z_m / (z_{r_0} + z_m)); \]
\[ i_{r_0} = i_{s_0} (1 + z_{s_0} / z_m) + v_0 / z_m; \]
\[ \text{torque} = r_r / s_0 \text{abs}(i_{r_0})^2; \]

end

% Display average torque and slip
%
 Torque, \( \theta_0, s_0 \)
%
% Calculate hunting frequency
%
\[ n = [1 : \text{max} \{ \text{size} (\theta_0) \}]; \]
\[ h = n / f; \]
%
% Solution of positive hunting frequency equivalent circuit
%
\[ z_{s_{\theta}} = r_s / (1 + h) + x_s i; \]
\[ z_{r_{\theta}} = r_r / (z_{s_0} + h) + x_r i; \]
\[ \nu_{r_{\theta}} = r_r / s_0 * i_{r_0} h / 2 / (z_{s_0} + h)^2; \]
\[ i_{s_{\theta}} = \nu_{r_{\theta}} / (z_{s_{\theta}} (1 + z_{s_{\theta}} / z_m)); \]
\[ i_{r_{\theta}} = i_{s_{\theta}} (1 + z_{s_{\theta}} / z_m); \]
\[ e_p = \text{real} (z_{r_{\theta}}) i_{r_{\theta}} - \nu_{r_{\theta}}; \]
%
% Solution of negative hunting frequency equivalent circuit
%
\[ z_{s_{\nu}} = r_s / (1 - h) + x_s i; \]
\[ z_{r_{\nu}} = r_r / (z_{s_0} - h) + x_r i; \]
\[ \nu_{r_{\nu}} = -r_r / s_0 * i_{r_0} h / 2 / (z_{s_0} - h)^2; \]
\[ i_{s_{\nu}} = \nu_{r_{\nu}} / (z_{s_{\nu}} + z_{r_{\nu}} (1 + z_{s_{\nu}} / z_m)); \]
\[ i_{r_{\nu}} = i_{s_{\nu}} (1 + z_{s_{\nu}} / z_m); \]
\[ e_n = \text{real} (z_{r_{\nu}}) i_{r_{\nu}} - \nu_{r_{\nu}}; \]
%
% Synchronising and damping torques
%
\[ \theta_s = \text{real} (i_{r_0} * \text{conj}(e_p + e_n) + (i_{r_0} + i_m) r_r / s_0 \text{conj}(e_p)); \]
\[ \theta_d = \text{imag} (i_{r_0} * \text{conj}(e_n - e_p) + (i_{r_0} - i_m) r_r / s_0 \text{conj}(e_p)) / h; \]
\[ u = \theta_s + i h \theta_d; \]
\[ v = u - h^2 e_p; \]
%
% Complex quantities calculated from synchronizing and damping torques
%
\[ \theta_{\text{tt}} = \theta_{\text{max}} \exp(i \theta_{\text{tang}}); \]
% complex shaft torque
Parameters of the DC Drives

PI controller parameters:

Speed PI (after auto-tuning) Kg = 1.92, ki = 2.93
Current PI (after auto-tuning) Kg = 0.16, ki = 40.0

P Controller Parameters:

Speed P Kp = 1.92, ki = 0.0
Current P Kp = 0.16, ki = 0.0

I Controller parameters:

Speed I Kg = 0.0, ki = 2.93
Current I Kg = 0.0, ki = 40.0

Drive Parameters:

Ramp-up and Ramp-down time → 0.05
Current Limit → 12 A
Thristor Gain → 25
Thyristor Delay → 8.34 ms

Machine Ratings:

DC Compound Motor: 220V, 17A, 1500rpm, 3kW; Field: 220V, 0.7A,
DC dynamometer: 230V, 10.5A, 1500rpm (Max 3400 rpm), 3kW;
Field: 220V, 0.9A

Machine Parameters:

Dynamometer Armature Resistance → 0.17 ohm
Dynamometer Armature Inductance → 10.53 mH
Dynamometer Torque Constant → 0.3769 Nm/A
Dynamometer EMF Constant → 1.5V/sec/rad
Load Machine Armature Resistance → 0.12 ohm
Load Machine Armature Inductance → 3.34 mH
Load Machine Torque Constant → 0.96 Nm/A
Load Machine EMF Constant → 1.1V/sec/rad
Friction Coefficient → 0.01 Nm/(rad/s)
Moment of inertia (MOI) → 0.5 Kgm²
Control System Analysis of PI Controller using MATLAB:

%********************************************************************
% DEFINE CONSTANTS:
%********************************************************************
clear all
clc
pack
wref = 1;
kp1 = 2.9; ki1 = 1.6;
kp2 = 0.16; ki2 = 40; m = 207.07;
gfg = 1.0; tfg = 0.00001;
kg = 1.0; ga = 6.0; b = 0.001; j = 0.5;
ws = 153; slope = 0.0;
kmw = 1.5; kc = 1.0; kw = 1.0; kmt = 0.3769;
%********************************************************************
% INPUT CALCULATIONS
%********************************************************************
load diary
t = diary (1:400, 1);
v = diary (1:400, 2);
%********************************************************************
% DEFINE NUMERATORS AND DENOMINATORS OF ALL BLOCKS:
%********************************************************************
n1 = wref;
d1 = 1;
n2 = [kp1 ki1];
d2 = [1 0];
n3 = 1;
d3 = 1;
n4 = [kp2 ki2];
d4 = [1 0];
n5 = m;
d5 = 1;
n6 = gfg;
d6 = [tfg 1];
n7 = kg;
d7 = 1;
n8 = ga;
d8 = 1;
n9 = 1;
d9 = [j b];
n10 = kw;
d10 = 1;
n11 = kc;
cc
d11 = 1;
n12 = 0;
d12 = 1;
n13 = 0;
d13 = 1;
n14 = slope;
d14 = 1;
n15 = kmw;
d15 = 1;
n16 = kmt;
d16 = 1;
n17 = 0;
d17 = 1;
n18 = ws;
d18 = 1;
n19 = 1/ws;
d19 = 1;

%********************************************************************
% SPECIFY NUMBER OF BLOCKS:
%********************************************************************

nblocs = 19;

%********************************************************************
% SPECIFIES INTERCONNECTION BETWEEN THE BLOCKS:
%********************************************************************

q = [
    0 0 0 0
    1 -10 0
    2 0 0
    3 -11 -12
    4 0 0
    5 17 13
    6 0 0
    7 15 0
    8 16 -14 0
    9 0 0
    10 8 0 0
    11 1 -10 0
    12 0 0
    13 0 0
    14 0 0
    15 -9 0 0
    16 0 0
    17 0 1
    18 0 1
    19 0 0 1
    19 18 -1 1
    ]

%********************************************************************

dd
% SPECIFY THE INPUT AND OUTPUT BLOCKS:
%******************************************************************************
u = [1];
y1 = [-9];
y2 = [5];
y3 = [6];
y4 = [6];
y5 = [14];
%******************************************************************************

% INTERCONNECTS:
%******************************************************************************
[ac1, bc1, cc1, dc1] = connect(a,b,c,d,q,lu,ly1);
sp = 1sim(ac1, bc1, cc1, dc1, v,t);
[ac2, bc2, cc2, dc2] = connect(a,b,c,d,q,lu,ly2);
va = 1sim(aa2, bc2, cc2, dc2, v,t);
[ac3, bc3, cc3, dc3] = connect(a,b,c,d,q,lu,ly3);
ifg = 1sim(ac3, bc3, cc3, dc3, v,t);
[ac4, bc4, cc4, dc4] = connect(a,b,c,d,q,lu,ly4);
ia = 1sim(ac4, bc4, cc4, dc4, v,t);
[ac5, bc5, cc5, dc5] = connect(a,b,c,d,q,lu,ly5);
tl = 1sim(ac5, bc5, cc5, dc5, v,t);
%******************************************************************************

% PLOTTING OF SEVERAL OUTPUTS
%******************************************************************************
subplot(211), plot(t,v),
old, subplot(211, plot(t, ysp, '9')
xlevel('times in 'second'))
ylabel('ilp and olp speeds (rad/s)')
title('PIC OUTPUT')
subplot(212), plot(t, -y+l, *ysp*3311000)
xlabel('time in seconds')
ylabel('Gen, output (kw)')
title('PIC OUTPUT')
figure
subplot(211), plot(t, yia)
xlevel('Time in second')
ylevel('Armature current (amp)')
title('PIC OUTPUT')
subplot(212), plot(t, -y+1)
xlabel('Time in second')
ylabel('generator input torque (Nm)')
title('PIC OUTPUT')
%******************************************************************************

% POST-PROCESSING OF SPEED OUTPUT:
%******************************************************************************
clc
```matlab
[num 1.den 1] = ss2 tf (ac1, bc1, cc1, dc1, lu);
w = logspace (-2,4);
[mag 1, phase 1] = bode (ac1,bc1,cc1,dc1,lu,w);
subplot (211), semilogx (w.mag1, 'w'), title ('MAGNITUDED RESPONSE OF SPEED PIC'))
grid
subplot (212), semilogx (w.phase1, 'w'), title ('Phase Response')
grid;
figure
subplot (211), plot (mag1, phase1, 'w'), title ('MODIFIED NICHOLS PLOT OF SPEED (PIC)')
grid
subplot (212), nyquist (ac1, bc1, cc1, dc1, lu, w), title ('MODIFIED 'NYQUIST PLOT'
grid
figure
J = 0.05; 01 : 01;
rl = rlocus (num1; den1);
subplot (221), plot (rl, 'w'), xlabel ('real'), ylabel ('imag')
title ('ROOT LOCUS (0, 05 < J < 0.1)'
grid
kil = 0.1; 01 : 1:6;
s1 = rlocus (num 1, den1, kil);
subplot (222), plot (s1, 'w'), xlabel ('real'), ylabel ('imag')
title ('ROOT LOCUS (0.1 < K1 < 1.6)')
grid
kp1 = 0.1; 0.5; 3.1;
tl = rlocus (num1, den1, kp1);
subplot (223), plot (tl, 'w'), xlabel ('real'), ylabel ('imag')
title ('ROOT LOCUS (0.1 < KPI < 3)')
grid
kw = 0.5:1:2;
ul = rlocus (num1, den1, kw):
figure
ga = 5.0; 5; 10.0;
rl = rlocus (num1, den1, j);
subplot (221), plot (rl, 'w*'), xlabel ('real'), ylabel ('imag')
title ('ROOT LOCUS (0.05< J<0.1)'
grid
kil = 0.1:0.1:1.6;
s1 = rlocus (num1, den1, ki2);
subplot (222), plot (s1, 'w*'), xlabel ('real'), ylabel ('imag')
title ('ROOT LOCUS (0.1 < K11 <1.6)')
grid
kp1 = 0.1:0.5:3.1;
tl = rlocus (num1, den1, kp1);
subplot (223), plot (tl, 'w*'), xlabel ('real'), ylabel ('imag')
```
title ('ROOT LOCUS (0.1 < KP1 <3)')
grid
kw = 0.5;1;2;
u1 = rlocus(num1, den1, kw);
subplot (224), plot (u1, 'w*'), xlabel ('real'), ylabel ('imag')
title ('ROOT LOCUS (0.5 < KW <2)')
grid
figure
ga = 5:0:5:10.0;
v1 = rlocus(num1, den1, ga);
subplot(221), plot(v1, 'w*'), xlevel('real'), ylevel('imag')
title ('ROOT LOCUS (5< GA <10)')
grid
ki2 = 30:5:50;
w1 = rlocus(num1, den1, ki2);
subplot(222), plot(w1, 'w*'), xlevel('real'), ylevel('imag')
title ('ROOT LOCUS (30< KI2 <50)')
grid
kp2 = 0.1:0:01:0.2
x1 = rlocus(num1, den1, kp2);
subplot(223), plot(x1, 'w*'), xlevel('real'), ylevel('imag')
title ('ROOT LOCUS (0.1< KP2 <0.2)')
grid
kc = 0.5:0:1:2;
y1 = rlocus(num1, den1, kc);
subplot(224), plot(y1, 'w*'), xlevel('real'), ylevel('imag')
title ('ROOT LOCUS (0.5< KC <2)')
grid
figure
tt1 = 0:0.005:0.5;
yy1 = step(num1, den1, tt1);
subplot (211), plot (tt1, yy1), title ('STEP RESPONSE OF SPEED (PIC)')
grid
yyy1 = impulse (num1, den1, tt1);
subplot (212), plot (tt1, yyy1), title ('IMPULSE RESPONSE')
grid

%***************************************************************************
% DIARY
%***************************************************************************
pack
delete pic3.ou1 ,
% diary pic3, ou1
disp ('DIARY FOR PIC SPEED OUTPUT OPENED')
num1
den1
dcgain (num1, den1)
printsys (num1, den1)
[gml, pm1, wcgl, wcp1] = margin (magl, phase1, w)
damp (den1)
damp (num1)
[p1, z1] = pzmap(ac1, bc1, cc1, dc1)
% diary off

**APPENDIX - U**

Generator parameter and equivalent circuit

Parameters of 55kW induction generator (SRIG)
Stator voltage (star) = $V = 415\, \text{V}$
Rotor voltage (star) = 260V
Stator Resistance per phase = $RS = 0.0181\, \text{ohm}$
Rotor Resistance per phase = $RR = 0.0334\, \text{ohm}$ (with no external resistance)
Rotor Resistance per phase = $RR = 0.717\, \text{ohm}$ (with one external resistance)
Stator Reactance per phase = $XS = 0.13\, \text{ohm}$
Rotor Reactance per phase = $XR = 0.16\, \text{ohm}$
Core-1088 Resistance per phase = $RM = 107.303\, \text{ohm}$
Magnetising Reactance per phase = $XM = 7.415\, \text{ohm}$
Frequency = 50 Hz.
No. of poles = 4
Rated speed = 1517 rpm

Fig. U1: steady state equivalent circuit of the induction machine
APPENDIX -V

TURBINE PARAMETERS and FORMULATION FOR

110 KW FIRST PROTOTYPE CONSTANT CHORD WELLS TURBINE

\[ TT = k \cdot C \cdot (V^2 - Vp^2) \]
\[ k = 0.57784 \]
\[ RTIP = 1.0m \]
\[ J = 330.0 \text{ kgm}^2 \]
\[ \tan = \frac{v_x}{w} \]
\[ \tan = \frac{v_x}{w} \]
\[ pr = \left( (0.4224 \cdot \tan) - 0.00393939 \right) \]
\[ delp = 4.12 \cdot pr \cdot w \cdot w \]
\[ \text{powerio} = (2 \cdot delp \cdot v_x) \]
\[ \text{torio} = \text{powerio} / w \]

55 KW SECOND PROTOTYPE TAPERED CHORD WELLS TURBINE

\[ TT = k \cdot C \cdot (V^2 - Vp^2)^2 \]
\[ k = 0.145968 \]
\[ RTIP = 0.5m \]
\[ J = 18.0 \text{ Kgm}^2 \]
\[ \tan = \frac{v_x}{(0.5 \cdot w)} \]
\[ pr = (0.471429 \cdot \tan) \]
\[ delp = 1.21 \cdot pr \cdot w \cdot w \]
\[ \text{powerio} = (1.006 \cdot delp \cdot v_x) \]
\[ \text{torio} = \text{powerio} / w \]

55 KW THIRD PROTOTYPE TAPERED CHORD WELLS TURBINE

\[ TT = K \cdot C \cdot (V^2 + (Vp/ngear)^2) \]
\[ k = 0.2066 \]
\[ qq = v_x \cdot \text{area} \]
\[ \text{area} = 0.3989 \]
\[ \text{var} = 0.4925 \cdot \text{ca/area} \cdot 1e3 \]
\[ delp = \text{var} \cdot ((0.4217 \cdot w/ngear)^2 + v_x^2) \]
\[ \text{powerio} = \text{abs}(delp \cdot qq) \]
\[ RTIP = 0.5m \]
\[ j\text{turb} = 16.0 \text{ kgm}^2 \]
\[ j\text{gen} = 2.0 \text{ Kgm}^2 \]
\[ J = (j\text{turb} / \text{ngear} \cdot \text{ngear}) + j\text{gen} \cdot 1.2 \]
Optimization results in MATLAB Workspace

Optimization of inertia (random input)
Setting up constraint window
Processing uncertainty information
Uncertainty turned off
Setting up call to optimization routine
Start time: 0, stop time: 60
There are 12006 constraints to be met in each simulation
There are 1 tunable variable
There are 1 simulation per cost function call

<table>
<thead>
<tr>
<th>f</th>
<th>COUNT</th>
<th>MAX (g)</th>
<th>STEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>18.6466</td>
<td>1</td>
<td>OK</td>
</tr>
<tr>
<td>6</td>
<td>16.5027</td>
<td>1</td>
<td>OK</td>
</tr>
<tr>
<td>9</td>
<td>8.69448</td>
<td>1</td>
<td>OK</td>
</tr>
<tr>
<td>12</td>
<td>7.58433</td>
<td>1</td>
<td>Mod Hess(2) OK</td>
</tr>
<tr>
<td>13</td>
<td>7.58431</td>
<td>1</td>
<td>Mod Hess(2) OK</td>
</tr>
</tbody>
</table>

Optimization converged successfully = 19.8119

Optimization of inertia (Sinusoidal input):
Processing uncertainty information
Uncertainty turned off
Setting up call to optimization routine
Start time: 0, stop time: 60
There are 12003 constraints to be met in each simulation
There are 1 tunable variable
There is 1 simulation per cost function call

<table>
<thead>
<tr>
<th>f</th>
<th>COUNT</th>
<th>MAX (g)</th>
<th>STEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>15.9264</td>
<td>1</td>
<td>OK</td>
</tr>
<tr>
<td>6</td>
<td>13.8477</td>
<td>1</td>
<td>OK</td>
</tr>
<tr>
<td>9</td>
<td>9.05396</td>
<td>1</td>
<td>OK</td>
</tr>
<tr>
<td>12</td>
<td>4.5188</td>
<td>1</td>
<td>OK</td>
</tr>
<tr>
<td>15</td>
<td>3.44722</td>
<td>1</td>
<td>OK</td>
</tr>
<tr>
<td>18</td>
<td>3.23744</td>
<td>1</td>
<td>OK</td>
</tr>
<tr>
<td>19</td>
<td>3.23744</td>
<td>1</td>
<td>OK</td>
</tr>
</tbody>
</table>

Optimization converged successfully = 49.8

Rotor Resistance Controller:
<table>
<thead>
<tr>
<th>f</th>
<th>COUNT</th>
<th>FUNCTION</th>
<th>MAX (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.36926</td>
<td>365.735</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>20.614</td>
<td>+0.99096</td>
<td>0.5</td>
</tr>
<tr>
<td>7</td>
<td>20.172</td>
<td>+0.20105</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>20.022</td>
<td>0.0227</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>20.0005</td>
<td>0.00048</td>
<td>1 mod Hess</td>
</tr>
<tr>
<td>13</td>
<td>202.309</td>
<td>e-007</td>
<td>1 mod Hess</td>
</tr>
<tr>
<td>14</td>
<td>207.603</td>
<td>e-013</td>
<td>1 mod Hess</td>
</tr>
</tbody>
</table>

Optimization converged successfully = [x s pin pfs] = 73.595 - 0.0500 20.000 0.8700

<table>
<thead>
<tr>
<th>f</th>
<th>COUNT</th>
<th>FUNCTION</th>
<th>MAX (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-0.1525</td>
<td>374.077</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>-2.3430</td>
<td>294.108</td>
<td>0.125</td>
</tr>
<tr>
<td>18</td>
<td>-34.706</td>
<td>230.984</td>
<td>1</td>
</tr>
<tr>
<td>27</td>
<td>-21.662</td>
<td>4.4231</td>
<td>1</td>
</tr>
<tr>
<td>36</td>
<td>-20.0257</td>
<td>0.0263</td>
<td>1</td>
</tr>
<tr>
<td>42</td>
<td>-204.1e-007</td>
<td>1 mod Hess</td>
<td>-</td>
</tr>
<tr>
<td>43</td>
<td>-208.1e-0013</td>
<td>1 mod Hess</td>
<td>-</td>
</tr>
</tbody>
</table>

Optimization converged successfully = [x s pin pfs] = 0.184 114.533 -0.1000 20.000 0.9206

[kk]
APPENDIX -X

Conduction pattern of controlled power devices and diodes between 0° to 360°

The experimental set-up of inductively loaded current controlled solid-state lead – lag VAR compensator is shown in fig. X.1. The waveforms are shown in fig.X.2 (a) to study the various current and voltage waveforms of the compensator at various important points. The experimentally obtained waveforms from eleven pair of points (shown in fig.X.2a) can be compared with the theoretically predicted waveforms which will assist in explaining the operation of the compensator. The block diagram of the control scheme consists of a synchronizing circuit, a microprocessor, RAM, ROM, a six channel programmable interval time, a programmable interrupt controller and an output interface circuit shown in fig.X.2(b).

X.1.1 sub-mode 0°- 60°, conduction pattern of controlled power devices and diodes: Both the start-up transient in voltages across commutating capacitors and steady state operators are described for switching delay angle 30° of controlled power devices.

X.1.1.1 Initiations of start-up transient in voltages across commutating capacitors:
When the devices T1 and T6 are simultaneously switched on, the voltages across of all the commutating capacitors gradually build-up. The reference polarities of the capacitor voltages etc are shown in fig. X.3 (a) as the voltage build up process is identical for the capacitors in both upper and lower section of the compensator, this build-up of voltage is shown in fig. X.3 (b) and described later in the section for the commutating capacitors only in the upper section of the compensator, 12 time intervals are identified to describe the stat-up transients in voltage across the capacitors c1,c3 and c5. All the time intervals are described below:

**Interval t0-t1:** At α =30° and t0 = 0sec, the device T1 and T6 is simultaneously switched on and current starts flowing through the path: phase A, D1-T1-T6-D6- Phase B. Here, Vc1=Vc3 = 0 = Vc5, and the diodes D3 and D5 are held reverse biased due to -ve instantaneous voltage of ebc and eca appearing across them.
Interval $t_1-t_2$: After $t_1$, $e_{ba} > 0$ and $D_3$ becomes conducting. $V_{c1} = e_{ba}$ and $V_{c5}$ and $V_{c3} = -1/2 e_{ba}$ till the instant $t_2$ is reached.

Interval $t_2-t_3$: At $t_2$ time, the $T_3$ is switched on and both the devices $T_3$ and $D_5$ starts conducting in the upper section. The $T_1$ and $D_1$ go to ‘OFF’ due to reverse biasing. In this interval $V_{c1}, V_3$ and $V_{c5}$ remain constant at the value equal to those of at the instant of $t_2$.

Interval $t_3-t_4$: At $t_3$, $V_{c3} = e_{cb}$ and $D_5$ starts conducting. In this interval $V_{c1}$ and $V_{c5}$ change to $-1/2 e_{cb}$ from the initial value of the voltages across each of them at the instant $t_3$. This variation in the capacitor voltage continuous up-to instant $t_4$.

Interval $t_4-t_5$: Now, $T_5$ is ‘ON’ so the $D_5$ starts conducting and $T_3$ and $D_3$ go into reverse bias condition or turn-OFF. $V_{c1}, V_c3$ and $V_{c5}$ remain constant during this interval.

Interval $t_5-t_6$: At the instant $t_5$, $V_{c5} = e_{ac}$ and $D_1$ starts conducting, $V_{c1}$ becomes equal to $e_{ac}$ those of across the capacitors $C_5$ and $C_3$ change with a magnitude = $(e_{ac})/2$, from their initial values at the instant $t_5$. This variation in the capacitor voltage continues till the instant $t_6$.

Interval $t_6-t_7$: At the instant $t_6$, the device $T_1$ is switched on and both the devices $T_1$ and $D_1$ conduct simultaneously in the upper section. Here $T_5$ and $D_3$ turn ‘OFF’. It is observed that, unlike the interval $(t_6-t_1)$ during interval $(t_6-t_7)$ all the capacitors hold constant voltages, this indicates that the voltages across all that capacitors have already been built up.

Interval $t_7-t_{12}$: During this interval, the voltage build-up is identical which is as follows: $(t_7-t_8) \rightarrow (t_1-t_2); (t_8-t_9) \rightarrow (t_2-t_3); (t_9-t_{10}) \rightarrow (t_3-t_4); (t_{10}-t_{11}) \rightarrow (t_4-t_5) \text{ and } (t_{11}-t_{12}); \rightarrow (t_5-t_6)$. Here the initial values of voltages of all the capacitor voltage in the former intervals between $t_8$ and $t_6$ are different from those of the corresponding later interval between $t_7$ and $t_{12}$.

Thus it is seen that within two cycles of the supply voltages from the instant the device $T_1$ is switched on, the voltage across capacitors in the upper section of the compensator come to steady – state value. It is also observed that the diodes start conducting prior to the instant at which the controlled power devices start conducting. Similar argument holds true for the conduction patterns of the devices in the lower section of the compensator. The conduction of diodes takes place with the sequences $D_1-D_2-D_3-D_4-D_5$.
D0-D1 and under steady state condition, the starting of conduction of a particular diode is delayed by an angle 60° from the previous instant of diode conduction (at a particular angular frequency of the input voltage). Based on the above discussion, the conduction pattern of the devices can be predicted and therefore steady-state operation of the compensator can be explained. Here α = 30° is a boundary case, where the diodes D1 and D4 tends to conduct simultaneously at same instants. Therefore, for greater clarity, the steady state condition patterns of the devices for switching delay angles between 0° and 30° (for α = 15°) and also between 30° and 60° (for α = 45°) are shown in figs and X.3 (c) & (d) separately in this sub-modes of operation.

X.1.1.2 Steady-state operation of the VAR compensator

The fig.X.4 describes the conduction pattern, voltage and current waveforms through thyristors, capacitors, diodes and dc side of compensator at different time at α = 30°. The timing choice (intervals) are based on the switching ‘on’ of one controlled power device ( T ) or one power diode ( D ). For complete analysis it is divided into six time intervals, which is explained below.

Interval (t0,t1): Fig. X.4 (a) shows the conduction pattern of thyristor and diodes prior to the instant to the diodes D1 and D3 are off as they are held reverse biased and the diode D5, along with T5 conducts to carry the dc current ‘I_d’ from the input phase. In fig.X.4 (c), the voltage across the D1 is monitored before the instant ‘t0’, it is observed that the diode voltage is the difference between the line voltage e and the capacitor voltage Vα (fig.X.4 (b)). Before, ‘t0’ this voltage is –ve and after ‘t0’ the anode potential of the diode is greater than its cathode potential and it starts conducting. Resultant circuit structure is shown in fig. X.4(f). Due to the simultaneous conduction of D1 and D5, current flows through C5 in parallel with series combination of the capacitors C1 and C3, which appears across the phase voltages e and eα. Here, iC1 = iC3 = iC5 = [2/3] iD, since D4 does not conduct current iD = iD1 = (3/2) C[deα /dt] ( shown in fig.X.4d), consequently the capacitor voltage profiles Vc1, Vc3 and Vc5 has been shown in fig. X.4 (b) here voltage rise in the C5 is observed when it is charged from the left. The current in T1 is absent, since it is not gated and it holds a voltage Vα (seen in fig. X.4(e)). The diode current iD
Fig. X.1. Experimental set up for the study of various wave-forms in inductively loaded current controlled solid-state lead-lag VAR compensator.
Fig. 2.1 Microprocessor based gating signal generator including analogue VAR reference and interface block.
Fig. X.3(a) Circuit diagram of the compensator marked with direction of currents and polarities of voltages.

Fig. X.3(b) Build up of voltages across commutating capacitors (upper section) of the compensator.
Fig. X.3 (c) Conduction pattern of power devices of the compensator for switching delay angle between 0° and 30°

Fig. X.3 (d) Conduction pattern of power devices of the compensator for switching delay angle between 30° and 60°
Fig. X.4 Conduction pattern of power devices and predicted wave-forms in sub mode
0° - 60° and at α = 30° (a) Conduction pattern of power devices (b) Voltage wave-forms
across power capacities C5, C1, C3 along with input voltage Waveform (c) Voltage wave
forms across power devices T1 and D1.
Fig. X.4 (d) Conduction pattern of power devices and predicted wave-forms in sub mode $0^\circ-60^\circ$ and at $\alpha = 30^\circ$. Current wave forms through TI, CI, C5, D1, D4 and Phase A.
Fig. X.4 (e) Conduction pattern of power devices and predicted wave-forms in sub mode 0°-60° and at α = 30°. Voltage and current wave forms at the dc-side of the compensator, V_d and I_d along with input voltage wave forms
Fig. X.4 (f) Conduction pattern of power devices and predicted wave-forms in sub mode 0°-60° and at α = 30°. Operational circuit structures at the upper section of the compensator at different time intervals.
Fig.X.5 (a) Experimental wave-forms at $\alpha = 30^\circ$ in the sub-mode $0^\circ - 60^\circ$, $R = 10$ ohm, horizontal axis 20 ms full scale, vertical axis, $e_{ac} = 40$ V peak, $V_{C5} = 20$ V Peak, $V_{DI} = 25$ V Peak; $i_p = 3.3$ A peak.

Fig.X.5 (b) Experimental wave-forms at $\alpha = 30^\circ$ in the sub-mode $0^\circ - 60^\circ$, $R = 100$ ohm, horizontal axis 50ms full scale, vertical axis, $i_{DI} = 3.3$ A peak; $IT_1 = 3.3$ A; peak; $i_c = 0.3$ A peak; $i_c = 0.3$ A(p)
Fig. X.5 (c) Experimental wave-forms at $\alpha = 30^\circ$ in the sub-mode $0^\circ$-$60^\circ$, $R = 320$ ohm, horizontal axis, 20 ms full scale. Vertical axis, $e_{ac} = 40$ V peak, $V_{cs} = 20$ V Peak; $V_{DI} = 25$ V peak; $i_b = 1.03(p)$

Fig. X.5 (d) Experimental wave-forms at at $\alpha = 30^\circ$ in the sub-mode $0^\circ$-$60^\circ$, $R = 100$ ohm, horizontal axis = 50 ms full scale vertical axis $i_{D1} = 3.3$ A peak; $i_{I1} = 3.3$ A peak; $i_{C3} = 0.3$ A peak
exists due to the simultaneous conduction of T5, D5 and T6, D6, this makes, $V_d = e_{bc}$ and consequently the current through dc inductor will be $I_d = 1/L \int e_{bc} \, dt$; here the R is neglected.

**Interval (t1-t2):** At $t_1$ the T1 is switched on, which commutates both the devices T5 and D5 by line commutation. In (t1-t2) duration, T1, D1, T6, D6 and D6 conduct. (as indicated in (fig.X.4(f). The resultant circuit structure is shown in fig.X.4 (f) the upper sections voltage; across all capacitors are constant D1 conducts. Here $i_{c1} = 0$ and $1_{T1} = i_{D1} = i_s = i_d$. The inductor current and voltage are $V_d = e_{bc}$ and $i_d = 1/L \int e_{bc} \, dt$. During this interval $V_{T1} = V_{D1} = 0$. All these waveforms are shown on figs.(X.4).

**Interval (t2-t3):** When $V_{c1} = e_{ba}$ forward bias condition starts and it begins to conduct along with the D1, then T1, D1, D3, T2 and D2 starts conducting simultaneously.(fig.X.4(a)). The D1 carries the dc current $I_d$ flowing through the power device T1 minus the line current due to the line voltage $e_{ba}$ due to the conduction of the D3 which carries a current, $iD3$. Thus the current $i_{D1}$, where, $i_{D3} = \left[\frac{3}{2}\right] C dE_{ba}/dt$. It is the slope $d e_{ba}/dt$. As the slope is maximum at $\omega t = 120^\circ$, therefore, $i_{D1}$, will have the smallest value at that instant (seen in fig. X.4 (d)) due to the simultaneous conduction of the diode D1 and D3 the capacitor C1 in parallel with series combination of the capacitors C3 and C5 appears across the phases B and A. Here, $i_{c1} = \left[\frac{3}{2}\right] C dE_{ba}/dt$. As slope is maximum, at $\omega t = 120^\circ$, therefore, $iD1$ will have the smallest value at that instant (seen in fig. X.4 (d)), due to the simultaneous conduction of the diode D1 and D3 the capacitor C3 in parallel with series combination D3 and capacitor C1 now, $V_{c1}$ rises and follows line voltage across the capacitor C1. $V_{c1}$ follows line Voltage $e_{ba}$ and the voltage across the capacitor $C5$; $V_{c5}$ rise in the capacitor voltage $V_{c1}$. During (t2-t3) the voltage across the capacitor C1; $V_{c1}$ rises and follows line voltage $e_{ba}$. It follows half the line voltage (-$e_{ba}/2$) profile and finally at $t_3$; $V_{c5}$ becomes zero. The voltage $Vc3$ allows to fall, as ($-e_{ba}/2$); half the line voltage and reaches maximum negative. From fig.X.4 (d) we can see that $iD1 = i_{T1} + iC5 + iC1 + iD1$; Here, the inductor current $I_d = 1/L \int e_{ac} \, dt$.

**Interval (t3-t4):** Here, T3 is triggered and commutates T1, D1 and T2, D2. The inductor current $I_d = 1/L \int e_{ac} \, dt$. During this period the devices T3 and D2 and T2, D2 and D4 Conduct (fig. 4.4 a) it’s resultant circuit structure is shown in fig.X.4(f) since any D3 conducts, so $V_{c5}, V_{c1}, V_{c3}$ are held constant at a value same as that at the instant $t_3$ (fig. X.4 b). Thus, $i_{c5}$
due to $D_4$'s conduction, $i_s$ becomes negative and equals $C \, \frac{d\epsilon_{ab}}{dt}$. Here $V_{d1} = (\epsilon_{ab} - V_{c1})$ (fig. X.4 (c)) and voltage across dc inductor $V_d \, \epsilon_{bc}$ and inductor current $i_d = \frac{1}{L} \int \epsilon_{bc} \, dt$.

**Interval** ($t_4$-$t_5$): At $t_4$, when $V_{c3} = \epsilon_{ab}$, the $D_5$ starts conducting along with $D_3$ during ($t_4$-$t_5$) period $T_3$, $D_3$, $D_5$, $T_4$ and $D_4$ conduct $D_1$ and $T_1$ do not conduct (seen in fig.X.4 (a)). The resultant configuration is shown in fig.X.4(f) since $D_3$ and $D_5$ conduct simultaneously, the $C_3$ in parallel with series combination of the capacitors $C_5$ and $C_1$ appear across the phase C and B.

Hence $i_5 = i_c = \frac{i_p5}{3}$ and $i_s/3 = [5/3]i_D$ and $i_D = [3/2] \, C \, \frac{d\epsilon_{ab}}{dt}$ flows between the $D_5$ and $D_3$ along the direction shown in fig.X.4(f) the direction of current in the $C_5$ and $C_1$ remains $-$ve hence it result in fall in the capacitor voltage $V_{c5}$ and $V_{c1}$ (shown fig.X.4b).

Here $i_D = i_{T1} = 0$, due to reverse bias. The reverse bias magnitude for $D_1$ is $V_{c1}$ and for $T_1$ is $(\epsilon_{ab} - V_{c1})$, Seen in fig.X.4(c). The $i_a = -i_D$ due to simultaneous conduction of $T_4$ and $D_4$. The dc inductor voltage $V_d = \epsilon_{ba}$ and inductor current $i_d = \frac{1}{L} \int \epsilon_{ba} \, dt$ shown in fig X.4 (d) and (e).

**Interval** ($t_5$-$t_6$): At the instant of $t_5$, $T_5$ is 'ON', which conducts $T_3$ and $D_3$ these outgoing devices are held reverse biased by line voltage $\epsilon_{ab}$ in this interval $T_5$, $D_5$ and $T_4$, $D_4$, and $D_6$ conduct (fig. X.4 (a)). The resultant circuit structure which is shown in fig. X.4f, here only $D_5$ conducts, therefore, $V_{c5} = V_{c1} = V_{c3}$ = Remain constant (fig. X.4b). Here $V_{d1} = \epsilon_{ac} - V_{c5}$ and $V_{T1} = V_{c5}$. Both devices hold reverse voltage across them (fig.X.4c in this interval, $i_5 = I_c = i_{c3} = i_{D1} = i_{T1} = 0$. Due to $D_4$ conduction, a $-$ve current of $[3/2 \, C] \, d\epsilon_{ab}/dt$ is flown in phase A. Here $i_d = \frac{1}{L} \int \epsilon_{ba} \, dt$ (shown in fig. X.4(c and d ). At $t = t_6$, $V_{c5}$ once again become equal to $\epsilon_{ac}$ and the $D_1$ begins to conduct after this instant, $V_d = \epsilon_{ac}$. The fig.X.5(a) shows the oscillogram of input supply voltage $\epsilon_{ac}$, $V_{c5}$, $V_{d1}$ and $i_s$ for the switching delay angle $\alpha = 30^0$ with external resistance of $10\Omega$ (for limiting the current) is connected in series with dc inductor. The figure shown in fig.X.5(a) closely matches with the corresponding waveforms are mainly due to presence of source inductance, which has been neglected in the predicted waveforms. In the experimental waveforms, the effect of inductances is included which is seen in the form of resonance between the source inductance and the commutating capacitors, (fig. X.5 pp
In the oscillogram various waveforms of current in the D1, T1. ic5, and ic1 are shown. It is important to note that the waveforms of ic5 and ic1 are negative of corresponding waveforms shown in fig. X.4. This is due to practical reasons, the reference direction of capacitor current are not followed while obtaining oscillogram. Therefore while comparing the theoretical and experimental waveforms, this point is to be kept in view and it is observed that both theoretically predicted and experimental current waveforms match closely. Fig.X.5 (c) shows the oscillograms when a 32 ohm external resistance is connected in series with the dc inductor of the compensator circuit.

Special features of sub-mode 0°- 60° of the compensator operation:

1. At any instant only one or two diode conducts in upper and lower section of the compensator.

2. D1 and D4 do not conduct simultaneously at any instant for the range of α = 0° to 30°. So, ia exists due to the conduction of either D1 or D4 in this range, but in the range of 30° to 60° both conduct simultaneously.

3. D1 starts conducting at, ‘t0’= -α (switching delay angle of thyristor)

4. For α = 0 commutating capacitors have zero charging or discharge periods as a result voltage across all capacitors remains zero.

5. For the range of operation between 0° and 60° the commutating capacitors charge and discharge for the duration equal to double of the switching delay angle period. In this period the capacitances holds constant voltages for a duration equal to 2. (60°- α). This duration of constant voltage across the commutating capacitors decrease with the increase in α.

(X.1.2) Submode 60°-90°:

For α = 60°, all the capacitors alternately charge or discharge for a duration of 120°. The duration of constant voltage across the capacitors is absent at this α. The compensator is on the verge of simultaneous conduction of three diodes in upper and lower sections. This leads to the beginning of another sub-mode from α = 60° onwards.

The conduction pattern of the controlled power devices and diodes are shown in fig. X.6(a). Here duration between successive switching ‘ON’ of the diode D1 has been subdivided into twelve time intervals. The α =75° has been considered for the analysis purpose here. The selection of intervals is based on the switching ‘ON’ of either thyristor
or diode of the compensator. The analysis is of voltage and current wave form patterns are given below for each interval.

(a) **Interval $t_0-t_1$:**

During this interval $T_3$, $D_1$, $D_3$, $D_5$, $T_4$, $D_4$ and $D_6$ conduct (shown in fig. X.6 a). Due to the conduction of the $D_5$, $D_3$, the $C_5$, $C_1$ and $C_3$ get charged. Due to reverse biasing, the $T_3$ and $D_3$ conduct to carry the dc current $i_d$ from the input phase $B$. If the voltage across the diode $D_1$ is monitored before the instant ‘$t_0$’ (seen in fig.X.6 c). It is observed that the diode voltage is the difference between the line voltage $e_{ac}$ and $V_{ac}$ (fig. X.6 (b)) before the instant ‘$t_0$’ this voltage is $-ve$. The anode potential of the diode is greater than its cathode potential and the diode starts conducting. The resultant circuit structure after $D_1$ is ‘ON’ which is shown in fig. X.6 (f). Due to the simultaneous conduction of the $D_1$, $D_3$, $D_5$, $C_5$, $C_1$, $C_3$ connected in $\Delta$ configuration are charged by the respective line voltages and the

\[ i_{c5} = C \frac{d\theta_{ba}}{dt}; \]

\[ i_{cl} = \frac{d\theta_{ba}}{dt}; \]

\[ i_{cl} = C \frac{d\theta_{cb}}{dt}, \]

flow through them and the current in the $T_1$ is zero, since it is not gated. (shown in fig.X.6 (d)).

$D_4$ conducts and $i_{ds} = \left[3/2\right] C \frac{d\theta_{ba}}{dt}$ and $i_a = \left( i_{D1} - i_{D4} \right)$. Here $V_{D1} = 0$ and $i_{D4} = id - \left[3/2\right] C. \frac{d\theta_{ba}}{dt}$ and $i_a = \left( i_{D1} - i_{D4} \right)$ the voltage across the dc inductor $V_d$ will be due to the simultaneous conduction of $T5$ and $D5$ and $T4$ and $D4$, this makes $V_d = e_{ca}$ and $i_d$ becomes $=\left[1/L\right] \int e_{ca} \, dt$; here $R$ is neglected (the waveforms are seen in fig. X.6 e).

**Interval $t_1-t_2$:** At time $t_1$ the $T_5$ is ON and $T3$ and $D3$ turns OFF. The $T5$, $D5$, $D1$, $T4$, $D4$ and $D6$ conduct (shown in fig. X.6f). Due to simultaneous conduction of $D1$ and $D5$ the $i_{c5}$ flows in parallel with series combination of $C1$ and $C3$ here $i_{cl} = i_{c3} = \left(3/2\right)C \frac{d\theta_{ca}}{dt} / \alpha = \left(i_{D1} / 3, \left(2i_{D1} / 3 = i_{c5} \right)$. Fig.X.6b and d show the waveforms of voltage profile of $Vc1$, $Vc3$, $Vc5$. Here $i_{T1} = 0$ since it is not yet gated and holds voltage across $C5$ (shown in fig. X.6c). Here $i_{D1} = \left(i_{D1} \right)$ currents in $T1 + C1 + C5$ (shown in fig.X.6d). The conducting devices in lower section are the same as given during interval $t_0-t_1$ above. The voltage across dc inductor $V_d$ will be due to simultaneous conduction of $T5$ and $D5$ and $T4$ and $D4$. Thus, $V_d = e_{ca}$ And $i_d = 1/L \int e_{ca} \, dt$ shown in fig. X.6c.

**Interval $t_2-t_3$:** During this period $T5$, $D5$ and $D1$ conduct therefore the analysis for the various waveforms, $V_{c5}$, $V_{c1}$, $i_{c5}$, $i_{c1}$, $i_{T1}$, $i_{D1}$, $V_{T1}$ and $VD1$ (fig. X.6) of the compensator remain the same as those described for the interval $t_1-t_2$ in this sub-mode. $T4$, $D4$, $D2$ and
Fig. X.6 Conduction pattern of power devices and predicted wave-forms in sub-mode
60°-90° and at α = 75° (a) Conduction pattern of power devices (b) Voltage wave-forms
across capacitors C5, C1, C3, along with input sup wave-forms (c) Voltage wave-forms
across power device T1, and D1

π-a
Fig. X.6 (d) Conduction pattern of power devices and predicted waveforms in submode 60°-90° and at $\alpha = 75^\circ$, current waveforms through $T_1, C_1, C_5, D_1, D_4$, and phase A.
Fig. X.6 (e) Conduction pattern of power devices and predicted Wave-forms in sub-mode 60°-90° and at α = 75°. Voltage and current wave-forms at the dc side of the compensator Vd and Id along with input voltage wave-forms.
Fig.X.6 (f) Conduction pattern of power devices and predicted wave-forms in sub-mode 60°-90° and at $\alpha = 75^\circ$. Operational circuit structures at the upper section of the compensator at different time intervals.
D6 conduct and a negative component of phase ‘A’ current is due to current in D4 which is sum of the capacitor currents $i_{c4}$ and $i_{c2}$ where, $Ic4 = C \frac{d\theta_{eb}}{dt}$ and $i_{c2} = C \frac{d\theta_{eb}}{dt}$; $Vd = \theta_{cb}$ which is due to the simultaneous conduction of T5, D4, T4, D5. The $i_d = \frac{1}{L} \int \theta_{cb} dt$ the wave forms are shown in fig. X.6 (e).

**Interval t3-t4:** Here T5, D5, and D1 conduct. The waveforms of $Vc5$, $Vc1$, $VD1$, $VT1$, $i_{d1}$, $i_{T1}$ and $i_{c5}$ are shown in fig.X.6 and it is the same as those described for the interval (t1,t2). The T6 is ON and responsible to OFF the T4 and D4. Here T6, D6 and D2 simultaneously conduct. Since during this interval D4 remains OFF hence no current is contributed to $i_a$ from the lower section of the compensator. Therefore $i_a = i_{d1}$ and $Vd = \theta_{cb}$ and $i_d = \frac{1}{L} \int \theta_{cb} dt$ (fig. X.6).

**Interval t4-t5:** At the instant $t_4$ the diode D3 comes out of reverse bias and starts conducting in a similar manner as the diode D1 began conduction at ‘$t_0$’ . Three diode conduction modes repeated during this time interval and T5, D5 and D1, D3 conduct. The analysis for the wave forms $V_{c5}$, $V_{c1}$, $i_{c5}$, $i_{c1}$, $i_{d1}$, $i_{T1}$, $VD1$ and $i_d$ in this interval remains the same as that for the interval ($t_0$. $t_1$) mode, discussed earlier. Here T1 does not conduct and is held reverse bias by voltage $V_{c5} = \theta_{cb}$. The T6, D6, and D2 still conduct and therefore the explanation for the contribution to the input current form this section as well as for DC voltage and current waveforms remains the same as that of during the interval t3-t4 (discussed earlier) All the waveforms during this interval are given in fig. X.5b to X.6e.

**Interval t5-t6:**
At starting of $t_5$, T1 is switched ON and T5 and D5 is turned OFF due to reverse bias voltage $\theta_{cb}$ appearing across both these devices, therefore T1, D1, D3, T6, D6 and D2 conduct simultaneously (seen in fig. X.6 (a)) the resultant circuit structure of the upper section is shown in fig X.5 (f). In this circuit the power device T1 carries the DC current $i_d$ and the diode D1 carries the difference of currents $i_d$ and $I_{D3}$, where $i_{D3}$ is due to the simultaneous conduction of the diodes D1 and D3 and equals to $-\frac{3}{2C} \frac{d\theta_{eb}}{dt}$ the slope $d\theta_{eb}/dt$ is maximum at $\omega = 120^0$. Hence $i_{D1}$ has the smallest value at this instant (fig. X.6(d)) Due to the simultaneous conduction of the diodes D1 and D3, the capacitor C1 in parallel with series combination of the capacitors C3 and C5 appears across the phases B and A. Here $i_{c5} = i_{c5} = iD3/3$ and C1 and C5 reverse. This results in fall in the capacitor.
voltage $V_{c5}$ and rise in the capacitor voltage $V_{c1}$. During the interval ($t_5$-$t_6$) the diode current $i_{D1} = i_{TI} + i_{c5} + i_{c1}$. From fig.X.6 (b) it is clear that the voltage $e_{ac}$ being greater than $V_{c5}$, keeps the diode $D_5$ held in reverse bias. No current will be contributed to $i_a$ from the lower section of the compensator because of the conduction of $T_6$, $D_6$, and $D_2$.

Thus current wave form in the input phase $A$ during this period $= i_{D1}$ the dc side inductor voltage $V_d = e_{ab}$ due to simultaneous conduction of power devices $T_1$, $D_1$, $T_6$ and $D_6$. Here $i_d = \frac{1}{2} \int e_{ab} \, dt$ (dt (fig. X.6 (e))

**t_6-t_7 interval:**
During this interval also devices $T_1$, $D_1$ and $D_3$ conduct in the upper section of the compensator therefore the analysis for the wave forms $V_{c5}$, $V_{c1}$, $V_{c3}$, $i_{c5}$, $i_{c1}$, $i_{D1}$, $i_{TI}$, $V_{D1}$ and $V_{T1}$ remain same as those for the interval $t_5$-$t_6$ of this sub-mode. $T_6$, $D_6$ and $D_4$ conduct, thus three-diode conduction takes place the $-ve$ component of phase $A$ current is due to current in the diode $D_4$ i.e. $i_{D4} = i_{c4} + i_{c2}$ where, $i_{c4} = C \frac{de_{ac}}{dt}$ and $i_{c2} = C. e_{ac}/dt$. In this interval $i_a$ still remains the difference of $i_{D1}$ and $j_d$. The expression of dc voltage is $V_d = e_{ab}$ and dc current is $i_d = \frac{1}{2} \int e_{ab} \, dt$. This is shown in fig. X.6(e).

**t_7-t_8 interval:**
Here, $T_1$, $D_1$ and $D_3$ still conduct, thus $V_{c5}, V_{c1}, i_{c1}, i_{TI}, i_{D1}, V_{T1}$ and $V_{D1}$ (fig. X.5) remain the same as of the interval $t_6$-$t_7$. Here the $T_2$ is switched ON resulting the ‘OFF’ of $T_6$ and $D_6$ and conduction of $T_2$, $D_2$ and $D_4$ simultaneously, the $i_d = \frac{3}{C} e_{ac}/dt$ and input current in phase $A$ = $i_a = i_{D1} - i_{D4}$ (seen in fig.X.6 (d) ) the dc voltage $V_d = e_{ac}$ since $T_2$, $D_1$, $T_2$ and $D_2$ conduct and the dc current is $i_d = \frac{1}{2} \int e_{ac} \, dt$ (shown in fig. X.6e).

**t_8-t_9 interval**
At the instant $t_8$ the diode $D_5$ comes conducting in the same way as the diodes $D_1$and $D_3$ began conducting at the instants $t_7$ and $t_1$ respectively. In the upper section of the compensator, the devices $T_1$, $D_1$, $D_3$ and $D_5$ conduct simultaneously; therefore, the analysis for the waveforms of $V_{c5}$, $V_{c1}$, $V_{c3}$, $i_{c5}$, $i_{c1}$ and $V_{D1}$ remains the same as those during the interval ($t_8$-$t_9$). All 3 diodes of upper section are conducting here, so the voltage across the device $T_1, V_{T1} = 0$ and the $T_2$, $D_2$ and $D_4$ still conduct here $i_{D1}$ = ($i_{c5}$ + $i_{c1}$ + $i_d$). The $T_2$, $T_2$ and $D_4$ still conduct so the $i_{D4}$ would be having some expression as given for ($t_7$-$t_8$) interval and $i_a = (i_{D1} - i_{D4})$ and $V_d = e_{ac}$ And the dc current $i_d = \frac{1}{2} \int e_{ac} \, dt$ (seen in fig. X.6e)
to-t10 interval:
Here the T3 is switched 'ON' and T1 and D1 turn off due to the reverse biasing voltage $e_{ba}$. Now T3, D3, D5, T2, D2 and D4 only conduct. The resultant circuit configuration is shown in fig.X.6(f) since D3 and D5 conduct simultaneously. The capacitor C3 is in parallel with series combination of C1 and C5. Thus $i_{c5} = i_{c1} = \frac{i_{D5}}{3}$ and $i_{D5} = \frac{[2i_{D5}]}{3}$ and $i_{D5} = \frac{[3/2]}{C}de_{cb}/dt$ which flows between the D5 and D3 (shown in fig.X.6f). It is observed that a negative direction of $i_{c5}$ and $i_{c1}$ resulting in fall in the $V_{c5}$ and $V_{c1}$ (shown in fig. X.6b) the current in the diode D1 and the controlled power device T1 are absent. The T2, D2 and D4 still conduct; therefore the relationship for $i_{D4}$ remains the same as that of during the interval $t_7$. The $i_a = i_{D1}, i_{D4}$ and dc voltage is $V_d = e_{bc}$ and $i_d = \frac{1}{L}\int e_{bc} \, dt$ (seen in fig.X.6(e)).

$t_{10}-t_{11}$ interval:
Like (t9-t10) interval, T3, D3 and D5 conduct. So the waveforms are also the same. The T2, D2, D4 and D6 conduct in lower section which gives $i_{D4} = (i_d + i_{c4} + i_{c2})$ where, $i_{c4}$ and $i_{c2}$, are $i_{c4} = C \frac{de_{cb}}{dt}$ and $i_{c2} = C \frac{de_{cb}}{dt}$ and $i_{a} = (i_{D1} - i_{D4})$ since the device T3, D3, T2 and D2 simultaneously conduct. Here DC Voltage $V_d = e_{bc}$ and $i_d = \frac{1}{L}\int e_{bc} \, dt$ as shown in fig. X.5 (b) to X.5 (e).

$t_{11}-t_{12}$ interval:
Here also the T3, D3, D5, conducts in upper section of the compensator like in the interval (t9-t10). So that the analysis for all waveforms will be same as per the (t9-t10) interval sub mode. Now, T4 is switched ON and T4, D4 and D6 conduct simultaneously. Therefore $i_{D4} = i_d + i_{c4} + i_{c2}$ where $i_{c4} = C \frac{de_{cb}}{dt}$ and $i_{c2} = \frac{C}{2} \frac{de_{cb}}{dt}$ and $i_{a} = i_{D1} - i_{D4}$; $V_d = e_{ba}$ and $i_d = \frac{1}{L}\int e_{ba} \, dt$ ( waveforms are shown in fig. X.6 )

Fig.X.7 (a) shows the oscillogram of the input voltage $e_{bc}$, $V_{c5}$, $V_{D1}$ and $i_a$ at $\alpha = 75^\circ$. Most of the waveforms shown in fig.X.7 (a) closely match with the corresponding waveforms in fig.X.6, the reason for the deviation from the waveforms has been discussed in previous sub-mode. The fig.X.7 (b) shows the oscillogram of the waveforms of the current in D1,T1, and C5 and C1 respectively. It is seen that, $i_{c5}$ and $i_{c1}$ are -ve in fig.X.6 due to similar reason as mentioned in the previous sub-section of
(0° - 60°) interval. It is observed that both theoretically predicted and experimental current waveforms match closely. Fig.X.7 (c) shows the waveforms of dc voltage and current along with input line voltage. There experimental waveforms also agree closely with the corresponding theoretically predicted waveforms shown in fig.X.6. The following special features have been identified for this sub-mode of compensator operation.

1. All diodes start conducting at an instant corresponding to \(-\alpha\), measured from the respective zero cross over of the line voltage.

2. Commutating capacitor are continuously being either charged or discharged by the input supply.

3. Normally the compensator in this sub-mode should draw lagging current whose magnitude depends on the value of \(\alpha\) and \(i_d\). However, it draws additional leading current from the source when all three diodes in a given section conduct simultaneously. When commutating capacitors with large capacitors are used and also if the resistance of the dc inductor is high, under these conditions it may be possible that the VAR from the source may be lagging. Therefore, if large commutating capacitors are used or dc resistance of the load inductor is high, it is expected that a structural change in the operation of the compensator would occur. When \(\alpha\) is changed from 0° to 90°; due to the simultaneously conduction of devices T1-D1, T2-D2, T6-D6 the displacement angle of the fundamental component of input current would draw maximum real power and zero lagging reactive power. As \(\alpha\) is increased, it could be predicted from the waveform analysis that the real power consumed by the compensator would decrease and reactive power consumption would increase. However, the peak reactive power (lag) and the delay angle \(\alpha\) at which it occurs cannot be predicted from this analysis. From the present analysis it is seen that for \(\alpha > 90°\); the commencement of conduction of the diode D1 can not occur at \(\omega t = \alpha\) any more, but it gets fixed to \(\omega t = 90°\) (seen in fig.X.8). This would happen because the potential difference between line voltage, \(e_{ac}\) and the voltage across the capacitor \(C_6, V_{c6}\) keeps the diode D1 under reverse biased condition before \(\omega t = 90°\). This leads to a new sub-mode from \(\alpha = 90°\) onward and is discussed in the following section.

**X.1.3 (90° - 150°) Submode:**

In the previous sub-mode (60°-90°), it has been observed that for the switching delay angle \(\alpha = 90°\) of the controlled power devices, the D1 would start conducting at \(\omega t\)
Fig. X.7 (a) Experimental wave-forms at $\alpha = 75^\circ$ in the sub-mode $60^\circ-90^\circ$; $R=10$ ohm, Horizontal Axis, 20ms full peak vertical axis, 0.3 A peak
$V_{ac}=38.6$V peak, $V_{D1} = 12$V peak; $i_a = 1.1$A peak; $i_T1 = 1.1$A peak

Fig. X.7 (b) Experimental wave-forms at $\alpha = 75^\circ$ in the sub-mode $60^\circ-90^\circ$; $R=10$ ohm, Horizontal axis = 50ms full Scale, vertical axis, $i_{o1} = 1.1$A peak; $i_{T1} = 1.1$A peak; $i_{CS} = 0.3$A peak.
Fig. X.7 (c) Experimental wave-forms at $\alpha = 75^\circ$ in the sub-mode $60^\circ$-90$^\circ$; $R=10$ ohm-
Horizontal axis $e_{ac} = 20$ ms full scale, Vertical axis $e_{ac} = 40$ V peak $V_d = 45$V peak to peak; $i_d$: 1.2A peak
It is also seen that when \( \alpha \) is further increased beyond \( 90^\circ \) even then, \( D_1 \) would begin its conduction at \( \omega t = -90^\circ \) only. We can explain it like as follows: Before the instant, \( D_1 \) starts conducting, \( D_5 \) has already started conducting and the voltage across the capacitor \( C_5 \) keeps the cathode potential of the diode \( D_1 \) below the voltage of the input phase \( C \), where as, the potential of anode of \( D_1 \) which is connected to phase \( A \), is equal to the line voltage \( (e_{ac} - V_{c5}) \) which is always negative for \( \omega t < -90^\circ \) and this keeps the diode \( D_1 \) reverse biased till \( \omega t \) reaches \(-90^\circ \) (seen in fig. X.8) when \( \omega t > (-90^\circ) \), now \( (e_{ac} - V_{c5}) \) becomes +ve and the diode \( D_1 \) becomes forward biased and goes into conduction. Therefore, the conduction of the \( D_1 \) always starts at \( \omega t = 270^\circ \) w.r.t. +ve zero cross over (or \( \omega t = -90^\circ \) w.r.t. the next positive zero cross over) of line voltage \( e_{ac} \).

Similarly the conduction of the \( D_3 \) and \( D_5 \) would start at angle \( 270^\circ \) with respect to +ve zero cross over of the live voltage \( e_{ba} \) and \( e_{cb} \) where \( D_1 \) is 'ON'. Now the phase \( A \) gets connected to the left hand plate of the capacitor (shown in fig. X.3a). For \( \omega t > -90^\circ \) both to line voltage \( e_{ca} \) and the capacitor voltage \( V_{pq} \) are positive. However, the magnitude of \( e_{ca} \) being smaller than the magnitude of the capacitor voltage \( V_{pq} \) (shown in fig. X.3a), \( D_5 \) is held reverse biased. Therefore, at \( \omega t = -90^\circ \). At \( 270^\circ \) from positive zero crossover of \( e_{ca} \) when the diode \( D_1 \) starts conducting at the same instant, the diode \( D_5 \) stops conducting. Similarly \( D_3, D_5 \) starts and \( D_1, D_3 \) stop conducting. Similar argument is true for the lower section of the compensator hence the conduction period of the diode in this interval is of \( 120^\circ \) duration. Since all the diodes conduct for \( 120^\circ \), therefore all the diode in the upper section \( D_1, D_3, \) and \( D_2 \) also can't conduct simultaneously. Due to symmetry \( D_1 \) & \( D_4 \) are in phase \( A \) and \( D_3 \) & \( D_4 \) in phase \( B \) and \( D_5 \) & \( D_2 \) in phase \( C \) will be conducting \( 180^\circ \) apart. Therefore, simultaneous conduction of the upper and lower diodes \( D_1-D_4, D_3-D_6 \) and \( D_5-D_2 \) will not be possible in this interval consequently the current waveforms are possible in this interval. Consequently the current waveforms in each phase coil have positive and negative half cycle of \( 120^\circ \) each and having an amplitude \( i_4 \).

The conduction pattern of the device in the input phase \( A \) for the switching delay of device \( T_1 \) of \( \alpha = 120^\circ \) is shown in fig. X.9 (a). It is shown in the figure that the \( D_1 \) starts conducting at \( \omega t = 270^\circ \) from the positive zero cross over of line voltage \( e_{ac} \) which occurs after the controlled power device \( T_1 \) is switched on. Here the conduction patterns of the diodes are fixed and only the instants of conduction of controlled power device

\[ \text{ww} \]
shift with positive change in switching delay angles. So, two consecutive instant of
initiation of conduction of any controlled power device have been considered here. It has
been divided into twelve, time intervals since the conduction and the waveforms of the
controlled power device T1 and D1 of the input phase A are only studied and the diode
D1 takes over conduction from D5, therefore the voltage and current waveforms in the
capacitor C5 are carefully monitored.

**Interval \( t_0-t_1 \):** At \( t = t_0 \), T1 is switched on at switching delay angle = \( \alpha \). The T1, D3, T6 and D6 conduct simultaneously, the resultant circuit structure is shown in fig.X.9 (a).

Due to the simultaneous conduction of T1 and D3 current flows through the C3 in
parallel with series combination of C3 and C5 and the \( i_{c3} = \frac{i_d}{3} \) which flow along the
direction shown in (fig.X.9 (b)). The \( V_{c1}, V_{c3}, V_{c5} \) are shown in fig. X.9 (b). The voltage
across the C5 falls from its peak value \( V_{c50} \) as the same rate as the voltage fall across the
C3. Fig.X.9 (b) also shows that \( V_{c1} \) rises at a rate twice of that as the rate of the fall in
\( V_{c5} \). Fig.X.9(a) shows the reference direction of all capacitors voltages. The voltage
waveforms across the power device T1 and the diode D1 are shown in fig.X.9 (c). Since
the device T1 is conducting, therefore \( V_{T1} = 0 \) and \( V_{D1} = (\varepsilon_{dc}-V_{c5}) \) (or \( Yar \)). All the current
waveforms are shown in fig.X.9(d) Here, \( i_{c5} = [1/3] i_d \) and \( i_{c1} = [2/3] i_s \) , \( i_{D1} = 0 \) since
only D4 in phase A is conducting thus \( i_s = -i_d \) (shown in fig.X.9(d)). The magnitude of the
current in \( C_4, C_6 \) and \( C_2 \) are \( 2/3 i_d \) and \( 1/3 i_c \) respectively. The rate of change of voltage
in the capacitor C4 will be twice as that of the change in the capacitors C6 or C2 (Seen in
Fig 4.9 (b)). From Fig 4.3 (a), \( V_d \) the DC inductor voltage = \( V_{yx} \) (seen in fig.X.9(b ) from
fig.X.3 (a), \( V_d = dc \) inductor voltage = \( V_{yx} = V_{in} + \varepsilon_{ba} V_{qr} \) and \( dc \) current is \( i_d = \frac{1}{L} \int V_d dt \).

**Interval \( t_1-t_2 \):** At \( t = t_1 \), D5 comes into conduction and D3 is turned ‘OFF’, as the reverse
bias across D5 is removed. Here D3 runs in reverse bias due to relative change in the
magnitude of voltage across the capacitor C3 and \( \varepsilon_{cb} \). After this T1, D5 and T6 and T6
and D4 conduct simultaneously. The resultant circuit structure is shown in fig.X.9 (f) due
to the simultaneous conduction of the devices T1 and D5 current flows through the
capacitor C5 in parallel with the series combination of C1 and C3 Thus \( i_{c1} = i_{c5} = \frac{i_d}{3} \) and
\( i_{c5} = \frac{2 i_d}{3} \) the direction is shown in fig.X.9 (f). The Voltage across the capacitor C1,
\( V_{c1} \) continuously rise at a rate which is half of the rate of fall in voltage across the C5
Fig X.8 Voltage waveform across the commutating capacitors in the upper section w.r.t. input voltage waveforms at $\alpha = 90^\circ$
Fig. X.9 Conduction pattern of power devices and predicted waveforms in sub-mode 90°-150° and at $\alpha = 120^\circ$ (a) Conduction pattern of power devices (b) Voltage waveforms across capacitances $C_2, C_3, C_4$ along with input voltage waveform (c) Voltage waveforms across power devices $T_1$ and $D_1$. 
Fig. X.9(d) Conduction pattern of power devices and predicted waveforms in sub-mode 90°-150° and at α = 120°. Current waveforms through T1, C1, C5, D1, D4, and Phase A.
Fig. X.9(e) Conduction pattern of power devices and predicted waveforms in sub-model 90°-150° and at α = 120°. Voltage and current waveforms at positive dc side of the compensator, Vd and Id along with voltage waveforms at the input and across commutating capacitors.
Fig. X.9 (f) Conduction pattern of power devices and predicted waveforms in sub-mode 90°-150° and at α = 120°, Operations circuit structures, at the upper section of the compensator, at different time intervals.
(shown in fig.X.9 b). Fig.X.9c, shows the voltage across the T1 and D1, due to conduction of T1, \( V_{T1} = 0 \), and \( V_{D1} = (e_{ac} - V_{c5}) \), since D5 conducts. Fig.X.9 (d) shows the wave forms of current of C1,C3, C5, T1 and D1 we can see that, \( i_{T1} = i_d \) and \( i_{D1} = 0 \) and \( i_t = -i_d \) since T6 and D4 conduct, fig.X.9e shows the waveforms of the conduction of T1, D5, T6, D4. Here \( V_d = (V_{T5} + e_{cb} + V_{q}) \) and \( i_d = \frac{1}{L} \int V_d \, dt \).

**Interval \((t_3-t_4)\):**

Here, T2 starts conduction and T6 goes ‘OFF’, resulting T1, D5, T2, D4 conduct simultaneously. It’s upper-section structure is same as \((t_1-t_2)\) section. Due to conduction of T2 and D4, \( V_{c4}, V_{c5} \) and \( V_{c2} \) change as shown in fig.X.9 (b) since D4 conduct so, \( i_t = -i_d \) (shown in fig.X.9 (e)). due to the conduction of the devices T1 and D5 and T2 and D4 the voltage across the dc inductor \( V_d = (V_{T5} + e_{cb} + V_{q}) \) and the dc inductor current \( i_d = \frac{1}{L} \int V_d \, dt \) (all waveforms are shown in fig. X.9).

**Interval \((t_4-t_5)\):**

Now D4 turns OFF and D6 is ON, so T1, D5, T2 and D6 starts conducting, simultaneously. The lower section waveforms pattern are similar to that of the interval \((t_2-t_3)\) since D1 to D4 do not conduct and the \( i_a \) remains zero and \( V_d = \) Voltage across dc inductor with T1, D5, T2, D2 being ‘ON’ which is \( = (V_{ac} + e_{cb} + V_{q}) \) and \( i_a = \frac{1}{L} \int V_d \, dt \) as shown in the fig. X.9 for all the waveforms.

**Interval \((t_5-t_6)\):**

T3 is switched on at \( t_4 \), T1 turns OFF due to the appearance of reverse voltage across it. Now T3, D5, T2 and D6 Conduct simultaneously. The resulting circuit is shown in fig. X.9 (f). Due to the simultaneous conduction of T3 and D5 the \( i_d \) flow through the C3 in parallel with series combination of C5 and C1 their currents are \( i_{c1} = i_{c2} = i_d / 3 \) and \( i_{c3} = 2i_d / 3 \) flow along the direction shown in fig. X.9 (f). Now voltages of C1and C5 fall but it rises for C3 with the rate of the twice as that of fall of former, across T1. Due to the conduction of the D5 the reverse voltage across non conducting diode D1 and \( V_{D1} \), the \( V_{D1} = (e_{ac} - V_{q}) \) Here \( i_{T1} = i_{D1} = 0 = i_a \) and \( V_d = V_{ac} + e_{cb} + V_{q} \) and dc inductor current \( i_a = \frac{1}{L} \int V_d \, dt \). Waveforms are shown in fig. X.9.

**Interval \((t_6-t_7)\):**

Now D1 starts and D5 stops, therefore T3, D1, T2 and D6 conduct simultaneously. The resultant circuit is shown in fig.X.9 (f). Due to the simultaneous conduction of T3 and D1
the current $i_d$ flows through the capacitor $C_1$ in parallel with the series combination of $C_5$ and $C_3$, their current are: $i_{c1} = 2i_d/3$ and $i_{c5} = i_d/3$. In fig. X.9 (f) its direction is also given. As a result of the direction of the capacitor current flow, the voltage across the capacitor $C_1$ rapidly falls at a rate twice as that of rise in the voltage across the capacitors $C_5$ and $C_3$. Since it is off and the device $T_3$ being in conduction state, the voltage across the $T_1 = V_{c1}$ and $i_{T1} = 0$, $i_D1 = i_{c1} + i_{c5}$ and the current in the input phase of $i_a = i_{D1} = i_d$ (shown in fig. X.9 (d). The voltage across the dc inductor $V_d$, due to the conduction of $T_3, D_1, T_2$ and $D_6$ is $V_{st} + e_{ab} + V_{ra}$; the $i_d = [1/L] \int V_d \, dt$ (waveforms are shown in fig. X.9).

**Interval $t_6 - t_7$:**
Here at $t = t_6$, $T_4$ is ON and $T_2$ is off. $T_3, D_1, T_4$ and $D_6$ conduct simultaneously, which is same as during the interval ($t_5, t_6$). Since, $T_3$ and $D_1$ conduct along with $T_4$ and $D_6$ the dc inductor voltage $V_d = (V_{st} + e_{ab} + V_{ra})$ and $i_d = [1/L] \int V_d \, dt$. [waveforms are shown in fig.X.9.]

**Interval $t_7 - t_8$:**

$D_2$ starts and $D_6$ stops at the starting of $t_7$. The waveforms are as same as of the duration of ($t_6-t_7$) or ($t_2-t_3$) because $T_3, D_1, T_4$ and $D_2$ conduct simultaneously which is same as in the upper section of ($t_5-t_6$) interval. Once again, since $D_1$ and $D_4$ do not conduct, $i_a = 0$ and $V_d = DC$ voltage $= (V_{st} + e_{ab} + V_{ra})$ and DC current is $i_d = [1/L] \int V_d \, dt$.

**Interval $t_8-t_9$:**

At the instant $t_8$, $T_5$ turns ON and $T_3$ turns OFF, The $T_5, D_1, T_4$ and $D_2$ conduct simultaneously. The resulting circuit is shown is fig.X.9 (f). Due to simultaneous conduction of $T_5$ and $D_1$, $i_d$ flows through $C_5$ in parallel with the series combination of the capacitors $C_1$ and $C_3$. Thus, $i_{c5} = 2i_d/3$ and $i_{c1} = i_{c5} = i_d/3$ flow along the direction shown in fig.X.9(f). Here $V_{c5}$ rises at a rate of twice as that of fall in the voltage across $C_1$ and $C_3$. During this interval current in the device $T_1$ the $i_{T1} = 0$ and $V_{T1} = V_{c5}$ and $i_{D1} = i_{c1} + i_{c5} = i_d$. The $A$-phase current $(i_a - i_{D1})$ and $V_d = V_{st} + e_{ab} + V_{p}$ due to conduction of $T_5, D_1$ and $T_4$ (waveforms are shown in fig.X.9).

**Interval $t_9-t_{10}$:**

$D_3$ starts and $D_1$ gives off due to appearance of reverse voltage across it. $T_5, D_3, T_4$ and $D_2$ conduct simultaneously during this interval. The resulting circuit is shown in fig. X.9. Due to simultaneous conduction of $T_5$ and $D_3$ the $i_d$ flows through $C_3$ which is in parallel with the series combination of the $C_1$ and $C_5$. The $i_{c3} = 2i_d/3$ and $i_{c1} = i_{c3} = i_d/3$ (fig. X.9 (f)). The $V_{c3}$ falls at a rate twice as that of rise of voltage in $C_1$ and $C_5$. Here, $i_{T1} = 0, V_{T1} = V_{c5}$. The $i_{D1} = 0$ and $V_{D1} = (e_{ab} - V_{c1})$.The voltage across dc inductor $V_d$, due to the conduction of $T_5, D_3, T_4$ and $D_2$ will be $(V_{st} + e_{bc} + V_{p})$ and the dc inductor current $i_d = [1/L] \int V_d \, dt$. The circuit structure and waveforms are given in fig. X.9 (f and b) respectively.
Interval $t_{10}$-$t_{11}$:

Now, T6 is switched ON and T4 OFF. Here T5, D2, D3 and T6 conduct. The upper sections characteristics will be same since its conduction sequence is same as per $(t_9-t_{10})$ interval. The waveforms of $V_{c1}$, $V_{c5}$, $i_{c3}$, $i_{c5}$, $i_{D1}$, $iT1$ and $iA$ will remain the same as that of during the interval $t_9$-$t_{10}$. Here D1 and D4 are non-conducting diodes, hence $i_d = 0$. The dc inductor voltage $v_d = V_{su} + e_{ac} + v_{pr}$ and $i_d = \int i_d \, dt$ the waveform are shown in fig.X.9.

Interval $t_{11}$-$t_{12}$:

Now, T6 is switched ON and D2 gets turn OFF. D3, T5, T6 and D4 conduct simultaneously. It has same characteristics like interval $t_9$-$t_{10}$. D4 conducts along with A-phase current $i_a = -i_d$ and $V_d = DC$ voltage $= (VTU + e_{ac} + V_p)$ due to conduction of T5, D3, T6 and D4. The $id = \int i_d \, dt$. The waveforms are shown in fig.X.9. The fig.X.10 (a) shows the oscillogram of input voltage $e_{ac}$, voltage across C5 and D1, $i_a$ for a switching delay angle of $120^\circ$. We observe that waveforms are matching with the predicted waveforms shown in fig.X.9. The deviation is due to the presence of source inductance which has been already explained in previous section. In fig.X.10 (b) it is observed that when the diode D1 starts conduction at an angle $270^\circ$ from positive zero, crossover as line voltage $e_{ac}$, which is confirmed by presence of the $i_a$ and also $V_{D1} = 0$ the diode D5 is "OFF" and remains OFF for a duration of $240^\circ$ corresponding to supply frequency. Fig.X.10 (c) shows the oscillogram of the waveforms of $i_{D1}$, $iT1$, $i_{c5}$ and $i_{c1}$. It is observed that both the theoretically predicted and experimental current waveforms match closely.

Fig X.10 (d) shows the $e_{ac}$ which is matching with the predicted one fig. X.10 (e) and (f) show the experimental waveforms and when an external resistance of 32 ohm is connected in series with the dc inductor (figures X.10 (a) and (b)). It is observed that the instant of beginning of conduction of the diode, when the external resistance is connected, is not $270^\circ$ from the positive zero cross over of line voltage $e_{ac}$ but is delayed for certain duration. All the voltage and current waveforms also shift for the same duration. As a result the compensator consumes more real power from the source while draws less capacitive VAR from the source, when external resistance is connected as compared to the case when no external resistance is connected with dc inductor. Here resonance (current due to source inductance is less dominant. The special features of compensator operations observed in sub-modes are:

1. Only the instant of commencement of conduction of thyristors change with switching delay angle $\alpha$.

2. Conduction of diodes always starts at, $\omega \cdot t = 90^\circ$ from the positive zero cross over or the corresponding line voltage which is equivalent to $\omega \cdot t = 270^\circ$, measured from the previous positive zero cross over of the line voltage. As a result, the fundamental component of the phase current leads the phase voltage by $90^\circ$. Due to this reason the
compensator starts working in the lead VAR region for $\alpha > 90^\circ$. Therefore this sub-mode is included in the lead VAR region of the operation.

3. There is no possibility for capacitors to hold constant voltage across them since T1-D1, T2-D2, T6-D6, D1-D4, D3-D6 and D5-D2 conduct simultaneously.

4. The duration of charge or discharge period of commutating capacitors change with '$\alpha$', but the rate of rise or fall in capacitor voltage remains constant. Thus it maintains the amplitude of dc and AC current at a certain level for all values of '$\alpha$' in this sub-mode.

5. It is observed that each half cycle of the input current is of $120^\circ$ duration.

6. Definite pattern of the transfer of diode conduction in upper and lower section of the compensator has been observed. In this sub-mode it has been observed that $\alpha$ is increased from $90^\circ$ onwards, the commencement of conduction of the controlled power device T1 moves closer to the commencement of conduction of the divide D5 and away from the diode D3. When $\alpha > 150^\circ$ the pattern of transfer of diode conduction in the upper section of the compensator will be D5-D1-D3 following the switching 'ON' of the T1. Similarly in the lower section, the transfer of diode conduction have the sequences D6-D2-D4 following the switching 'ON' the controlled power device T2. This operational phenomena lead to be developed of a new sub-mode (150$^\circ$ - 270$^\circ$), described below.

X.1.4 Sub-mode (150$^\circ$ - 270$^\circ$):
In the previous sub-mode it has been described that till the delay angle $\alpha = 150^\circ$, when T1, is switched on, T5 gets switched off and the diode D5 and T1 start conducting simultaneously. Therefore, T1-D1, T2-D2 - T6 - D6 conduction is not possible in that sub-mode. Consequently all the capacitors always carry charging or discharging current and get charged or discharged all the time. For $\alpha$ beyond 150$^\circ$, for a certain duration, the devices T5 and D5 conduct simultaneously and during this period, the capacitors in the upper section of the compensator do not carry current and hold fixed voltage across them. The devices and the capacitor in the lower section on the compensator operate in a similar manner. The nature of conduction of devices and the capacitors separates this sub-mode from the previous sub-mode (90$^\circ$-150$^\circ$). For $\alpha = 200^\circ$; the conduction pattern of the power devices are shown in fig. X.11 (a). This period is once again subdivided into twelve time intervals based on the circuit structure taken up by the compensator due to the conduction of various devices in that time period. All the circuit structure for these intervals is shown in fig. X.11 (f).
Fig.X.10(a) Experiment waveforms at $\alpha=120^\circ$ in the sub-mode $90^\circ$-150$^\circ$, $R=2$ ohm, Horizontal axis 20ms full scale, vertical axis $E_{ac}=40V$ peak; $V_{cs}=53V$ peak; $V_{D1}=50V$ peak; $i_s=0.53A$ peak; $V_{D1}=50V$ peak; $i_s=0.53A$ peak.

Fig.X.10(b) Experiment waveforms at $V_{DS}=-50$ peak, with same conditions as given in fig. X.10(a).
Fig. X.10(c) Experiment waveforms with $i_d = i_l = 0.53$ A peak; $i_{es} = 0.36$ V peak = ic, with same conditions as given in fig. X.10(a).

Fig. X.10(d) Experiment waveforms with $V_d = 85$ V peak; $I_d = 0.53$ A(peak), $\omega_m = 40$ A(p) with same conditions as given in fig. X.10(a).
Fig.X.10(e) Experiment waveforms with $\alpha = 120^\circ$, $R = 32$ ohms, horizontal axis 20ms full scale, Vertical axis - $e_{ac} = 40$ V, $V_{ds} = 45$V peak; $V_{ds1} = -40$ V peak; $i_s = 0.3$ A peak

Fig.X.10(f) Experiment waveforms with horizontal axis $= 50$ms full-scale. Vertical axis. $i_{d1} = 0.3A = i_{\gamma1}$ & $i_{ds} = i_{os} = 10.2A$ peak. other conditions are same as given in fig. X.10(e)
Fig.X.11 Conduction pattern of power devices and predicted wave-forms in sub-mode 150°-270° and at α = 200° (a) Conduction pattern of power devices (b) Voltage wave-forms across capacitors C₅, C₁, C₃ along with input voltage waveform (c) Voltage wave-forms across power devices T₁ and D₁
Fig.X.11(d) Conduction pattern of power devices and predicted wave-forms in sub-model 150°-270° and at α = 200°, current wave-forms through T12, C1, C5, D1, D4 and phase A.
Fig. X.11 (e) Voltage and current wave-forms at the dc side of the compensator Vd and Id along with voltage-forms at the input and across commutation capacitors.
Fig X.11(f) operational circuit structures at the upper section of the compensator at different intervals.
Interval $t_0-t_1$:

At the instant to the controlled power device $T_1$ is switched on and consequently the previously conducting device $T_5$ turns OFF due to application of a reverse voltage due to the voltage across capacitor $C_5$. The devices $T_1$ and $D_5$, $T_6$ and $D_4$ belonging to the upper and lower sections of the compensator simultaneously conduct during to ($\alpha$ - 150°). The capacitor $C_5$ holds peak voltage of $V_{c0}$ at the instant when the device $T_1$, is switched on. Due to simultaneous conduction of $T_1$ and $D_5$ in the upper section (fig. X.11(f)), the dc current $i_d$ flows through the capacitor $C_5$ in parallel with the series combination of the capacitors $C_1$ and $C_3$. Consequently the capacitor currents of magnitude, $i_c$ and $i_{c1} = i_{c3}$ flow along the direction shown in fig. X.11 (f). As a result, the voltage across $C_5$ falls linearly at a rate twice as that of the voltage rise across two other capacitors $C_1$ and $C_3$. The capacitor voltage $V_{c5}$, $V_{c1}$ and $V_{c3}$ are shown in fig.X.11 (b). The $T_1$ is conducting and voltage across $D_1$ is ($e_{ac}$ - $V_{c5}$) which are shown in fig.X.11 (c). The waveforms of current flowing through the $C_5$, $C_1$ and $C_3$ are shown in fig X.11 (e). Since $D_1$ is OFF so the $i_{D1} = 0$; $i_{T1} = i_d$ is shown in fig.X.11 (d). Here $i_c = i_d$ since diode $D_4$ conducts. The voltage across the dc inductor is governed by the conduction of $T_1$ and $D_5$ and $T_6$ and $D_4$. The dc voltage is governed by the conduction of $T_1$ and $D_5$ and $T_6$ and $D_4$. The dc voltage is $V_d = (V_{us} + e_{ac} + V_{ap})$ and resulting dc current $i_d = [1/L] \int v_d dt$ as shown in fig. X.11 (e).

Interval $t_1-t_2$:

At $t = t_1$, $D_6$ becomes ON and $D_4$ is OFF. The $T_1$ and $D_5$ conduct in the upper section of the compensator. The waveforms are same as that of given for the interval ($t_0-t_1$) above of $V_{c5}$, $V_{c1}$, $V_{c3}$, $i_{c1}$, $i_{c5}$, $i_{D1}$, $i_{T1}$, $V_{D1}$ and $V_{T1}$. Waveforms are remaining same like the waveforms as ($t_0-t_1$) interval. Here $i_d = 0$ since $D_1$ and $D_4$ are OFF. Due to the simultaneous conduction of the devices $T_1$ and $D_5$ and $T_2$ and $D_6$ the voltage across the dc inductor becomes $V_d = (V_{us} + e_{ac} + V_{ap})$ and resulting dc inductor current $i_d = [1/L] \int V_d dt$ (fig.X.11).

Interval $t_2-t_3$:

Here $T_2$ is starting and $T_1$, $D_5$, $T_2$ and $D_6$ conduct simultaneously. Still the $T_1$, $D_5$ conduct. Here $i_d = 0$ since $D_1$ and $D_4$ are Off. Due to the simultaneous conduction of the devices $T_1$ and $D_5$ and $T_2$ and $D_6$ the voltage across the dc inductor becomes $V_d = (V_{us} + e_{ac} + V_{ap})$ and resulting dc inductor current $i_d = [1/L] \int V_d dt$ (as shown in fig.X.11).

Interval $t_3-t_4$:

At the instant $t_3$, the reverse bias across the $D_1$ is removed due to the instantaneous value of the $V_{c5}$ which is equalizing $e_{ac}$. The conducting devices in the lower section remain unchanged. Consequently the devices $T_1$ and $D_1$, $D_2$, $T_2$ and $D_6$ conduct simultaneously during this interval. Since $T_1$ and $D_1$ conduct simultaneously, the $C_1$, $C_3$ and $C_5$ do not carry current. Therefore the voltage across them remains
constant at the respective value attained at the instant \( t_3 \) during interval as shown in fig. X.11 (b). Thus the capacitance currents, \( i_{c5} = i_{c1} = i_{c3} = 0 \). Here \( i_a = i_{d1} = i_{T1} = i_d \) (shown in fig. X.11(d)) and \( V_{T1} = V_{D1} = 0 \) (fig. X.11 (b)). Voltage across dc inductor \( V_d = (V_{ua} + e_{ab}) \) and \( i_d = [1/L] \int V_d dt \) (fig. X.11 (e)).

**Interval \( t_3 - t_5 \)**

Now \( T_3 \) is switched ON and hence the previously conducting device \( T_1 \) is commutated since \( V_{C1} \) appears as reverse bias across \( T_1 \). Here \( T_3, D_1, T_2 \) and \( D_6 \) conduct simultaneously and the diode current \( i_d \) flows through \( D_1 \) and \( T_3 \). Consequently the \( i_{c1} = 2i_d/3 \) and \( i_{c5} = i_{c3} = i_d/3 \). Since the capacitors from a configuration with \( C1 \) being in parallel with series combination of \( C5 \) and \( C3 \). This results \( Vc1 \) falls rapidly at a rate twice as that of rise in voltage in the capacitors \( C5 \) and \( C3 \). The \( Vc5, Vc1 \) and \( Vc3 \) waveforms are shown in fig. X.11(b). Here \( V_{D1} = 0 \) and \( V_{T1} = V_{c1} \) as shown in fig. X.11(c). The \( i_{c1} = [2/3] i_d \) and \( i_{c5} = i_d/3 \), current through the \( D_1, i_{d1} = i_d \) and \( i_{T1} = 0 \) (fig. X.11(d)) \( i_a = i_{d1} \), thus \( i_a = i_d \) (Fig. X.11(d)).

In fig. X.11(e), \( V_d \) and \( i_d \) is shown. Due to simultaneous conduction of \( T3, D1, T2, D6 \). The \( V_d = (V_{ua} + e_{ac} + V_{rr}) \) and \( i_d = \) inductor current = \([1/L] \int V_d dt \) (seen in fig. X.11(e))

**Interval \( t_5 - t_6 \)**

At the instant, \( t_5 \) the \( D2 \) starts conducting and \( D6 \) goes into off-state. Here \( T3, D1, T2 \) and \( D2 \) conduct simultaneously. The waveforms of \( Vc5, Vc3, ic1, iD1, iT1, i_d, VD1, \) and \( VT1 \) remain the same as per \((t_4-t_5)\) waveforms since \( T3 \) and \( D1 \) conduct like the \((t_4-t_5)\) interval. Again due to the simultaneous conduction of \( T3, D1, D2 \) and \( D3 \), the voltage across the dc inductor is given by \( V_d = (e_{ac} + V_{rr}) \) and DC inductor current is given by \( i_d = [1/L] \int V_d dt \). All these waveforms are shown in fig. X.11. Again due to the simultaneous conduction of \( T3 \) and \( D1, D2 \) and \( D2 \) the voltage across the dc inductor is given by \( V_d = (e_{ac} + V_{rr}) \) and DC inductor current is given by \( i_d = [1/L] \int V_d dt \). All these waveforms are shown in fig. X.11.

**Interval \( t_6 - t_7 \)**

At the instant \( t_6 \), the controlled power device \( T_4 \) is switched on and as a result the device \( T2 \) gets commutated. During this interval the device \( T3, D1, T4 \) and \( D2 \) conduct simultaneously. Still the same device \( T3 \) and \( D1 \) conduct in the upper section of the compensator, therefore the analysis for the waveform of \( Vc5, Vc1, Vc3, ic1, ic5, iD1, \)
iT1, ia, vD1 and VT1 remain the same as that of the internal (t4 - t5) given above in this sub-mode. The dc side inductor voltage Vd is given by Vd = (Vst + eac + Var) and dc inductor current is given by id. Here, \( id = \frac{1}{L} \int V_d \, dt \). All these waveforms are given in fig.X.11.

**Interval t7 - t8:**

Now, at \( t = t_7 \), D3 goes into forward bias and it starts conducting and the diode D1 gets turned OFF due to reverse bias across it. As a result, the devices T3 and D3 conduct simultaneously in the upper section and the devices T4 and D2 conduct in the lower section of the compensator during this interval. Since T3 and D3 conduct simultaneously so ic1 = ic3 = ic5 = 0. Therefore, during this interval the voltages across all these capacitors remain constant at the values corresponding to that of the instant t3. The voltage across the diode D1 = \( e_{ab} - Vc1 \). The T1 holds a reverse biased voltage which is equal to Vc1 (seen in fig.X.11(b & c)). Thus we can say that ic5 = ic1 = ic3 = 0 since neither D1 nor D4 conduct during this interval. Therefore \( id = 0 \) also iD1 = iT1 = 0 (fig. X.11(d)). Due to the simultaneous conduction of T3 and D3 and T4 and D2 the voltage across the DC inductor Vd = (eac + Vst) and dc inductor current is given by \( id = \frac{1}{L} \int V_d \, dt \) (shown in fig. X.11(e)).

**Interval t9 - t10:**

At the instant \( t_9 \), the controlled power device T5 is switched on and as a result the device T3 is commutated by the capacitor voltage Vc3. Here T5, D3, T4 and D2 conduct and T5 and D3 conduct simultaneously so the dc current \( id \) flows through C3 which is parallel with the series combination of C1 and C5. Hence the ic3 = \( \frac{2}{3} \) id and ic1 = ic5 = \( \frac{1}{3} \) id. The direction of current is shown in fig. X.11(f). The Vc3 fall linearly at a rate twice as that of voltage rise across two other capacitors C1 and C5 (shown in fig.X.11(b)). Fig X.11(c) shows the voltage across D1 equals to the difference of \( e_{ab} \) and Vc1 and that of across the controlled power devices T1 = Vc5. Fig.X.11(d) shows the current waveforms through the C1, C5, D1 and T1. Both iD1 and iT1 = 0, as these devices are not conducting and also, since neither the diode D1 nor the diode D4 conducts, the current in input phase A is shown in fig. X.11(d) to be equal to zero. Due to simultaneous condition of devices T4, D2, T5 and D3 the voltage across the dc inductor Vd = (Vst + ebo + VD1) and the resulting dc current \( id = \frac{1}{L} \int V_d \, dt \) (fig. X.11(e)).
Interval $t_9 - t_{10}$:

Here the D4 is ON and D2 is OFF. Therefore the devices T4 and D4 conduct simultaneously. Still the T5 and D3 conduct as they were conducting in $(t_8 - t_9)$. So the analysis for the waveforms of $Vc_5$, $Vc_1$, $Vc_3$, $ic_1$, $ic_5$, $ic_3$, $iT_1$, $iD_1$, and $VT_1$ remain the same as that of $(t_8 - t_9)$. Due to D4 conduction $i_a = i_d$ and $Vd = (e_{be} + Vpr)$. The simultaneous conduction of T5 and D3 and T4 and D4 and DC inductor results current, $i_d = \frac{1}{L} \int V_d \, dt$ as shown in fig. X.11.

Interval $t_{10} - t_{11}$: At the instant of $t_{10}$, the devices T6 is switched ON and as a result T4 gets turned OFF. The T6 and D4 conduct simultaneously. Still the same device T5 and D3 conduct during $(t_{10} - t_{11})$ in the upper section of the compensator as were conducting during $(t_8 - t_9)$. Therefore, the analysis for various waveforms, $Vc_5$, $Vc_1$, $Vc_3$, $ic_1$, $ic_3$, $iD_1$, $iT_1$, $VD_1$ and $VT_1$ remain the same as that of during the interval $(t_8 - t_9)$ of this sub-mode. Since the diode conducts, so $i_a = - i_d$ and $Vd = (Vt_1 + e_{be} + Vpr)$. Due to the simultaneous conduction of devices T5 and D3, T6 and D4 and dc current is given by $i_d = \frac{1}{L} \int V_d \, dt$. All the waveforms are shown in fig. X.11.

Interval $t_{11} - t_{12}$: At the instant $t_{11}$, D5 starts conducting and D3 goes to ‘OFF’, therefore T5, D5, T6, and D4 conduct. Due to the simultaneous condition of the device T5 and D5. The capacitors do not carry charging current. Thus all these capacitances remain constant equal to the value at $t_{11}$ instant. Hence, it can be concluded that $ic_5 = ic_1 = ic_3 = 0$. At $t_{11} = 150$ degree, the $Vc_5 = Vcos$ the peak capacitor voltage. Since D4 conducts, the input phase current=$i_a = - i_d$ and the current waveforms of it are shown in fig. X.11(d). The voltage across the dc inductor $Vd = (e_{ca} + Vpr)$ due to the simultaneous conduction of the device T5, D5, T6, D4 and $i_d = \frac{1}{L} \int V_d \, dt$. All the waveforms are shown in fig. X.11.Fig. X.12(a) shows the oscillogram of input voltage $e_{ca}$, $Vc_5$, $VD_1$ and $i_a$ at $a = 200^\circ$, when no external resistance is connected in series with the dc inductor. It is observed that the waveforms shown in fig. X.12(a) are closely matching with the corresponding waveform in fig. X.11.Fig. X.12(d) and (e) show the experimental waveforms corresponding to fig. X.12(a) and (b) respectively at external resistance of 32 ohm connected in series with the dc inductor at the time of taking oscillogram of figs.X.12(d) and (e). When figures X.12(d) and (e) are compared with those of X.10(e) and (f) respectively, it is observed
that in this sub-mode also the other is delayed for certain duration and also the other waveforms shift for the same duration. The magnitudes of currents decrease due to the presence of external resistance being connected to the dc-side. The simultaneous conduction of two diodes of either upper or lower section of the compensator is of very short duration and as a result, the resonance current due to presence of source inductance is not much prominent in this sub mode.

Special features of the compensator operation:
1. Input ac current increases with ‘α’.
2. For α lying between 210° and 270°, a new feature of compensator operating can be observed as compared to that of ‘α’ lying between 150° and 210°. For α = 150°-210°, the transfer of diode conduction in the upper section of the compensator will not be over before the instant the controlled power device is switched on in the other section of the compensator. A conduction overlap occurs between the diode of upper and lower sections of the compensator. However α = 210°; the transfer of diode conduction in any particular section of the compensator is over before the instant the controlled power device in the other section of compensator is switched ON.
3. All the switching delay angel at α = 270°, in ideal case, where dc resistance and source inductance is neglected, the diode and the controlled power device having the same number. Mainly due to presence of large dc current, for switching delay angle beyond 270°, the pattern of the diode conduction w.r.t. corresponding controlled power device changes, which leads to a new sub mode 270°- 360°, discussed in the following section.

X.1.5 Sub-mode 270° – 360°
In an ideal situation, when dc resistance and source inductance are neglected the diode D1 starts conducting at an angle 270° from the positive zero crossover of eac and the voltage cross the capacitor C5, Vc5 = eα. For α = +270°, the T1 is switched ‘ON’, the voltage across C5, reverse biases the conducting device T5 consequently gets turned OFF. Then the capacitor is charged in the reverse direction within a finite time frame. Here Vc5 = eα and D1 starts conducting. Unlike the previous conduction of beyond 270° from positive zero crossover of voltage eα. The delay in diode conduction from 270° depends on
(a) Magnitude of switching delay angle beyond 270° i.e. \( \alpha = 270° \) and

(b) The time taken to change to \( C5 \) in the reverse direction so that \( Vc5 = e_{ca} \) at which the \( D1 \) starts conducting. This time is very small.

Here \( \alpha = 300° \) has been considered. The conduction pattern of the power devices are shown in fig. X.13(a). The conduction of \( T1 \) has been sub divided into 12 time interval based on the circuit structure taken by the compensator due to the conduction of various power devices. All the waveforms and circuit structures during these intervals are shown in fig. X.13(b) to X.13(f). From the conduction patterns and waveforms during a particular time interval, the operation of the compensator can be explained in the same manner as that of for \( \alpha = 200° \) given in the previous section for sub-mode 150° – 270°.

Figures X.14(a) to X.14(e) show the oscillograms of various voltage and current waveforms for switching delay angle 300° of the controlled power devices of the compensator. These waveforms correspond to those of figures X.12(a) to X.12(e) respectively of the sub mode 150° – 270°. Figure X.14(a) to X.14(c) closely agrees to the corresponding waveforms shown in figure X.13. The switching angle of the power devices in this sub mode is beyond 270°, therefore all the waveforms shift accordingly. Due to excessive dc as well as ac current in the compensator, the effect of source inductance is more prominent in both current and voltage waveforms. All other arguments given for sub mode 150°-270° hold true in this sub-mode also.

The following special features of compensator operation are observed in this sub-mode:

1. The diode conduct at \( \alpha \) beyond 270° and this instant is dependent on the value of \( \alpha \).
2. The transfer of diode conduction in a particular section the compensation is over before the instant of the controlled power device in the other section of the compensator which is switched ‘ON’.
Fig. X.12(a) Experimental waveforms at $\alpha = 200'$ in the sub-mode $150'-270'$; $R = 2$ ohm. Horizontal axis 20ms, full scale. Vertical axis: $e_{ac} = 100$ V (peak); $V_{os} = 110$ V peak; $VD1 = -170$ V. Peak, $i_a = 2.42$ A peak

Fig. X.12(b) Experimental waveforms at $\alpha = 200'$ in the sub-mode $150'-270'$; $R = 2$ ohm. Horizontal axis 50ms, full scale. Vertical axis: --in; $2.42$ A (peak); $i_{in} = 2.42$ A (peak); $i_{oc} = 1.6$ A (peak); $i_c = 1.6$ A peak
Fig. X.12(c) Experimental waveforms at $\alpha = 200^\circ$ in the sub-mode 150°-270°; with vertical axis $Vd = 160 \text{ V (peak)}$; $id = 2.42 \text{ A}$, $e_{ac} = 100 \text{ A (peak)}$; $id = 2.42 \text{ A}$, $e_{ac} = 100 \text{ A (peak)}$ $R = 2 \text{ ohm}$. Horizontal axis 50ms, full scale. Vertical axis --- $id$; 2.42A (peak); $i_{dc} = 2.42 \text{ A (peak)}$; $i_{dc} = 1.6 \text{ A (peak)}$; $ic = 1.6 \text{ A peak}$.
Fig. X.12(d) Experimental waveforms at \( \alpha = 200^\circ \) in the sub-mode 150°-270°; \( R = 2 \) ohm.
Horizontal axis 20 ms, full scale at \( \alpha = 200^\circ \), \( R = 32 \) ohm, Horizontal axis = 20 ms full scale; vertical axis \( e_{es} = 100 \) V peak, \( V_{ds} = 85 \) V peak; \( V_{01} = -73 \) V; \( i_a = 1.1 \) Amp. (peak)

Fig. X.12(e) Experimental waveforms at \( \alpha = 200^\circ \) in the sub-mode 150°-270°; \( R = 2 \) ohm.
Horizontal axis 50 ms, full scale at \( \alpha = 200^\circ \), \( R = 32 \) ohm, Horizontal axis = 20 ms full scale; vertical axis \( e_{es} = 100 \) V peak, \( V_{ds} = 85 \) V peak; \( V_{01} = -73 \) V; \( i_a = 1.1 \) Amp. (peak)
\( i_{01} = 1.1 \) A (peak)
Fig.X.13 Conduction pattern of power devices and predicted wave-forms in sub-mode
270°- 360° and α = 300° (a) Conduction pattern of power devices (b) Voltage wave-forms across capacitors C₅, C₁, C₃ along with input voltage waveform (c) Voltage wave-forms across power devices T₃ and D₁
Fig. X.13 (d) Conduction pattern of power devices and predicted waveforms in sub-mode 270°-360° and α = 300°. Current waveforms through T1, C1, C5, D1, D4 and phase A.
Fig.X.13 (e) Conduction pattern of power devices and predicted wave-forms in sub-mode 270° - 360° and α = 300°. Voltage and current waveforms at dc side of the compensator Vd and Id along with voltage waveforms at the input and across commutating capacitors.
Operational circuit structures at the upper section of the compensator at different intervals.

Conduction pattern of power devices and predicted wave-forms in sub-mode $270^\circ - 360^\circ$ and $\alpha = 300^\circ$
Fig. X.14 (a) Experimental waveforms at $\alpha = 300^\circ$ in sub-mode 270° - 360°. $R = \text{ohm}$, horizontal axis 20 ms full scale, vertical axis $v_{ac} = 40V$, peak; $V_{ds} = 40V$ peak; $VD1 = -70V$ peak; $i_a = 19.5A$ peak.

Fig. X.14 (b) Experimental waveforms at $\alpha = 300^\circ$ in sub-mode 270° - 360°. Experimental waveform at $\alpha = 300^\circ$; $R = 20 \text{ ohm}$, horizontal axis 50 ms (full scale) Vertical axis. $I_{D1}; 19.5 \text{ A (peak)}$; $I_{T1} = 19.5 \text{ peak}$; $I_{a1} = 12.9A \text{ peak}$; $i_{a1} = 12.9 \text{ A (peak)}$. 

hhb-h
Fig. X.14 (c) Experimental waveforms at $\alpha = 300^\circ$ in sub-mode $270^\circ - 360^\circ$. At $\alpha = 300^\circ$.

$R = 20$ ohm, Horizontal axis, 20 ms (full scale) vertical axis $V_d = 106$ V, $V_{peak}$ to peak;

$id = 19.5$A peak $e_{ac} = 40$ V
Fig.X.14 (d) Experimental waveform at $\alpha = 300^\circ$ in the sub-mode $270^\circ-360^\circ$, $R = 32$ ohm, horizontal axis. 20 ms (full scale). Vertical axis $e_a = 40V$ peak, $Vc_5 = 32V$ peak; $V_{D1} = -49V$ peak; $i_a = 5.5A$.

Fig.X.14 (e) Experimental waveform at $\alpha = 300^\circ$ in the sub-mode $270^\circ-360^\circ$, $R = 32$ ohm, horizontal axis. 50 ms (full scale). Vertical axis $e_a = 40V$ peak, $Vc_5 = 32V$ peak; $V_{D1} = -49V$ peak; $i_a = 5.5A$. Vertical axis $iD1 = 5.5$ amp. (peak); $i_{T1} = 5.5$ amp. peak; $i_{c5}$: 3.7 A, peak $i_{d1} = 3.7$ amp. peak.
APPENDIX Y

Conduction pattern and circuit performance of power devices of the operational circuits for all the intervals in all the sub-modes

The time reference for obtaining the circuit waveforms in each sub-mode corresponds to the instant when the controlled power device T1 is switched ON. Here \( e_a, e_b \) and \( e_c \) having constant amplitude, \( E_{mp} \) and angular frequency, \( \omega \) can be expressed below:

\[
e_a = E_{mp}\sin (\omega t + 30^\circ + \alpha); e_b = E_{mp}\sin (\omega t - 90^\circ + \alpha); e_c = E_{mp}\sin (\omega t + 150^\circ + \alpha) \quad (Y.1)
\]

When this 3-phase supply is connected to the compensator the phase voltages, after the source inductance across the compensator terminals can be expressed by the following equations:

\[
V_a = e_a - L_s p_i_a, \quad V_b = e_a - L_s p_i \quad \text{and} \quad V_c = e_a - L_s p_i_c \quad \text{(Y.2)}
\]

The polarities of voltage and currents of all the phases, commutating capacitors, dc side inductor and power devices are shown in fig. Y.1

(a) sub-mode \((60^\circ - 90^\circ)\):

Fig.Y.2 shows the conduction pattern of power device. The reference time instant is synchronized with the switching time instant of the controlled power device T1, the sub-mode is sub-divided into twelve time intervals. Depending on operations circuit at each instant the operational equations are given below:

a1. Operational Circuit – 1 for interval \([t_0 - t_1]\)

Fig.Y.3 shows the operational circuit 1 for the duration \((t_0 - t_1)\) the conduction pattern is already given in fig.Y.2. The equations are as follows:-

(x) Differential Equations:

\[
pV_{c1} = -(2/3) C i_2; \quad pV_{c2} = (1/3) C i_2; \quad pV_{c3} = (1/3) C i_2; \quad pV_{c4} = (1/3) C i_2; \quad pV_{c5} = -(1/3) C i_2; \quad pV_{c6} = -(2/3) C i_2; \quad \text{(Y.3)}
\]

(y) Algebraic Equations:

\[
\begin{align*}
    j_a &= i_1 + i_2; \quad j_b = -i_1; \quad j_c = -i_2; \quad j_d = i_1; \quad V_d = L p i_1 + R i_1; \quad V_{DI} = V_{eb} + V_{Cl};
    V_{T1} &= 0.0; \quad i_{T1} = i_d; \quad p i_1 = [2L_s(e_a - e_b - R i_1) - L_s(e_a - e_c + V_{Cl} - V_{Cs})]/\Delta \quad \text{and} \\
    P_{I2} &= [-L_s(e_a - e_b - R i_1) - B_l(e_a - e_c + V_{Cl} - V_{Cs})]/\Delta \quad \text{(Y.4)}
\end{align*}
\]

iii
Where, \( BI = 2L_4 + L \) and \( \Delta = 3L_4^2 + 2L_4L \) \hfill (Y.5)

(z) Conditional Equation: The diode \( D_4 \) starts conducting when the following voltage condition across it occurs, \( V_{C4} \geq V_{ab} \) \hfill (Y.6)

(a2) **Operational Circuit – 2 interval \([t_1 - t_2]\):**

Fig (Y.4) shows operational circuit; here \( T_1, D_1, D_2, T_6, D_6, D_2 \) and \( D_4 \) conduct simultaneously. The operational equations are:

|x| Differential Equations: | \( pV_{c1} = -(1/6C) i_2; pV_{c2} = -(7/12C) i_2; pV_{c3} = (1/12C) i_2; \)
|---|---|---|
|\( pV_{c4} \) | \( [1/6C] i_2; pV_{c5} = [1/12C] i_2; pV_{c6} = [5/12C] i_2; \)
\( p_i1 \) | \( [2L_4 (e_a - e_b - R_{i1}) - L_4 (e_a - e_c - V_{c4} - V_{c6})] / \Delta \)
\( p_i2 \) | \( [-L_4 (e_a - e_b - R_{i1}) + BI (e_a - e_c - V_{c4} - V_{c6})] / \Delta \) \hfill (Y.7)
(The \( i_3 = [7/12] i_2; \) which is used to derive the above equations)

(y) Algebraic Equations: \( i_a = i_1 + i_3; i_b = -i_1; i_c = -i_2; i_d = i_1; V_d = Lp_i1 + R_{i1}; \)

\( V_{D1} = 0.0; V_{T1} = 0.0 \) \hfill (Y.8)

(z) The transfer to the next operational circuit occurs when the device \( T_2 \) is switched ON at an instant: \( t = T_2 \) \hfill (Y.9)

where, \( T_2 = T_4 + [1/6] (T_F); T_F = \) period of supply frequency (1/\( f \); \( f = 50Hz. \))

(a3) **Operational Circuit – 3 for \([t_2 - t_3]\) interval:**

Fig. Y.5 shows it. \( T_1, D_1, D_3, T_2, D_2, D_4 \) simultaneously conduct.

|x| Differential Equations: | \( pV_{c1} = (2/3 C) i_2; pV_{c2} = -(2/3 C) i_2; pV_{c3} = -(1/3 C) i_2; \)
|---|---|---|
|\( pV_{c4} \) | \( (1/3 C) i_2; pV_{c5} = -(1/3 C) i_2; pV_{c6} = (1/3 C) i_2; \)
\( p_i1 \) | \( [2L_4 (e_a - e_c - R_{i1}) - L_4 (e_b - e_c - V_{c1} - V_{c2})] / \Delta \)
\( p_i2 \) | \( [-L_4 (e_a - e_b - R_{i1}) + BI (e_b - e_c - V_{c1} - V_{c2})] / \Delta \) \hfill (Y.10)
BI and \( \Delta \) are given in (Y.5)

(y) Algebraic Equations: \( i_a = i_1; i_b = i_2; i_c = -(i_1 + i_2); i_d = i_1; V_d = Lp_i1 + R_{i1}; V_{D1} = 0.0; \)

\( V_{T1} = 0.0; i_{T1} = i_d; \) \hfill (Y.11)

(z) Conditional Equation: The transfer to the next operational circuit will start when \( D_5 \) starts conducting. It is possible when \( V_{c4} \geq (-V_{c5}) \) \hfill (Y.12)
Circuit configuration of inductively loaded current controlled solid-state lead-lag VAR compensator

Fig. Y.1

Conduction pattern of power devices in submode 60°-90°

Fig. Y.2
Fig. Y.3 Operational circuit-1 of submode 60°-90°

Fig. Y.4 Operational circuit-2 of submode 60°-90°

Fig. Y.5 Operational circuit-3 of submode 60°-90°

Fig. Y.6 Operational circuit-4 of submode 60°-90°
Fig. Y.7 Operational circuit-5 of submode 60°-90°

Fig. Y.8 Operational circuit-6 of submode 60°-90°

Fig. Y.9 Operational circuit-7 of submode 60°-90°

Fig. Y.10 Operational circuit-8 of submode 60°-90°
Fig. Y.11 Operational circuit-9 of submode 60°-90°

Fig. Y.12 Operational circuit-10 of submode 60°-90°

Fig. Y.13 Operational circuit-11 of submode 60°-90°

Fig. Y.14 Operational circuit-12 of submode 60°-90°
(a4) **Operational Circuit – 4 int. ($t_3$ – $t_4$):**

Fig. Y.6 shows the simultaneous conduction of T1, D1, D3, T2, D2, and D4. Their operational equations are:

(x) **Differential Equations:**
- $pV_{c1} = (5/12 C) i_2$; $pV_{c2} = -(1/6 C) i_2$; $pV_{c3} = -(7/12 C) i_2$
- $pV_{c4} = (1/12 C) i_2$; $pV_{c5} = (1/6 C) i_2$; $pV_{c6} = (1/12 C) i_2$
- $pi_1 = [2L_s (e_b - e_c - R_{i1}) - L_s (e_b - e_a - V_{C1} - V_{C3})] / \Delta$
- $pi_2 = [-L_s (e_b - e_c - R_{i1}) + BI (e_b - e_a - V_{C3})] / \Delta$

(y) **Algebraic Equations:**
- $i_3 = i_1$; $i_5 = i_2$; $i_6 = -(i_1 + i_2)$; $i_4 = i_1$; $V_d = L p i_1 + R_{i1}$; $V_{p1} = 0.0$
- $V_{T1} = 0.0; i_{T1} = i_3$; and $i_3 = [7/12] i_2$

(z) **Conditional Equation:** When T3 is switched ON then next operation start at:
- $t = T_3$

where, $t_3 = t_2 + [1/6] T_p$

(a5) **Operational Circuit – 5 ($t_4$ – $t_5$):** Here T3, D3, D5, T2, D2 and D4 conduct simultaneously (shown in fig. Y.7)

(x) **Differential Equations:**
- $pV_{c1} = [1/3C] i_2$; $pV_{c2} = [2/3C] i_2$; $pV_{c3} = [-2/3C] i_2$
- $pV_{c4} = [1/3C] i_2; pV_{c5} = [1/3C] i_2; pV_{c6} = [-1/3C] i_2$
- $pi_1 = [2L_s (e_b - e_c - R_{i1}) - L_s (e_b - e_a + V_{C3} - V_{C2})] / \Delta$ and
- $pi_2 = [-L_s (e_b - e_c - R_{i1}) + BI (e_b - e_a + V_{C3} - V_{C2})] / \Delta$

(y) **Algebraic Equation:**
- $i_3 = i_2$; $i_5 = i_1 + i_2$; $i_6 = -i_1$; $i_4 = i_1$; $V_d = L p i_1 + T_{i1}$
- $V_{D1} = V_{sc} - V_{C5}; V_{T1} = V_{C1}$ and $i_{T1} = 0$

(z) **Conditional Equation:** Next operation starts when D6 is ON, when
- $V_{C6} >= V_{bc}$

(a6) **Operational Circuit – 6 ($t_5$ – $t_6$):**

In fig. Y.8, the conduction of T3, D3, D5, T2, D4 and D6 is performed simultaneously.

(x) **Differential Equation:**
- $pV_{c1} = [1/12C] i_2$; $pV_{c2} = [5/12C] i_2$; $pV_{c3} = [-1/6C] i_2$
- $pV_{c4} = [-7/12C] i_2$; $pV_{c5} = [1/12C] i_2$; $pV_{c6} = [-1/6C] i_2$
- $pi_1 = [2L_s (e_b - e_c - R_{i1}) - L_s (e_b - e_a - V_{C6} - V_{C2})] / \Delta$ and
- $pi_2 = [-L_s (e_b - e_c - R_{i1}) + BI (e_b - e_a - V_{C6} - V_{C2})] / \Delta$

(y) **Algebraic Equations:**
- $i_3 = -i_2$; $i_5 = -i_1 + i_2$; $i_6 = -i_1$; $i_4 = i_1$; $V_d = L p i_1 + R_{i1}$
- $V_{D1} = V_{sc} - V_{C5}; V_{T1} = V_{C1}$ and $i_{T1} = 0$

(z) **Conditional Equation:** At, $t = T_4$

-vvv
Next operation starts where, \( T_4 = T_3 + [1/6] T_P \).

(a7) **Operational Circuit - 7** \( t_6 - t_7 \): Fig. Y.9, shows the circuit. Here \( T_3, D_3, D_5, T_4, D_4 \) and \( D_6 \) conduct simultaneously.

(x) Differential Equations:
\[
\begin{align*}
pV_{c1} &= -[1/3C] i_2; \\
pV_{c2} &= [1/3C] i_2; \\
pV_{c3} &= [2/3C] i_2; \\
pV_{c4} &= -[2/3C] i_2; \\
pV_{c5} &= -[1/3C] i_2; \\
pV_{c6} &= [1/3C] i_2; \\
p_i_1 &= [2L_s (e_b - e_a - R_i_1) - L_s (e_c - e_a - V_{C3} - V_{C4})] / \Delta \\
p_i_2 &= [-L_s (e_b - e_a - R_i_1) + B I (e_c - e_a - V_{C3} - V_{C4})] / \Delta \\
\end{align*}
\]

(y) Algebraic Equations are:
\[
\begin{align*}
i_a &= -(i_{1} + i_2); \\
i_b &= i_1; \\
i_c &= i_2; \\
i_d &= i_1; \\
V_d &= L p_i_1 + R_i_1; \\
V_D &= V_C; \\
V_{T1} &= V_C i_{T1} = 0.0 \\
(2) \text{Conditional Equation: Next operation starts as } D_1 \text{ is ON, at, } V_{ab} >= (-V_{C1}) \\
\]

(a8) **Operational Circuit - 8** \( t_7 - t_8 \): Fig. Y.10, shows the circuit. Where \( T_3, D_3, D_5, T_4, D_4 \) and \( D_6 \) conduct simultaneously.

(x) Differential Equations are:
\[
\begin{align*}
pV_{c1} &= [1/6C] i_2; \\
pV_{c2} &= [1/12C] i_2; \\
pV_{c3} &= -[5/12C] i_2; \\
pV_{c4} &= -[1/6C] i_2; \\
pV_{c5} &= -[7/12C] i_2; \\
pV_{c6} &= [1/12C] i_2; \\
p_i_1 &= [2L_s (e_b - e_a - R_i_1) - L_s (e_c - e_a - V_{C3} - V_{C4})] / \Delta \\
p_i_2 &= [-L_s (e_b - e_a - R_i_1) + B I (e_c - e_a - V_{C3} - V_{C4})] / \Delta \\
\end{align*}
\]

(y) Algebraic Equations are:
\[
\begin{align*}
i_a &= -(i_{1} + i_2); \\
i_b &= i_1; \\
i_c &= i_2; \\
i_d &= i_1; \\
V_d &= L p_i_1 + R_i_1; \\
V_D &= V_C; \\
V_{T1} &= V_C i_{T1} = 0.0 \\
(2) \text{Conditional Equation: Starting of next operation occurs when } D_2 \text{ starts conducting when, } V_{C2} >= V_{C4} \\
\]

(a9) **Operational Circuit - 9** \( t_8 - t_9 \): Fig. Y.11 shows the circuit. Where \( T_5, D_5, D_1, T_4, D_4 \) and \( D_6 \) conduct simultaneously.

(x) Differential Equations are:
\[
\begin{align*}
pV_{c1} &= [1/3C] i_2; \\
pV_{c2} &= -[1/3C] i_2; \\
pV_{c3} &= [1/3C] i_2; \\
pV_{c4} &= [2/3C] i_2; \\
pV_{c5} &= -[2/3C] i_2; \\
pV_{c6} &= -[1/3C] i_2; \\
p_i_1 &= [2L_s (e_b - e_a - R_i_1) - L_s (e_c - e_b + V_{C5} - V_{C4})] / \Delta \\
p_i_2 &= [-L_s (e_b - e_a - R_i_1) + B I (e_c - e_b + V_{C5} - V_{C4})] / \Delta \\
\end{align*}
\]

(y) Algebraic Equations are:
\[
\begin{align*}
i_a &= -i_1; \\
i_b &= i_2; \\
i_c &= i_1 + i_2; \\
i_d &= i_1; \\
V_d &= L p_i_1 + R_i_1; \\
V_D &= 0.0; \\
V_{T1} &= V_C i_{T1} = 0.0 \\
(2) \text{Conditional Equation:} \\
\]

(3) \text{Conditional Equation: Starting of next operation occurs when } D_2 \text{ starts conducting when, } V_{C2} >= V_{C4} \\

(3) \text{Conditional Equation:} \\

III
Fig. Y.15 Conduction pattern of the power devices in submode 90°-150°

Fig. Y.16 Operational circuit-1 of submode 90°-150°

Fig. Y.17 Operational circuit-2 of submode 90°-150°
(a10) **Operational Circuit – 10 \(t_9 - t_{10}\):** Fig. Y.12, shows the operational circuit, Where T5, D5, D1, T4, D4, D2, D6 conduct simultaneously. Their operational equations are as follows:

(x) Differential Equations are:

\[ \begin{align*}
\text{pV}_c &= \frac{1}{12C} i_2; \\
\text{pV}_c &= \frac{1}{6C} i_2; \\
\text{pV}_c &= \frac{1}{12C} i_2; \\
\text{pV}_c &= \frac{5}{12C} i_2; \\
p_i &= \left[ 2L_a (e_a - e_b - R_1) - L_a (e_a - e_b - V_{C5} - V_{C6}) \right] / \Delta \\
p_i &= \left[ -L_a (e_a - e_b - R_1) + B_1 (e_a - e_b - V_{C5} + V_{C6}) \right] / \Delta
\end{align*} \]

(y) Algebraic Equations are:

\[ \begin{align*}
i_a &= i_b; \\
i_b &= i_a; \\
i_c &= i_d = i_1; \\
V_d &= L_p i_1 + R_i_1; \\
V_{D1} &= 0.0; \\
V_{T1} &= 0.0; \\
i_{T1} &= -V_{C5}
\end{align*} \]

(z) Conditional Equations: At starting \(t_9\), next operation starts as \(T6\) is ON, at \(t = t_6\)

where \(t_6 = (T5 + [1/6] T_P)\).

(a11) **Operational Circuit – 11 \(t_{10} - t_{11}\):**

It is shown in fig. Y.13, here T5, D5, D1, T6, D2, D6 conducts simultaneously.

(x) Differential Equations are:

\[ \begin{align*}
\text{pV}_c &= -\frac{1}{3C} i_2; \\
\text{pV}_c &= \frac{1}{3C} i_2; \\
\text{pV}_c &= -\frac{1}{3C} i_2; \\
\text{pV}_c &= \frac{2}{3C} i_2; \\
p_i &= \left[ 2L_a (e_a - e_b - R_1) - L_a (e_a - e_b - V_{C5} + V_{C6}) \right] / \Delta \\
p_i &= \left[ -L_a (e_a - e_b - R_1) + B_1 (e_a - e_b - V_{C5} + V_{C6}) \right] / \Delta
\end{align*} \]

(y) Algebraic Equations are:

\[ \begin{align*}
i_a &= i_2; \\
i_b &= (i_1 + i_2); \\
i_c &= -i_1; \\
i_d &= i_1; \\
V_d &= L_p i_1 + R_i_1; \\
V_{D1} &= 0.0; \\
V_{T1} &= -V_{C5}, \\
i_{T1} &= 0.0
\end{align*} \]

(z) Conditional Equations: At starting \(D3\), next operation starts

when, \(V_{bc} \geq (-V_{C3})\)

(a12) **Operational Circuit – 12 \(t_{11} - t_{12}\):**

Fig.Y.14 shows the circuit where simultaneous conduction occurs in: T5, D5, D1, D3, T6, D6 and D2.

(x) Differential Equations are:

\[ \begin{align*}
\text{pV}_c &= -\frac{7}{12C} i_2; \\
\text{pV}_c &= \frac{1}{12C} i_2; \\
\text{pV}_c &= \frac{1}{6C} i_2; \\
\text{pV}_c &= \frac{5}{12C} i_2; \\
p_i &= \left[ 2L_a (e_a - e_b - R_1) - L_a (e_a - e_b - V_{C5} + V_{C6}) \right] / \Delta \\
p_i &= \left[ -L_a (e_a - e_b - R_1) + B_1 (e_a - e_b - V_{C5} + V_{C6}) \right] / \Delta
\end{align*} \]

(y) Algebraic Equations are:

\[ \begin{align*}
i_a &= i_2; \\
i_b &= -(i_1 + i_2); \\
i_c &= i_1; \\
i_d &= i_1; \\
V_d &= (L_p i_1 + R_i_1);
\end{align*} \]
(Y.38)

(2) Conditional Equations: The transfer to the operational circuit -1 will occur when the device T1 is switched ON again at the instant \( t = T1 \)  

Where, \( T1 = T6 + \frac{1}{6} T_p \).

b. **Sub-mode (90° - 150°)**

The conduction pattern of the power devices in the compensator over the input cycle for sub mode 90° - 150° is shown in fig. Y.15. The reference time instant is synchronized with switching instant of controlled power device T1. Depending upon the set of devices under conduction, the submode is sub divided into 18 time intervals. Here, 6 additional intervals are incorporated, due to resonance, while transferring conduction from one side to another, these are:

\[ t_1 - t_2, \ t_4 - t_5, \ t_7 - t_8, \ t_{10} - t_{11}, \ t_{13} - t_{14} \text{ and } t_{16} - t_{17} \]

The relevant differential, algebraic and conditional equations for operational circuit of each time interval are given below:

**(b1) Operational Circuit - 1:**

Fig.Y.16 shows the operational circuit of b1. Here T1, D3, T6 and D4 conduct simultaneously.

(x) Differential Equations: 
\[ \text{pV}_{C1} = \frac{2}{3} C i; \text{pV}_{C2} = \frac{1}{3} C i; \text{pV}_{C3} = -\frac{1}{3} C i; \]
\[ \text{pV}_{C4} = -\frac{2}{3} C i; \text{V}_{C5} = \frac{1}{3} C i; \]
\[ \text{p} = \frac{1}{6} (e_b - V_{C1} - e_d - R_i + V_{C4}) \]  \( \text{Y.40} \)

(y) Algebraic Equations: 
\[ i_a = -i; \ i_b = i; \ i_c = 0.0; \ i_d = i; \ V_d = L_p i + R_i; \]
\[ \text{V}_{D1} = V_{ab} + V_{C1}; \ V_{T1} = 0.0, \ i_{T1} = i_d \]  \( \text{Y.41} \)

(2) Conditional Equations: Next operational circuit starts when D5 is ON at,
\[ V_{eb} >= V_{C3} \]  \( \text{Y.42} \)

**(b2) Operational Circuit - 2:**

In fig.Y.17, the D5 starts conducting along with the D3 the resonant circuit is active till the diode D3 turns OFF. In this interval the devices T1, D3, D5 and D4 simultaneously conduct.
Fig. Y.22 Operational circuit-7 of submode 90°-150°

Fig. Y.23 Operational circuit-8 of submode 90°-150°

Fig. Y.24 Operational circuit-9 of submode 90°-150°

Fig. Y.25 Operational circuit-10 of submode 90°-150°
Differential Equations:

\[ pV_{c1} = [1/3C] (i_1 + i_2); \]
\[ pV_{c2} = [1/3C] i_3; \]
\[ pV_{c3} = - [1/3C] (2i_1 - i_3); \]
\[ pV_{c4} = - [2/3C] i_3; \]
\[ pV_{c5} = [1/3C] (2i_3 - i_1); \]
\[ pV_{c6} = [1/3C] (2i_3 - i_1); \]
\[ pV_{c7} = - [2/3C] i; \]
\[ pi_1 = [1/3C] (e_b - e_c + V_{C5}) + L_a (e_c - e_t + V_{C5} - R_i + V_{C4})/\Delta \] and
\[ pi_3 = [2L_a (e_c - e_a + V_{C3} - R_i + V_{C4}) + L_a (e_c - e_c + V_{C3})]/\Delta \] (Y.43)

Algebraic Equations:

\[ i_a = -i_3; \]
\[ i_b = i; \]
\[ i_c = i_3 - i_1; \]
\[ i_d = i_3; \]
\[ V_d = Lp_i_3 + R_i; \]
\[ V_{D1} = V_{ab} + V_{C1}; \]
\[ V_{T1} = 0.0 \] (Y.44)
\[ i_2 = 1/3 (i_1 + i_3) \] (Y.45)

Conditional Equations: The resonance will be over when the current through the diode D3 = 0 and D5 allows full DC side current. At this instant the transfer to the next operational circuit takes place. \( i_1 = 0 \) (Y.46)

(b3) Operational Circuit – 3 \([t_4 - t_5]\)

In Fig.Y.18, circuit is shown. Here T1, D5, T6 and D4 conduct simultaneously.

Differential Equations:

\[ pV_{c1} = [1/3C] i_2; \]
\[ pV_{c2} = [1/3C] i_3; \]
\[ pV_{c3} = [1/3C] i_3; \]
\[ pV_{c4} = - [2/3C] i; \]
\[ pi = (1/BI)(e_c - e_a - V_{C5} - R_i + V_{C4}) \] (Y.47)

Algebraic Equations:

\[ i_a = -i; \]
\[ i_b = i; \]
\[ i_c = 0; \]
\[ i_d = i \] (Y.48)

Conditional Equations: At the starting of T2 the operating starts, \( t = T_2 \) (Y.49)

Where \( T_2 = T_1 + [1/6] T_p \) and \( T_1 = 0.0 \) at the instant when T1 is switched ON for the first time.

(b4) Operational Circuit – 4 \([t_5 - t_6]\):

Fig.Y.19, shows the circuit, here T1, D5, T2 and D4 conduct.

Differential Equations:

\[ pV_{c1} = [1/3C] i_2; \]
\[ pV_{c2} = [2/3C] i; \]
\[ pV_{c3} = [1/3C] i; \]
\[ pV_{c4} = -[p(1/3C) i; \]
\[ pV_{c5} = -[2/3C] i; \]
\[ pV_{c6} = -[1/3C] i; \]
\[ pi = (1/BI)(e_c - e_a + V_{C5} - R_i + V_{C2}) \] (Y.50)

Algebraic Equations:

\[ i_a = -i; \]
\[ i_b = 0.0; \]
\[ i_c = i; \]
\[ i_d = i; \]
\[ V_d = LP_i + R_i; \]
\[ V_{D1} = V_{ac} - V_{C5}; \]
\[ V_{T1} = 0.0 \] (Y.51)

Conditional Equations: Next operation starts when diode D6 is ON. For this,
\[ -V_{C4} >= V_{ba} \] (Y.52)
(b5) Operational Circuit – 5:
Fig.Y.20, shows the operational circuit – 5. Here the D6 starts conducting along with the D4. This resonant circuit is active till the D4 turns ON. Now T1, D5, T2, D4 and D6 conduct simultaneously.

(x) Differential Equations: \( pV_{c1} = [1/3C] i_1; pV_{c2} = [1/3C] (2i_1 - i_3); pV_{c3} = [1/3C] i_1; pV_{c4} = [1/3C] (2i_3 - i_1); pV_{c5} = [-2/3C] i_1; pV_{c6} = [-1/3C] (i_1 + i_3) \)
\[
pi_1 = \frac{2L_s (e_c - e_a + V_{C5} - Ri_1 - C_{V2}) + L_s (e_b - e_b - V_{C4})}{\Delta} \\
pi_3 = \frac{[Bl (e_a - e_b - V_{C4}) + L_s (e_c - e_a + V_{C5} - Ri_1 - V_{C2})]}{\Delta} \tag{Y.53}
\]
(y) Algebraic Equations: \( i_a = i_3 - i_1; i_b = i_3; i_c = i_1; i_d = i_1; V_d = Lp_1 + Ri_1; V_{T1} = 0.0 \)
and \( i_2 = -[1/3] (i_1 + i_3) \) which is derived from fig. 5.20
(z) Conditional Equations: AT \( i_{D4} = 0; \) resonance is over and D6 allow full dc-side current. This starts next operation. Here, \( i_1 = i_3 \) \( \tag{Y.55} \)

(b6) Operational Circuit – 6 \((t_3 - t_4)\):
Fig.Y.21, here T1, D5, T2 and D6 conduct simultaneously.

(x) Differential Equations: \( pV_{c1} = [1/3C] i_1; pV_{c2} = [1/3C] i; V_{c3} = [1/3C] i; pV_{c4} = [1/3C] i; pV_{c5} = [-2/3C] i; pV_{c6} = [-2/3C] i\)
\[
pi = \frac{(1/Bl) (e_c - e_b + V_{C5} - Ri - V_{C6})}{\Delta} \tag{Y.56}
\]
(y) Algebraic Equations: \( i_a = 0.0; i_b = -i; i_c = i; V_d = Lp_1 + Ri; V_{D1} = V_{ac} - V_{C5}; V_{T1} = 0.0 \)
\( \tag{5.57} \)
(z) Conditional Equations: Next operation starts when T3 switches ON at the instant, \( t = T_3 \) \( \tag{Y.58} \)
where \( T_3 = T_2 + T_p / 6; T_p \) is the time period of supply frequency.

(b7) Operational Circuit – 7 \((t_5 - t_6)\):
Fig.Y.22 shows the circuit. Here T3, D5, T2 and D6 conduct simultaneously.

(x) Differential Equations: \( pV_{c1} = [-1/3C] i_1; pV_{c2} = [1/3C] i_1; pV_{c3} = [2/3C] i_1; pV_{c4} = [1/3C] i; pV_{c5} = [-1/3C] i_1; pV_{c6} = [-2/3C] i_1\)
\[
pi = \frac{(1/Bl) (e_c - e_b + V_{C3} - Ri + V_{C6})}{\Delta} \tag{Y.59}
\]
(y) Algebraic Equations: \( i_a = 0.0; i_b = -i; i_c = i; V_d = Lp_1 + Ri; V_{D1} = V_{ac} - V_{C5}; V_{T1} = V_{C1} \)
\( \tag{5.60} \)
Conditional Equations: The transfer to the next operational circuit will take place when the D1 starts conducting when: \( V_{av} \geq V_{C5} \)  

(b8) Operational Circuit – 8 \([t_7- t_8]\):

Fig.Y.23 shows it. When D1 starts conducting with D5, the resonant circuit becomes active and it is operative till D5 turn off. Here T3, D5, D1, T2 and D6 conduct simultaneously.

(x) Differential Equations:
\[
\begin{align*}
pV_{c1} &= -[1/3C] (i_1 + i_3); \quad pV_{c2} = [1/3C] i; \quad pV_{c3} = [1/3C](2i_3 - i_1); \\
pV_{c4} &= [1/3C] i_3; \quad pV_{c5} = [1/3C] (2i_1 - i_3); \\
pV_{c6} &= [2/3C] i_3; \\
p_i &= \left[ BI (e_a - e_c - V_{C5}) + L_4 (e_a - e_c - V_{C3} - Ri_3 + V_{C6}) \right] / \Delta \quad \text{and} \\
p_{i3} &= \left[ 2L_4 (e_a - e_b - V_{C3} - Ri_3 + V_{C6}) + L_4 (e_a - e_c - V_{C3}) \right] / \Delta 
\end{align*}
\]

(y) Algebraic Equations: \( i_a = i_1; \quad i_b = -i_3; \quad i_c = i_3 - i_1; \quad i_d = i_3; \quad V_d = Lpi_3 + Ri_3; \quad V_{D1} = V_{av} - V_{C5}; \quad V_{T1} = V_{C1} \) and \( i_2 = (i_1 + i_3) / 3 \)

(z) Conditional Equations: The next operation starts at \( D5 = 0; \) \( i_{D1} \) starts conducting.
\( i_3 = i_1 \)

(b9) Operational Circuit – 9 \([t_8- t_9]\):

Fig.Y.24 shows the operational circuit. Here T3, D1, T2 and D6 simultaneously conduct.

(x) Differential Equations:
\[
\begin{align*}
pV_{c1} &= [-2/3C] i; \quad pV_{c2} = [1/3C] i; \quad pV_{c3} = [1/3C] i; \\
pV_{c4} &= [1/3C] i; \quad pV_{c5} = [1/3C] i; \quad pV_{c6} = [2/3C] i; \\
p_i &= (1/BI) (e_a - e_b + V_{C1} - Ri + V_{C6}) 
\end{align*}
\]

(y) Algebraic Equations: \( i_a = i; \quad i_b = -i_t; \quad i_c = 0.0; \quad i_d = i; \quad V_d = Lpi + Ri; \quad V_{D1} = 0.0; \quad V_{T1} = V_{C1}; \quad i_{T1} = 0.0 \)

(z) Conditional Equations: The transfer to the next operational circuit will occur when the T4 is switched ON at the instant \( t = T4 \)

Where, \( T4 = T3 + [1/6] T_P; \) where \( T_P \) = time period of supply frequency.

(b10) Operational Circuit – 10 \([t_9- t_{10}\):  

Fig. Y.25, here T3, D1, T4 and D6 conduct simultaneously.

(x) Differential Equations:
\[
\begin{align*}
pV_{c1} &= [-2/3C] i; \quad pV_{c2} = [1/3C] i; \quad pV_{c3} = [1/3C] i; \\
pV_{c4} &= [2/3C] i; \quad pV_{c5} = [1/3C] i; \quad pV_{c6} = [-1/3C] i; \\
p_i &= (1/BI) (e_a - e_b + V_{C1} - Ri - V_{C4}) 
\end{align*}
\]

(y) Algebraic Equations: \( i_a = i; \quad i_b = -i_1; \quad i_c = 0.0; \quad i_d = i; \quad V_d = Lpi + Ri; \quad V_{D1} = 0.0; \quad V_{T1} = 0.0; \quad i_{T1} = 0.0 \)
(z) Conditional Equations: Next operations circuit operates when D2 starts conducting. It happen when \( V_{c6} = V_{c8} \) \( \text{ (Y.70)} \)

(b11) Operational Circuit – 11 \([t_{i0} - t_{i1}]\):

Fig. Y.26 shows the operational circuit - 11. Here D2 starts conducting along with D6. This resonant circuit is active till the diode D6 turns off. Here D1, D6, D2, T4 and T3 conduct simultaneously.

(x) Differential Equations: \( pV_{c1} = -[2/3C] i_3; \) \( pV_{c2} = -[1/3C] (i_1 + i_3); \) \( pV_{c3} = [1/3C] i_3; \) \( pV_{c4} = [1/3C] (2i_3 - i_1); \) \( pV_{c5} = -(2/3C0 i_3; \) \( pV_{c6} = (1/3C) (2i_1 - i_3); \)

\[ p_{i1} = \left[ Bi (e_b - e_c - V_{c6}) + L_s (e_c - e_b + V_{c1} - R_{i3} - V_{c6}) \right] / \Delta \text{ and} \]

\[ p_{i3} = \left[ 2L_s (e_a - e_b + V_{c1} - R_{i3} - V_{c4}) + L_s (e_b - e_c - V_{c6}) \right] / \Delta \] \( \text{ (Y.71)} \)

(y) Algebraic Equations: \( i_a = i_3; \) \( i_b = i_1 + i_3; \) \( i_c = -i_1; \) \( i_d = i_3; \) \( V_d = L_{pi3} + R_{i3}; \)

\( V_{DI} = 0.0; \) \( V_{T1} = V_{CI}; \) \( i_{T1} = 0.0 \) \( \text{ (Y.72)} \)

From (Fig. 5.26); \( i_2 = -1/3 (i_1 + i_3) \) \( \text{ (Y.73)} \)

(z) Conditional Equations: Resonance is over when \( I_w = 0 \) and D2 allows full DC side current. Next instant operation I2 starts, as \( i_3 = i_1 \). \( \text{ (Y.74)} \)

(b12) Operational Circuit – 12 \([t_{i1} - t_{i2}]\):

Fig. Y.27 shows the circuit in which one can find that, T3, D1, T4 and D2 conduct simultaneously.

(x) Differential Equations: \( pV_{c1} = -[2/3C] i_3; \) \( pV_{c2} = -[2/3C] i_3; \) \( pV_{c3} = [1/3C] i_3; \)

\( pV_{c4} = [1/3C] i_3; \) \( pV_{c5} = [1/3C] i_3; \) \( p_{i1} = (1/B) (e_a - e_c + V_{c1} - R_{i3} + V_{c2}) \)

(y) Algebraic Equations: \( i_a = i_3; \) \( i_b = 0.0; \) \( i_c = -i; \) \( i_d = i_3; \) \( V_d = L_{pi} + R_{i}; \)

\( V_{DI} = 0.0; \) \( V_{T1} = V_{CI}; \) \( i_{T1} = 0.0 \) \( \text{ (Y.75)} \)

(z) Conditional Equations: The transfer to next operational circuit occurs when T5 is switched ON at \( t = T5 \)

Where, \( T5 = T4 + [1/6] T_P; \) Where \( T_P \) = time period of supply frequency. \text{ (Y.76)}

(b13) Operational Circuit – 13 \([t_{i2} - t_{i3}]\):

Fig. Y.28 shows it. Here D1, D5, T4 and D2 conduct simultaneously.

(x) Differential Equations: \( pV_{c1} = -[1/3C] i_3; \) \( pV_{c2} = -[2/3C] i_3; \) \( pV_{c3} = -[1/3C] i_3; \)

\( pV_{c4} = [1/3C] i_3; \) \( pV_{c5} = [2/3C] i_3; \) \( pV_{c6} = [1/3C] i_3; \)
Fig. Y.26 Operational circuit-11 of submode 90°-150°

Fig. Y.27 Operational circuit-12 of submode 90°-150°

Fig. Y.28 Operational circuit-13 of submode 90°-150°

Fig. Y.29 Operational circuit-14 of submode 90°-150°
(b14) Operational Circuit – 14 \( t_{13} - t_{14} \):

Fig. Y.29, shows the operational circuit - 14 where D3 and D1 conductings together. This resonant circuit is active till the D1 turns off. Here T5, D1, D3, T4 and D2 conduct simultaneously.

(x) Differential Equations: \( pV_{c1} = -[1/3C] (2i_1 - i_2); pV_{c2} = -[1/3C] i_2; \)
\( pV_{c3} = [1/3C] i_3; pV_{c5} = [1/3C] i_1 + i_3; pV_{c6} = [1/3C] i_3; \)
\( pi = [BI (e_a - e_b + V_{C1}) + L_5 (e_b - e_c + V_{C3} - Ri_3 + V_{C2})] / \Delta \) and
\( pi_3 = [2L_5 (e_b - e_c + V_{C3} - Ri_3 + V_{C2}) + L_4 (e_a - e_b + V_{C1})] / \Delta \)  \( \text{(Y.81)} \)

(y) Algebraic Equations: \( i_a = i_1; i_b = (i_1 + i_2); i_c = -i_3; i_d = i_3; V_d = Lpi_3 + Ri_3; \)
\( V_D1 = 0.0; \)  \( \text{(Y.82)} \)

From (Fig. 5.29); \( i_2 = 1/3 (i_1 + i_2) \)

(z) Conditional Equations: The resonance will be over when the current of D1 = 0 and D3 allows to pass full dc side current. At this instant the transfer to the next operational circuit takes place. \( i_1 = 0 \)  \( \text{(Y.83)} \)

(b15) Operational Circuit – 15 \( t_{14} - t_{15} \):

Fig. Y.30 shows it. The T5, D3, T4 and D2 conduct simultaneously.

(x) Differential Equations: \( pV_{c1} = [1/3C] i; pV_{c2} =-[2/3C] i; pV_{c3} =-[2/3C] i; \)
\( pV_{c4} = [1/3Cp] i; pV_{c5} = [1/3C] i; pV_{c6} = [1/3] i; \)
\( pi = (1/BI) (e_b - e_c + V_{C3} - Ri + V_{C2}) \) \( \text{(Y.84)} \)

(y) Algebraic Equations: \( i_a = 0.0; i_b = i; i_c = -i; i_d = i; V_d = Lpi + Ri; \)
\( V_D1 = V_{ab} + V_{C1}; i_{t1} = 0.0; V_{T1} = -V_{c5} \) \( \text{(Y.85)} \)

(z) Conditional Equations: The transfer to the next operational circuit occurs when the device T6 is switched ON at the instant, \( t = T_6 \) \( \text{(Y.86)} \)

where, \( T_6 = T_5 + [1/6] T_P. \)
(b16) **Operational Circuit – 16 [t₁₅– t₁₆]:**

Fig.Y.31 shows the circuit. Here T5, D3, T6 and D2 conduct simultaneously.

(x) Differential Equations:
\[ pV_{c₁} = [1/3C] i; \]
\[ pV_{c₂} = -[1/3C] i; \]
\[ pV_{c₃} = -[2/3C] i; \]
\[ pV_{c₄} = [1/3C] i; \]
\[ pV_{c₅} = [1/3C] i; \]
\[ pV_{c₆} = [2/3C] i; \]
\[ Pi = (1/BI) (e_b - e_c + V_{C₁} - R_i + V_{C₆}) \]  \hspace{1cm} (Y.87)

(y) Algebraic Equations:
\[ i_a = 0.0; i_b = i; i_c = -i; i_d = i; V_d = Lpi + R_i; \]
\[ V_{D₁} = V_{ab} + V_{C₁}; i_{T₁} = 0.0; V_{T₁} = -V_{C₃} \]  \hspace{1cm} (Y.88)

(z) Conditional Equations: The transfer to the next operational circuit will take place when the diode D4 starts conducting. This occurs when,
\[ -V_{C₂} >= V_{ac} \]  \hspace{1cm} (Y.89)

(b17) **Operational Circuit – 17 [t₁₆– t₁₇]:**

Fig.Y.32, Resonant circuit is developed due to conduction of D4 and D2. Here the T5, D3, T6, D2 and D4 conduct simultaneously.

(x) Differential Equations:
\[ pV_{c₁} = [1/3C] i; \]
\[ pV_{c₂} = [1/3C] (2i₁ - i₃); \]
\[ pV_{c₃} = -[2/3C] i₃; \]
\[ pV_{c₄} = [1/3C] (i₁ + i₃); \]
\[ pV_{c₅} = [1/3C] i₃; \]
\[ pV_{c₆} = [1/3C] (2i₃ - i₁); \]
\[ pi = [BI (e_c - e_a + V_{C₂}) + L₅ (e_b - e_c + V_{C₃} - R_{i₃} + V_{C₅})] / \Delta \] \hspace{1cm} (Y.90)
\[ pi₃ = [2L₆ (e_b - e_c + V_{C₃} - R_{i₃} + V_{C₅}) + L₆ (e_c - e_a - V_{C₂})] / \Delta \]

(y) Algebraic Equations:
\[ i_a = -i₁; i₆ = i₃; i_c = i₁ - i₃; i_d = i₃; V_d = Lpi₃ + R_{i₃}; \]
\[ V_{D₁} = V_{ab} + V_{C₁}; V_{T₁} = -V_{C₅}; i_{T₁} = 0 \] \hspace{1cm} (Y.91)

The i2 (from Fig. 5.32) is equal to[1/3 ](i₁ + i₃)

(z) Conditional Equations: The resonance will be over when the I_{D₂} = 0 and D4 allows full dc side current. Here the transfer to the next operational circuit takes place. Now
\[ i₃ = i₁ \] \hspace{1cm} (Y.92)

(b18) **Operational Circuit – 18 [t₁₇– t₁₈]:**

Fig.Y.33. Here T5, D3, T6 and D4 conduct simultaneously.

(x) Differential Equations:
\[ pV_{c₁} = [1/3C] i; \]
\[ pV_{c₂} = [1/3C] i; \]
\[ pV_{c₃} = -[2/3C] i; \]
\[ pV_{c₄} = [1/3C] i; \]
\[ pV_{c₅} = [1/3C] i; \]
\[ pV_{c₆} = [1/3C] i; \]
\[ pi = (1/BI) (e_b - e_c + V_{C₃} - R_i + V_{C₄}) \] \hspace{1cm} (Y.93)
(y) Algebraic Equations: 
\[ i_a = -i; \quad i_b = i; \quad i_c = 0; \quad i_d = i; \quad V_d = L p_i + R i; \]
\[ V_{D1} = V_{ab} + V_{C1}; \quad V_{T1} = -V_{C5} \]  \hspace{1cm} (Y.94)

(z) Conditional Equations: The transfer to the operational circuit occurs when the device 
T1 is switched ON at the instant. \( t = T_1 \) \hspace{1cm} (Y.95)
Where, \( T_1 = T_6 + [1/6 ]T_p. \)

Thus at the end of one input cycle, transfer of circuit topology takes place from the 
operational circuit 18 to the operational circuit – 1.

c. Sub-mode 150° – 210°:

In fig. Y.34, the conduction pattern has been shown. The reference time instant is 
synchronous with the switching instant of the controlled power device T1. Depending 
upon the set of devices under conduction, this sub-mode is also sub-divided into 18 time 
intervals. Their operational equations are given below:

(c-1) Operational Circuit – 1 \([t_0 - t_1]\): 
Fig. 5.35 shows the operational circuit – 1. Here D5, D4, T1 and T6 conduct 
simultaneously its circuit is shown in fig. Y.35

(x) Differential Equations: 
\[ p V_{C1} = [1/3C] i; \quad p V_{C2} = [1/3C] i; \quad p V_{C3} = [1/3C] i; \]
\[ p V_{C4} = -[2/3C] i; \quad p V_{C5} = -[2/3C] i; \quad p V_{C6} = [1/3C] i; \]
\[ p V_{C7} = -[2/3C] i; \quad p V_{C8} = -[2/3C] i; \quad p V_{C9} = [1/3C] i; \]
\[ p I_1 = (1/B I) (e_c - e_b + V_{C5} + V_{C5}) \]  \hspace{1cm} (Y.96)

(y) Algebraic Equations: 
\[ i_a = -i; \quad i_b = 0.0; \quad i_c = i; \quad i_d = i; \quad V_d = L p_i + R i; \]
\[ V_{D1} = V_{ab} - V_{C5}; \quad V_{T1} = 0.0; \quad i_{T1} = i_d \]  \hspace{1cm} (Y.97)

(z) Conditional Equations: 
Next operational circuit starts when D6 starts conducting. This 
condition is achieved when, \(-V_{C4} > V_{ba}\) \hspace{1cm} (Y.98)

(c-2) Operational Circuit – 2 \([t_1 - t_2]\): Fig. Y.36 shows the operational circuit. Here the 
D6 conduct along with D4. The resonant circuit is active till the D4 turns OFF. During 
this interval the T1, D5, T6, D6 and D4 conduct simultaneously.

(x) Differential Equations: 
\[ p V_{C1} = [1/3C] i_2; \quad p V_{C2} = [1/3C] (i_2 - i_1); \quad p V_{C3} = [2/3C] i_2; \]
\[ p V_{C4} = -[2/3C] (i_2 - i_1); \quad p V_{C5} = -[2/3C] i_2; \quad p V_{C6} = [1/3C] (i_2 - i_1); \]
\[ p I_2 = [2L_s (e_c - e_b + V_{C5} + C_{V4} - R_{i_2}) + L_s (e_a - e_b - V_{C4})] / \Delta \] and
\( p_i = \left[ B_l (e_a - e_b + V_{C4}) + L_5 (e_c - e_a + V_{C5} + V_{C4} - R_i i) \right] / \Delta \) \hspace{1cm} (Y.99)

(y) Algebraic Equations: \( i_a = i_1 - i_2; \quad i_b = -i_1; \quad i_c = i; \quad i_d = i_2; \quad V_d = L p i_2 + R i_2; \)
\( V_{D1} = V_2 - V_{C5}; \quad V_T1 = 0.0 \) \hspace{1cm} (Y.100)

(z) Conditional Equations: Resonance will be over when the current through the diode D4 goes to zero and the D6 allows full dc side current at this instant. The transfer to the next operational circuit will take place when \( i_2 = i_1 \) \hspace{1cm} (Y.101)

(c-3) **Operational Circuit – 3 \([t_2 - t_3]:**

Fig. Y.37 shows the operational circuit. Here T1, D5, T6 and D6 conduct simultaneously.

(x) Differential Equations: \( p V_{Cl} = [1/3C] i; \quad p V_{c3} = [1/3C] i; \quad p V_{c5} = -2/3C i; \)
\( p_i = (1/B_l) (e_c - e_b + R i + V_{C5} - R_i) \) \hspace{1cm} (Y.102)

(y) Algebraic Equations: \( i_a = 0.0; \quad i_b = -i; \quad i_c = i; \quad i_d = i; \)
\( V_{D1} = V_{ac} - V_{C5}; \quad V_d = L p i + R i; \quad V_T1 = 0.0; \quad i_T1 = i_d \) \hspace{1cm} (Y.103)

(z) Conditional Equation: When T2 is switched ON at the instant \( t = T_2; \)
Where \( T_2 = T_1 + 1/6 T_P \) (the next operation starts) \hspace{1cm} (Y.104)
\( T_P \) is the time period of supply frequency and \( T_1 = 0 \) at the instant when the device T1 is switched ON for the first time.

(c-4) **Operational Circuit – 4 \([t_2 - t_4]:**Fig. Y.38, Here T1, D5, T2 and D6 conduct simultaneously.

(x) Differential Equations: \( p V_{c1} = [1/3C] i; \quad p V_{c2} = [1/3C] i; \quad p V_{c3} = [1/3C] i; \)
\( p V_{c4} = -[2/3C] i; \quad p V_{c5} = -[2/3C] i; \quad p V_{c6} = -[2/3C] i; \)
\( p_i = (1/B_l) (e_c - e_b + V_{C5} - R_i + V_{C6}) \) \hspace{1cm} (Y.105)

(y) \( i_a = 0.0; \quad i_b = -i; \quad i_c = i; \quad i_d = i; \)
\( V_d = L p i + R i; \quad V_{D1} = V_{ac} - V_{C5}; \quad i_T1 = i_d; \quad V_T1 = 0.0 \) \hspace{1cm} (Y.106)

0(z) Conditional Equation: As D1 starts next operational circuit begin when \( V_{ac} \geq V_{C5} \) \hspace{1cm} (Y.107)

(c-5) **Operational Circuit – 5 \([t_4 - t_5]:**

When the D1 starts conducting along with D5, the resonant circuit is active till the D5 turn OFF. Here the T1, D1, D5, T2 and D6 conduct simultaneously. Its circuit is shown in fig. Y.39

(x) Differential Equations: \( p V_{c1} = [1/3C] (i_2 - i_1); \quad p V_{c2} = [1/3C] i_2; \quad p V_{c3} = [1/3C] (i_2 - i_1); \)
\( p V_{c4} = [1/3C] i_2; \quad p V_{c5} = -[2/3C] (i_2 - i_1); \quad p V_{c6} = -[2/3C] i_2; \)
Fig. Y.34 Conduction pattern of power devices in submode 150°-210°

Fig. Y.35 Operational circuit-1 of submode 150°-210°

Fig. Y.36 Operational circuit-2 of submode 150°-210°
\[\pi_1 = \frac{[BI (e_a - e_c - V_{C5}) + L_5 (e_c - e_b + V_{C5} + V_{C6})]}{\Delta}\]

\[\pi_2 = \frac{[2L_4 (e_c - e_b - R_i + V_{C5} + V_{C3}) + L_4 (e_a - e_c - V_{C5})]}{\Delta}\]

(y) Algebraic Equations: \(i_a = i_1; i_b = -i_2; i_c = i_2; i_d = i_2; V_d = L_{pi2} + R_i;V_{D1} = 0.0; i_{T1} = i_d, \ V_{T1} = 0.0\) \[(Y.108)\]

(z) Conditional Equations: When \(i_{DS} = 0; \) Resonance is over and \(D1\) allows full DC side current at this instant the transfer to the next operational circuit take place.

Here \(i_2 = i_1\) \[(Y.110)\]

(c-6) **Operational Circuit – 6 \([t_5 - t_6]\):** Fig. 5.40 shows the circuit, in this \(T1, D1, T2\) and \(D6\) conduct simultaneously.

(x) Differential Equations: \(pV_{c1} = [1/3C] i; pV_{c2} = [1/3C] i; pV_{c3} = [2/3C] i;\)

\[p_i = \frac{1}{(BI)} (e_a - e_b - R_i + V_{C6})\]

(y) Algebraic Equations: \(i_a = i; i_b = -i; i_c = 0.0; i_d = i; V_d = L_{pi} + R_i; V_{D1} = 0.0; i_{T1} = i_d\)

(c-7) **Operational Circuit – 7 \([t_6 - t_7]\):**

Fig. 5.41 shows the circuit. Here \(D1, T4, D6\) conduct simultaneously.

(x) Differential Equations: \(pV_{c1} = [-2/3C] i; pV_{c2} = [1/3C] i; pV_{c3} = [1/3C] i;\)

\[pV_{c4} = [1/3C] i; pV_{c5} = [1/3C] i; pV_{c6} = [-2/3C] i;\]

\[p_i = \frac{1}{(BI)} (e_a - e_b + V_{C1} - R_i + V_{C6})\]

(y) Algebraic Equations: \(i_a = i; i_b = i; i_c = 0.0, i_d = i;\)

\[V_d = L_{pi} + R_i; V_{D1} = 0.0; V_{T1} = V_{C1}; i_{T1} = i_d\]

(c-8) **Operational Circuit – 8 \([t_7 - t_8]\):** Fig. Y.42 shows the operational circuit. The \(D2\) starts conducting along with \(D6\). This resonant circuit is active till the diode \(D4\) turns OFF. Here \(T3, D1, T2\) and \(D6\) conduct simultaneously.

(x) \(pV_{c1} = [-2/3C] i_2; pV_{c2} = [1/3C] (i_2 - i_1); pV_{c3} = [1/3C] i_2;\)

\[pV_{c4} = [1/3C] i_2; pV_{c5} = [1/3C] i_2; pV_{c6} = [-2/3C] (i_2 - i_1);\]

\[p_{II} = \frac{[BI (e_b - e_c - V_{C6}) + L_5 (e_a - e_b + V_{C1} - R_{i2} + V_{C5})]}{\Delta} \text{ and}\]
\[ \pi_2 = \frac{[2L_s (e_a - e_b + V_{c1} - R_{l2} + V_{c3}) + L_s (e_b - e_c - V_{c3})]}{\Delta} \quad (Y.117) \]

(y) Algebraic Equations: \( i_a = i; \ i_b = i_1 - i_2; \ i_c = i_1; \)

\[ V_d = L p_i_2 + R_{i2}; \ V_{D1} = 0.0; \ V_{T1} = V_{C1} \quad (Y.118) \]

(z) Conditional Equation: Resonance will be over when \( i_{D6} = 0 \) and the diode D2 allows full DC side current, at this instant transfer to the next operational circuit will take place at \( i_1 = i_3 \) \( \quad (Y.119) \)

(c-9) **Operational Circuit - 9 \[t_9 - t_6\]:**

Fig. Y.43, Here T3, D1, T2, D2 conduct simultaneously.

(x) Differential Equations: \( pV_{c1} = -[2/3C] i; \ pV_{c2} = [1/3C] i; \ pV_{c3} = [1/3C] i; \)

\[ p_1 = (1/BI) (e_a - e_c + V_{c1} - R_i) \quad (Y.120) \]

(y) Algebraic Equations: \( i_a = i; \ i_b = 0.0; \ i_c = i; \)

\[ V_d = L p_i + R_i; \ id = I; \ V_{T1} = V_{C1}; \ i_{T1} = 0.0; \ V_d = 0.0 \]

(z) Conditional Equation: Next operation starts when \( T_4 \) is switched on at the instant \( t = T_4 \)

Where \( T_4 = T_3 + [1/6 \ T_p; \ T_p \) is the time period of supply frequency.

(c-10) **Operational Circuit - 10 \[t_9 - t_6\]:**

Its circuit is shown in fig. Y.44. Here T3, D1, T4, D2 conduct simultaneously.

(x) Differential Equations: \( pV_{c1} = -[2/3C] i; \ pV_{c2} = -[2/3C] i_2; \ pV_{c3} = [1/3C] i_2; \ pV_{c4} = [1/3C] i; \ pV_{c5} = [1/3C] i; \ pV_{c6} = [1/3C] i; \)

\[ p_1 = -(1/BI) (e_a - e_c + V_{c1} - R_i + V_{c2}) \quad (Y.123) \]

(y) Algebraic Equations: \( i_a = i; \ i_b = 0.0; \ i_c = i; \ i_d = i; \)

\[ V_d = L p_i + R_i; \ V_{D1} = 0.0; \ id = 0; \ V_{T1} = V_{C1} \quad (Y.124) \]

(z) Conditional Equation: The transfer to the next operation is taken place when D3 starts conducting and this condition comes when, \( V_{bc} >= V_{c1} \)

(c-11) **Operational Circuit - 11 \[t_1 - t_{11}\]:**

Fig. Y.45 shows the circuit. Here D3 conducts along with D1 so, resonant circuit works till D1 turns OFF. Here T3, D2, D1, T4 and D2 conduct simultaneously.

(x) Differential Equations: \( pV_{c1} = -[2/3C] (i_2 - i_1); \ pV_{c2} = -[2/3C] i_2; \ pV_{c3} = [1/3C] (i_2 - i_1); \ pV_{c4} = [1/3C] i_2; \ pV_{c5} = [1/3C] (i_2 - i_1); \ pV_{c6} = [1/3C] i_2; \)

\[ p_{i1} = [B (e_a - e_a - V_{c1}) + L_s (e_a - e_c + V_{c1} - R_i + V_{c3})] / \Delta \] and

\[ p_{i2} = [L_s (e_a - e_b + V_{c1}) + 2L_s (e_a - e_c + V_{c2})] / \Delta \quad (Y.126) \]
Fig. Y.41 Operational circuit-7 of submode 150°-210°

Fig. Y.42 Operational circuit-8 of submode 150°-210°

Fig. Y.43 Operational circuit-9 of submode 150°-210°

Fig. Y.44 Operational circuit-10 of submode 150°-210°
Fig. Y.45 Operational circuit-1 of submode 150°-210°

Fig. Y.46 Operational circuit-2 of submode 150°-210°

Fig. Y.47 Operational circuit-3 of submode 150°-210°

Fig. Y.48 Operational circuit-4 of submode 150°-210°
(y) Algebraic Equations: \( i_a = i_2 - i_1; i_b = i; i_c = -i_2; i_d = i_2; V_d = Lp_i + R_i \); 
\[ V_{D1} = 0.0; i_{T1} = i_2; V_{T1} = V_{C1} \] (Y.127) 
(z) Conditional Equations: After \( i_{D1} = 0 \); resonance circuit is over and D3 allows dc side current. Here next operation is started, when \( i_2 = i_1 \) (Y.128) 
(c-12) **Operational Circuit - 12** \([t_{11} - t_{12}]\): 
Fig. Y.46 shows the circuit. Here T3, D3, T4 and D4 conduct simultaneously. 
(x) Differential Equations: \( pV_{c1} = [2/3C] i; pV_{c2} = [2/3C] i; pV_{c3} = [1/3C] i; \) 
\[ p_i = (1/BI) (e_b - e_c - R_i + V_{c2}) \] (Y.129) 
(y) Algebraic Equations: \( i_a = 0.0; i_b = i; i_c = -i; i_d = i \); 
\[ V_d = Lp_i + R_i; V_{D1} = V_{ab} + V_{C1}; V_{T1} = V_{C1}; i_{T1} = 0 \] (Y.130) 
(z) Conditional Equation: The transfer to the next operational circuit occurs when T5 is switched on at \( t = T_3 \) (Y.131) 
where \( T_3 = T_a + [1/6] T_p \); where \( T_p \) is time period of supply frequency. 
(c-13) **Operational Circuit - 13** \([t_{12} - t_{13}]\): 
Fig. Y.47, Here D3, D2, T5 and T4 conduct simultaneously. 
(x) Differential Equations: \( pV_{c1} = [1/3C] i; pV_{c2} = [2/3C] i; pV_{c3} = [2/3C] i; \) 
\[ pV_{c4} = [1/3C] i; pV_{c5} = [1/3C] i; pV_{c6} = [1/3C] i; \) 
\[ p_i = (1/BI) (e_b - e_c + V_{c3} - R_i + V_{c2}) \] (Y.132) 
(y) Algebraic Equations: \( i_a = 0.0; i_b = i; i_c = -i; i_d = i \); 
\[ V_d = Lp_i + R_i; V_{D1} = V_{ab} + V_{C1}; V_{T1} = -V_{C3}; i_{T1} = 0.0 \] (Y.133) 
(z) The transfer to the next operational circuit will take place when the diode D4 starts conducting. This will occur when the following condition is met. \(-V_{c2} >= V_{ac}\) (Y.134). 
(c-14) **Operational Circuit - 14** \([t_{13} - t_{14}]\): 
Fig.Y.48 shows the operational circuit. The D4 starts conducting along with the diode D2. This resonant circuit is active till the diode D2 turns off. Here T5, D3, T4, D4 and D2 conduct simultaneously. 
(x) Differential Equations: \( pV_{c1} = [2/3C] i; pV_{c2} = -[2/3C] (i_2 - i_1); pV_{c3} = [2/3C] i; \) 
\[ pV_{c4} = [1/3C] (i_2 - i_1); pV_{c5} = [1/3C] i; pV_{c6} = [1/3C] (i_2 - i_1); \) 
\[ p_i = [BI (e_c - e_a - e_{c2}) + L_s (e_b - e_c + V_{c3} - R_i + V_{c2})] / \Delta \] and 
\[ p_i2 = [L_s (e_c - e_a - V_{c2}) + 2L_s (e_b - e_c - V_{c3} - R_i + V_{c2})] / \Delta \] (Y.135) 
(y) Algebraic Equations: \( i_a = -i; i_b = i_2; i_c = i_1 - i_2; i_d = i_2; V_d = Lp_i + R_i \);
(z) Conditional Equations: As \( i_{D2} = 0 \) the resonance is over and D4 allows full dc side current. At this instant, the transfer to next operational circuit is being taken place. This condition can be mathematically shown as as: \( i_1 = i_2 \) (Y.137)

(c-15) **Operational Circuit - 15** \([t_{t4} - t_{t5}]\):  
Fig. Y.49, Here T5, D3, T4 and D4 conduct simultaneously.

(x) Differential Equations: \( pV_{c1} = [1/3C]i \); \( pV_{c3} = [1/3C]i \); \( pV_{c5} = [1/3C]i \); \( pV_d = [-2/3C]i \); \( pV_{C5} = [1/3C]i \); \( pV_{C5} = [1/3C]i \); \( pi = (1/BI) (\epsilon_b - \epsilon_a + V_{C3} - R_i) \) (Y.138)

(y) Algebraic Equations: \( i_a = i_b = i_c = 0.0; i_d = i_i \); \( V_d = Lpi + Ri; V_{D1} = V_{ab} + V_{C1}; V_{T1} = V_{C5}; i_{T1} = 0 \) (Y.139)

(z) Conditional Equations: Next operation starts when T6 is switched on at the instant \( t = T_6 \) (Y.140)

where, \( T_6 = T_5 + [1/6] \) \( T_p \) is the time period of supply frequency.

(c-16) **Operational Circuit - 16** \([t_{t5} - t_{t6}]\):  
Fig. Y.50 shows the circuit, where the D4, D3, T5 and T6 conduct simultaneously.

(x) Differential Equations: \( pV_{c1} = [1/3C]i \); \( pV_{c3} = [1/3C]i \); \( pV_{c3} = [-2/3C]i \); \( pV_{c4} = -2/3C i \); \( pV_{C5} = 1/3C i \); \( pV_{C5} = 1/3C i \); \( pi = (1/BI) (\epsilon_b - \epsilon_a + V_{C3} - R_i + V_{C4}) \) (Y.141)

(y) \( i_a = -i; i_b = i; i_c = 0.0; i_d = i; V_d = Lpi + Ri; V_{T1} = -V_{C5}; V_{D1} = V_{ab} + V_{C1} \) (Y.142)

(z) Next step starts when D5 conduction is ON. For this condition is: \( V_{ab} >= V_{C3} \) (Y.143)

(c-17) **Operational Circuit - 17** \([t_{t6} - t_{t7}]\):  
Fig. Y.51 shows the operational circuit. Here D5 starts conducting along with the diode D3. This resonant circuit is active till the diode D3 turns OFF. Here D3, D5, T5, T6 and D4 conduct simultaneously.

(x) Differential Equations: \( pV_{c1} = [1/3C]i_2 - i_1 \); \( pV_{c2} = [1/3C]i_2 - i_1 \); \( pV_{c3} = [-2/3C]i_2 - i_1 \); \( pV_{c5} = [1/3C]i_2 \); \( pV_{C5} = [1/3C]i_2 \); \( pi_1 = [BI (\epsilon_c - \epsilon_b - V_{C5}) + L_5 (\epsilon_b - \epsilon_a + V_{C3} - R_{C2} + V_{C4})] / \Delta \) and \( pi_2 = [L_4 (\epsilon_c - \epsilon_b - V_{C3}) + 2L_4 (\epsilon_b - \epsilon_a + V_{C3} - R_{C2} + V_{C4})] / \Delta \) (Y.144)

(y) Algebraic Equations: \( i_a = -i_2; i_b = i_2 - i_1; i_c = i_1; i_d = i_2; V_d = LPI_2 + Ri_2 \);
\[ V_{D1} = V_{ab} + V_{C1}; i_{T1} = 0.0; V_{T1} = V_{CS} \]  \( \text{(Y.145)} \)

(z) Conditional Equations: The resonance will be over when \( i_{D3} = 0 \) and D5 allows full dc side current. Next operation starts when \( i_1 = i_2 \) \( \text{(Y.146)} \).

(c-18) **Operational Circuit – 18** \([t_{17} - t_{18}]\):  

Fig. Y.52 shows the circuit. Here the T5m D5, T6 and D4 conduct simultaneously.

(x) Differential Equations: \( pV_{C1} = [1/3C] i; pV_{C4} = [-2/3C] i; pV_{C6} = [1/3C] i; \)  
\[ p_1 = (1/BI) (e_c - e_a - R_i + V_{C4}) i_{T1} = 0 \]  \( \text{(Y.147)} \)

(y) Algebraic Equations: \( i_a = -i; i_b = 0.0; i_c = i; \)  
\[ V_d = Lpi + Ri; i_d = i; V_{D1} = V_{ac} - V_{CS}; V_{T1} = -V_{CS}; \]  \( \text{(Y.148)} \)

(z) Conditional Equations: The transfer to the operational circuit – 1 occurs when the device T1 is switched ON at the instant, \( t_1 = T_1 \)  \( \text{(Y.149)} \)

Where, \( T_1 = (T_6 + [1/6] T_P) \) where \( T_P \) is the time period of the supply frequency. There at the end of one input cycle, transfer of circuit topology take place from the operational circuit – 18 to the operational circuit – 1.

d. **Sub mode** \((210^\circ - 360^\circ)\):

Conduction pattern is shown in fig. Y.53. The reference time instant is synchronized with the switching instant of the controlled power device T1. Here also the conduction mode is sub divided in 18 time interval, depending upon the set of devices for conduction. The relevant (x) differential (y) Algebraic and (z) Conditional Equations for each operational circuit are given below:

(d1) **Operational Circuit – 1** \([t_{9} - t_{11}]\):

Fig. Y.54 shows the operational circuit – 1 during the interval \( t_9 - t_{11} \), where the devices T1, T6, D5 and D6 simultaneously conduct.

(x) Differential Equations: \( pV_{C1} = [1/3C] i; pV_{C3} = [1/3C] i; pV_{C5} = [-2/3C] i; \)  
\[ p_1 = (1/BI) (e_c - e_a + V_{CS} - R_i) \]  \( \text{(Y.150)} \)

(y) Algebraic Equations: \( i_a = -i; i_b = -i; i_c = i; \)  
\[ V_a = Lpi + Ri; i_d = i; V_{D1} = V_{ac} - V_{CS}; i_{T1} = 0; V_{T1} = 0 \]  \( \text{(Y.151)} \)

(z) Conditional Equation: Next operational circuit will take place when the D1 starts conducting. This occur when, \( V_{ac} >= V_{CS} \)  \( \text{(Y.152)} \)
Fig.Y.55 shows the circuit. When D1 starts conducting along with the D5, the resonant circuit is active till the diode D5 turns OFF. Here, the T1, D1, D5, T6 and D6 conduct simultaneously.

(x) Differential Equations: \( pV_{e1} = \frac{1}{3C} (i_2 - i_1) \); \( pV_{e3} = \frac{1}{3C} (i_2 - i_1) \);
\( pV_{e5} = -\frac{2}{3C} (i_2 - i_1) \);
\( pi_1 = \left[ B_1 (e_a - e_b - V_{C3}) + L_5 (e_b - e_a - V_{C5} - R_{i2}) \right] / \Delta \)
\( pi_2 = \left[ 2L_6 (e_a - e_b + V_{C5} - R_{i2}) + L_5 (e_a - e_b - V_{C5}) \right] / \Delta \)

(y) Algebraic Equations: \( i_a = i_1; i_b = i_2 - i_1; i_c = -i_2; i_d = i_2; V_d = Lpi_2 - Ri_2 \);
\( V_{D1} = V_{ab} + V_{C1}; V_{D1} = 0; V_{T1} = 0; i_{T1} = i_d \)

(z) Conditional Equations: The resonance will be over when the \( i_{D5} = 0 \) and D1 allows full dc side current. At this instant the transfer to the next operational circuit takes place at \( i_1 = i_2 \).
Fig. Y.49 Operational circuit-15 of submode 150°-210°

Fig. Y.50 Operational circuit-16 of submode 150°-210°

Fig. Y.51 Operational circuit-17 of submode 150°-210°

Fig. Y.52 Operational circuit-18 of submode 150°-210°
Fig. Y.53 Conduction pattern of power devices in submode 210°-360°
Fig. Y.56 Operational circuit-3 of submode 150°-210°

Fig. Y.57 Operational circuit-4 of submode 210°-360°

Fig. Y.58 Operational circuit-5 of submode 210°-360°

Fig. Y.59 Operational circuit-6 of submode 210°-360°
Fig.Y.60 Operational circuit-7 of submode 210°-360°

Fig.Y.61 Operational circuit-8 of submode 210°-360°

Fig.Y.62 Operational circuit-9 of submode 210°-360°

Fig.Y.63 Operational circuit-10 of submode 210°-360°
(d-5) **Operational Circuit – 5 \(t_5-t_6\)**: Fig.Y.58 shows the operational circuit. Here D2 starts conducting along with D6. This resonance circuit is active till D6 turns OFF. Here, T1, D2, T2, D2 and D6 conduct simultaneously.

(x) \(pV_{d2} = [1/3C] (i_3 - i_1); pV_{d4} = [1/3C] (i_2 - i_1); pV_{d6} = -[2/3C] (i_2 - i_1); p_i = [B_I (e_b - e_c - V_{C6}) + L_s (e_a - e_c + V_{C6} - R_{d2})] / \Delta\)

(y) Algebraic Equations: \(i_a = i_2; i_b = (i_1 - i_2); i_c = -i_1; i_d = i_2; V_d = L p_i + R i_2; V_{D1} = 0; V_{T1} = 0; i_{T1} = 0\)

(z) Conditional Equation: The transfer to the next operational circuit will be taken place when \(i_{d6} = 0\) and D2 allows full dc side current. At this instant the transfer to the next operational circuit takes place.

Here \(i_1 = i_2\)

(Y.164).

(d-6) **Operational Circuit – 6 \(t_5-t_6\)**: Fig.Y.59 shows the operational circuit. Here T1, D1, T2 and D2 conduct simultaneously.

(x) Differential Equations: \(p_i = (1/B_I) (e_a - e_c - R_i)\)

(y) Algebraic Equations: \(i_a = i; i_b = 0; i_c = -i; i_d = i; V_a = L p_i + R i_2; V_{D1} = 0; V_{T1} = 0; i_{T1} = i_d\)

(z) Conditional Equation: The transfer to the next operational circuit occurs when the device T3 is ON at the instant, \(t = T_3\)

Where, \(T_3 = T_2 + [1/6] T_p\)

(d-7) **Operational Circuit – 7 \(t_5-t_6\)**: Fig.Y.60 shows the operational circuit. Here T1, D1, T2 and D2 conduct simultaneously.

(x) Differential Equations: \(pV_{C1} = [-2/3C] i; pV_{C3} = [1/3C] i; pV_{C5} = [1/3C] i; and p_i = (1/B_I) (e_a - e_c + V_{C1} - R_i)\)

(y) Algebraic Equations: \(i_a = i; i_b = 0; i_c = -i; i_d = i; V_a = L p_i + R_i; V_{T1} = V_{C1}\)

(z) Conditional Equation: The transfer to the next operational circuit will be taken place when D2 starts conducting of \(V_{ba} >= V_{C1}\)
Operational Circuit – 8 [t7 – t8]: Fig.Y.61 shows the circuit, when D2 starts conducting along with the D1, the resonant circuit is active till D1 turns OFF. Here, D1, D3, D2, T2 and T3 conduct simultaneously.

(x) Differential Equations: 
\[ pV_{c1} = -\left[ \frac{2}{3}C \right] i; \quad pV_{c3} = \left[ \frac{1}{3}C \right] i; \quad pV_{c5} = \left[ \frac{1}{3}C \right] i; \]
\[ \pi_1 = \left[ BI (e_a - e_b + V_{C1}) + L_a (e_b - e_c - R_{i2}) \right] / \Delta \]
\[ \pi_2 = \left[ 2L_a (e_b - e_c - R_{i2}) + L_a (e_a - e_b + V_{C1}) \right] / \Delta \]  

(y) Algebraic Equations: 
\[ i_a = i_1; \quad i_b = i_2 - i_1; \quad i_c = i_2; \quad i_d = i_2; \quad V_d = L\pi_2 + R_i; \]
\[ V_{D1} = 0; \quad V_{T1} = V_{CI}; \quad i_{T1} = 0 \]  

(z) Conditional Equations: The resonance will be over when the current through D1 goes to zero and D2 allows full dc side current, at this instant the transfer to the next operational circuit takes place when \( i_1 = i_2 \)  

Operational Circuit – 9 [t8 – t9]: Fig.Y.62 shows the circuit. Here the T3, D3, T2 and D2 conduct simultaneously.

(x) Differential Equations: 
\[ \pi_1 = (1/BI) (e_b - e_c - R_i) \]  

(y) Algebraic Equations: 
\[ i_a = 0; \quad i_b = i; \quad i_c = i; \quad i_d = i; \]
\[ V_d = L\pi_1 + R_i; \quad V_{D1} = V_{ab} + V_{T1}; \quad V_{T1} = V_{CI}; \quad i_{T1} = i_d \]  

(z) Conditional Equation: The transfer to the next operational circuit occurs when the T4 is switched ON at the time, \( t = T_4 \)  

Where, \( T_4 = T_3 + \left[ \frac{1}{6} \right] T_p; \quad T_p \) is the time period of supply frequency.

Operational Circuit – 10 [t9 – t10]: Fig.Y.63 shows the circuit. Here T3, D3, T4 and D2 conduct simultaneously.

(x) Differential Equations: 
\[ pV_{c2} = -\left[ \frac{2}{3}C \right] i; \quad pV_{c4} = \left[ \frac{1}{3}C \right] i; \quad pV_{c5} = \left[ \frac{1}{3}C \right] i; \]
\[ \pi_1 = \left[ (1/BI) (e_b - e_c + V_{C2} - R_i) \right] (Y.178) \]
\[ \pi_1 = \left[ (1/BI) (e_b - e_c + V_{C2} - R_i) \right] \]
\[ V_d = L\pi_1 + R_i; \quad V_{D1} = V_{ab} + V_{C1}; \quad V_{T1} = V_{CI}; \quad i_{T1} = 0 \]  

(z) Conditional Equation: \( t = T_5 \)  

Operational Circuit – 11 [t10 – t11]: Fig.Y.64 shows the circuit of d-11. When D1 starts conducting along with the D2, the resonant circuit becomes active till the diode D2 turns OFF. Here the T3, D3, T4, D4 and D2 conduct simultaneously.

(x) Differential Equations: 
\[ pV_{c2} = -\left[ \frac{2}{3}C \right] (i_2 - i_1); \quad pV_{c4} = \left[ \frac{1}{3}C \right] (i_2 - i_1); \]
\[ pV_c = \left[ 1/3C \right] (i_2 - i_1); \]

\[ p_{i_1} = \left[ B1 (e_b - e_a - V_{C2}) + L_d (e_b - e_c - R_{i2} + V_{C2}) \right] / \Delta \]

\[ p_{i_2} = \left[ 2L_d (e_b - e_a - R_{i2} + V_{C2}) + L_d (e_c - e_a - V_{C2}) \right] / \Delta \quad (Y.181) \]

(y) Algebraic Equations: \[ i_b = -i_1; \quad i_b = i_2; \quad i_c = (i_1 - i_2); \quad i_d = i_2; \quad V_d = L_{pi} + R_{i2}; \]

\[ V_{D1} = V_{ab} + V_{C1}; \quad V_{T1} = V_{C1}; \quad i_{T1} = i_d \quad (Y.182) \]

(z) Conditional Equations: The resonance will be over when the \[ i_{D2} = 0 \] and the diode D4 allows full dc side current. At this instant the transfer to the next operational circuit takes place. This condition can be mathematically shown as below:

\[ i_1 = i_2 \quad (Y.183). \]

(d-12) **Operational Circuit – 12 \([t_{11} - t_{12}]\):**

Fig.Y.65 shows the circuit diagram. Here T3, D3, T4 and D4 conduct simultaneously.

(x) Differential Equations: \[ pV_{C1} = [1/3C] i; \quad pV_{C3} = [-2/3C]; \quad pV_{C5} = [1/3C] i; \]

and

\[ p_{i_1} = (1/B1) (e_b - e_a + V_{C3}) \quad (Y.186) \]

(y) Algebraic Equations: \[ i_a = -i; \quad i_b = i; \quad i_c = 0; \quad i_d = i; \]

\[ V_d = L_{pi} + R_i; \quad V_{D1} = V_{ab} - V_{C5}; \quad V_{T1} = V_{C5}; \quad i_{T1} = 0 \quad (Y.187) \]

(z) Conditional Equation: The transfer to the next circuit occurs when the T5 is switched ON at the time, \[ t = T_5 \quad (Y.185) \]

Where, \[ T_5 = T_4 + [1/6 ]T_p; \]

\[ T_P \] is the time period of supply frequency.

(d-13) **Operational Circuit – 13 \([t_{12} - t_{13}]\):**

Fig.Y.66 shows the circuit. Here T5, D3, T4 and D4 conduct simultaneously.

(x) Differential Equations: \[ pV_{C1} = [1/3C] i; \quad pV_{C3} = -[2/3C]; \quad pV_{C5} = [1/3C] i; \]

and

\[ p_{i_1} = (1/B1) (e_b - e_a + V_{C3}) \quad (Y.186) \]

(y) Algebraic Equations: \[ i_a = -i; \quad i_b = i; \quad i_c = 0; \quad i_d = i; \]

\[ V_d = L_{pi} + R_i; \quad V_{D1} = V_{ab} - V_{C5}; \quad V_{T1} = V_{C5}; \quad i_{T1} = 0 \quad (Y.187) \]

(z) Conditional Equation: The transfer to the next operational circuit will take place when the D5 starts conducting. This will occur when the following condition is met:

\[ V_{C5} >= V_{C3} \quad (Y.188). \]

(d-14) **Operational Circuit – 14 \([t_{13} - t_{14}]\):**

Fig.Y.67 shows the circuit. When D5 starts conducting along with D3, the resonant circuit is active till the D3 turns OFF. During this interval T5, D5, D3, T4 and D4 conduct simultaneously.
(x) Differential Equations: \( pV_1 = \frac{1}{3}C_i \); \( pV_3 = \frac{-2}{3}C_i \); \( pV_5 = \frac{1}{3}C_i \);
\[
p_i = \frac{[BI (e_b - e_a + V_{C2}) + L_s (e_c - e_a - R_{i2})]}{\Delta}
\]
\[
p_i = \frac{[2L_s (e_c - e_a - R_{i2}) + L_s (e_b - e_c)]}{\Delta}
\]  
(Y.189)

(y) Algebraic Equations: \( i_a = -i_2; i_b = i_1 \); \( i_c = i_2 - i_1; i_d = i_2; V_d = Lp_i_2 + R_i_2 \);
\[
V_{D1} = V_{ac} - V_{CS}; V_{T1} = -V_{CS}; i_{T1} = i_d
\]  
(Y.190)

(z) Conditional Equation: The resonance will be over when the current through the diode D3 goes to zero and D5 allows full dc side current. Next operation starts at this moment. Here \( i_1 = 0 \)  
(Y.191).

(d-15) Operational Circuit – 15 \( t_{14} - t_{15} \):
Fig.Y.68 shows the circuit. Here T5, D5, T4 and D4 conduct simultaneously.

(x) Differential Equations: \( pV_i = (1/BI) (e_c - e_a - R_i) \)  
(Y.192)

(y) Algebraic Equations: \( i_a = i; i_b = 0; i_c = i; i_d = i \);
\[
V_{D1} = V_{ac} - V_{CS}; V_{T1} = -V_{CS}; i_{T1} = i_d
\]  
(Y.193)

(z) Conditional Equation: The transfer to the next operational circuit occurs when the devices T6 is switched ON at the time, \( t = T_6 \)  
(Y.194)

Where, \( T_6 = T_5 + [1/6]T_P; T_P \) is the time period of supply frequency.

(d-16) Operational Circuit – 16 \( t_{15} - t_{16} \):
Fig.Y.69 shows the circuit. Here D5, T5, T6 and D4 conduct simultaneously.

(x) Differential Equations: \( pV_{c2} = [1/3]C_i \); \( pV_{c4} = \frac{-2}{3}C_i \); \( pV_{c6} = [1/3]C_i \); and
\[
p_i = \frac{[BI (e_b - e_a + V_{C4} - R_i)]}{\Delta}
\]  
(Y.195)

(y) Algebraic Equations: \( i_a = -i; i_b = 0; i_c = i; i_d = i \);
\[
V_d = Lp_i + R_i; V_{D1} = V_{ac} - V_{CS}; V_{T1} = -V_{CS}; i_{T1} = i_d
\]  
(Y.196)

(z) Conditional Equation: Next operation starts when D6 in ON. This will occur when the \( V_{C4} \geq V_{ba} \)  
(Y.197).

(d-17) Operational Circuit – 17 \( t_{16} - t_{17} \):
Fig.Y.70 shows it. When D6 starts conducting along with the D4, the resonance circuit is active till the diode D4 turns OFF. Here T5, D5, T6, D6 and D4 conduct simultaneously.

(x) Differential Equations: \( pV_{c2} = [1/3]C_i (i_2 - i_1) \); \( pV_4 = -[2/3]C_i (i_2 - i_1) \); \( pV_6 = [1/3]C_i (i_2 - i_1) \);
\[
p_i = [BI (e_b - e_a + V_{C4}) + L_s (e_c - e_b - R_{i2})] / \Delta
\]
\[
p_i = [2L_s (e_c - e_b - R_{i2}) + L_s (e_b - e_a + V_{C4})] / \Delta
\]  
(Y.198)
(y) Algebraic Equations: \( i_a = -i_1; i_b = i_1 - i_2; i_c = i_2; i_d = i_2 \) and
\[ V_d = Lp i_2 + R i_3; \ V_{D1} = V_{ac} - V_{CS}; \ V_{T1} = V_{CS}; \ i_T1 = i_d \] (Y.199)

(z) Conditional Equations: The resonance will be over when the current through D4 goes to zero and D6 allows full dc side current. At this instant transfer to the next operational circuit takes place. Here \( i_1 = i_2 \) (Y.200)

(d-18) **Operational Circuit – 18 \([t_{17} – t_{18}]\):**
Fig. Y.71 shows the circuit. Here T5, D5, T6 and D6 conduct simultaneously.

(x) Differential Equations: \( p i = (1/BJ) (e_c - e_b - R i) \) (Y.201)

(y) Algebraic Equations: \( i_a = 0; i_b = i; i_d = i \); (Y.202)

(z) The transfer to the operational Circuit – 1 is taken place when the T1 is switched ON for the first time at; \( t = T_1 \) (Y.203).

Where, \( T_1 = T_6 + [1/6] T_p \); where \( T_p \) is the time period of supply frequency. Thus at the end of one input cycle, transfer of current topology takes place from the operational circuit – 18 to the operational circuit – 1. Conduction of all devices, operational circuits and corresponding equations would repeat from now onward for operational circuit – 1 to the operational circuit – 18 in a similar manner as expressed earlier.

***