Chapter-4

THERMOELASTIC ANALYSIS OF SKEW LAMINATES WITH CUTOUTS
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4.1 INTRODUCTION

In the existing literature, various methods such as i) Classical Lamination Plate Theory, ii) First-order Shear Deformation Theory, iii) Higher-order Shear Deformation Theory, iv) Theory of Elasticity and v) Finite element methods have been used for the analysis of laminated composite plates. From the review of literature it is observed that there exists no Thermoelastic analysis of skew plates with cutouts using elasticity theory. The Theory of Elasticity based finite element method is used for the thermoelastic analysis of Skew laminated composite plates with circular cutouts.

This chapter presents the finite element modeling for the heat transfer analysis and thermoelastic analysis of skew laminated composite plates with a circular hole at the geometric centre of the plate. A 20 node brick element is used for the three-dimensional heat conduction analysis and the similar element designed based on the 3-D elasticity theory is used for thermoelastic analysis to create the finite element mesh. Convergence tests are made and the results of a five-layered square laminate are compared with the results of the exact temperature distribution available in literature. The results due to the pressure load are also compared with exact elasticity theory to test the validity of the finite element method. The finite element
method is then used to determine the transverse deflection and stresses (including the inter-laminar stresses at the free edge of the hole) of a skew laminated plate with hole for various cases by varying
i) skew angle for cross-ply laminate, ii) fiber angle for cross-ply laminate, iii) fiber angle for angle-ply laminate, iv) fiber angle and skew angle for the laminate with fibers parallel to the sides of the plate and v) fiber angle and skew angle for the laminate with fibers parallel to the diagonals of the plate.

4.2 LAMINATE NOMENCLATURE

Laminated composite or laminate is a stack of layers made of different materials or of same material with different orientations of principal material directions. In the engineering terminology of composite material, a layer also called lamina is a resin impregnated sheet of reinforcement cured in a flat or curved shape. In a laminate the layers differ one from the other by the variation of

- fiber volume fraction,
- fiber orientation,
- thickness and
- fiber material.

When the constituent materials of all the lamina are same, the layered structure is called simply laminate, when the constituents are different for different layers, the stacked structure is called a hybrid laminate. The process of lamination yields a constructional flexibility by means of which directional and locational distribution of the
material is permitted to accommodate the varied nature of the loading in a structure. A specific situation of this nature is a sandwich.

For the purpose of analysis, it is required to establish a coordinate system for specifying locations through the thickness of the laminate as well as along the length and across the width. For flat plates, an $x$-$y$-$z$ global Cartesian coordinate system is useful in describing a laminate.

![Diagram](image)

Fig. 4.1 Laminate nomenclature

Fig. 4.1 illustrates a global Cartesian coordinate system and a general laminate consisting of $N$ layers. The upper portion of the figure is a cross-sectional view in the $x$-$z$ plane ($y = 0$ plane), and the lower portion is a plan form view. The laminate thickness is denoted by $H$ and the thickness of an individual layer by $h$. Not all layers necessarily have the same thickness, so the thickness of the $k^{th}$ layer...
is denoted as \( h_k \). The geometric mid plane may be within a particular layer or at an interface of adjacent layers. Herein the \( +z \) axis will be downward and the laminate extends in the \( z \)-direction from \(-H/2\) to \( H/2\). The layer at the extreme negative \( z \) location is referred as layer 1, the next layer is layer 2, the layer at an arbitrary location as layer \( k \), and the layer at the extreme positive \( z \) position as layer \( N \). The locations of the layer interfaces are denoted by a subscript to \( z \); the first layer is bounded by locations \( z_0 \) and \( z_1 \), the second layer by \( z_1 \) and \( z_2 \), the \( k \)th layer by \( z_{k-1} \) and \( z_k \), and the \( N \)th layer by \( z_{N-1} \) and \( z_N \).

To identify the fiber angles of the various layers, the fiber angle relative to the \( +x \) axis of each layer is specified. The specification starts with layer 1, the layer at the extreme negative \( z \) location.

4.2.1 Symmetric Laminates

Fig. 4.2 \([0/90]_s\) Laminate
The laminate in Fig. 4.2 is denoted as a [0/90/90/0] laminate. The x-axis is oriented in the lengthwise direction of the laminate. The leftmost entry in the laminate notation refers to the orientation of layer 1. In cases where the stacking sequence to the one side of \( z = 0 \) plane, the laminate geometric mid-plane, is a mirror image of the stacking sequence on the other side of the \( z = 0 \) plane, the stacking notation can be abbreviated by referring to only one-half of the laminate and subscripting the stacking notation with an \( s \), which means symmetric. The laminate of Fig. 4.2 can thus be denoted by [0/90] \( s \). With this notation, the leftmost entry in stacking specification is either layer 1 or layer \( N \); that is the stacking specification starts with the outer layer on each side of the laminate.

To categorize a laminate as symmetric, it is imperative that the material properties, fiber orientation and thickness of the layer at a specific location to one side as the geometric mid-plane be identical to the material properties, fiber orientation and thickness of the layer at the mirror image location on the other side. Otherwise the laminate is not truly symmetric. In fact symmetric laminates have been emphasized to such an extent that if one encounters the notations [0/90/90/0] in a discussion of composites, one usually assumes that an eight layer laminate is being discussed, with stacking sequences [0/90/90/0/0/90/90/0]. To emphasize that in deed the complete laminate is being specified by the subscript \( T \) for total is some times used. Thus the laminate of Fig. 4.2 could be denoted as [0/90/90/0] \( T \).
When the stacking sequence involves adjacent layers of opposite orientations, as is often the case, short hand notation is used. For example, if a six layer Laminated has the stacking sequence $[+45/-45/0/0/-45/+45]/_r$, it would be abbreviated as $[\pm 45/0]_s$. Hence the $\pm$ is used to contract the notation and indicates there is a layer with its fibers oriented at $+45^0$ with respect to the $+x$ axis and adjacent to it another layer with its fibers oriented at $-45^0$ with relative to $+x$ axis. Next to the $-45^0$ layer is a layer with its fibers aligned with the $x$-axis, as shown in Fig 4.3.

When a stacking sequence of a subset consisting of several layers is repeated within a laminate, further shorthand notation is often used. If a 12-layer laminate has a stacking arrangement of $[+45/-45/0/+45/-45/0/-45/+45/0/-45/+45/0]/_r$, it can be contracted to read $[(\pm 45/0)_2]_s$.

4.2.2 Unsymmetric Laminate

If a laminate is not symmetric, then it is referred to as an unsymmetric laminate.
4.2.3 Balanced Laminate

A laminate is said to be balanced if for every layer with a specified thickness, specific material properties and specific fiber orientation, there is another layer with the identical thickness, material properties, but opposite fiber orientation somewhere in the laminate. The layer with opposite fiber orientation does not have to be on the opposite side of the reference surface, nor immediately adjacent to the other layer, nor anywhere in particular. The other layer can be anywhere within the thickness. A laminate does not have to be symmetric to be balanced. The symmetric $[\pm 30/0]$ s laminate and the unsymmetric $[\pm 30/0]_T$ laminate are both balanced laminates.
4.2.4 Symmetric Balanced Laminate

A laminate is said to be a symmetric balanced laminate if it needs both the criterion for being symmetric and the criterion for being balanced are satisfied.

4.2.5 Angle-ply Laminate

A laminate is said to be an angle ply laminate if the adjacent laminae have opposite signs of the angle of orientation of the principal material properties with respect to the laminate axis.

4.2.6 Cross-Ply Laminate

A laminate is said to be a cross-ply laminate if every layer has its fiber oriented at either 0° or 90°. In the present work, symmetric cross-ply and angle-ply laminates are considered for analysis.
4.3 PROBLEM MODELING

4.3.1 Geometric Modeling

The Fig 4.4 shows the in-plane dimensions of the laminate considered for the present analysis. The dimensions for 'l' and 'b' are taken as 20 units of length. The value of 'd' is determined from the ratio of d/l and the skew angle $\alpha$ is varied from 0 to 50°, the thickness of the plate is fixed from the length to thickness ratio $l/h$ (s) which is taken as 10. The individual layers are arranged so that the total thickness of the odd number of layers is equal to the total thickness of the even number of layers in case of five-layered laminate.
4.3.2 Finite Element Modeling

The finite element mesh for the heat transfer analysis is generated using a three-dimensional brick element ‘SOLID90’, a 20-node thermal element that is applicable to a 3-D, steady state or transient thermal analysis. The element has 20 nodes with a single degree of freedom, temperature, at each node. The 20-node elements have compatible temperature shapes and are well suited to model curved boundaries.

The finite element mesh for structural analysis is generated using a three-dimensional brick element ‘SOLID 95’. This element is a structural solid element designed based on three dimensional elasticity theory and is used to model orthotropic solids. The element is defined by 20 nodes having three degrees of freedom u, v and w per node which are translations in the nodal x, y, and z directions respectively. The element may have any spatial orientation.

4.3.3 Boundary Conditions

i) Thermal:

a) Uniform Temperature (UT)

In this case all the points of the laminated plate are subjected to uniform temperature rise of 100°C.
b) *Linearly varying Temperature (LVT)*

In this case a temperature of $100^\circ C$ on the top surface and $25^\circ C$ on the bottom surface of the laminate are applied and the sides of the laminate are insulated so that the variation of temperature across the thickness of the laminate is linear.

\[ \text{c) Non-linearly varying Temperature (NVT)} \]

A temperature of $100^\circ C$ on the top surface, and $25^\circ C$ on the bottom surface and side faces of the laminate are applied. The surface of the hole is subjected to convection with film coefficient $h=5\text{W/m}^2\text{°C}$ and bulk temperature $25^\circ C$. In this case the variation of the temperature across the thickness is observed as non-linear.

ii) **Structural:**

All the edges of the skew plate are clamped i.e. all the three degrees of freedom (Displacements in global x-, y- and z- directions) of the nodes attached to the side faces of the plate are constrained.

**4.3.4 Loading**

i) The output from the thermal analysis is applied as thermal loading while doing structural analysis.

ii) A transverse pressure of 1MPa is applied on the top surface of the plate in addition to the change in temperature in case of combined loading.
4.3.5 Material Properties (Graphite-Epoxy)

The material properties for graphite-epoxy are taken as below from the reference Tungikar and Rao (238).

\[
\begin{align*}
K_L &= 36.42 \text{ W/m K} & K_T &= 0.96 \text{ W/m K} \\
E_L &= 172.72 \text{ GPa}, & E_T &= 6.909 \text{ GPa} \\
G_{LT} &= 3.45 \text{ GPa}, & G_{TT} &= 1.38 \text{ GPa}, \\
\nu_{LT} - \nu_{TT} &= 0.25 & \alpha_L &= 0.57 \times 10^{-6} / \degree \text{C} & \alpha_T &= 35.6 \times 10^{-6} / \degree \text{C}
\end{align*}
\]

where \( L \) signifies the direction parallel to the fibers, \( T \) the transverse direction and \( \nu_{LT} \) is poisson's ratio measuring strain in the \( T \) direction under uniaxial normal stress in the \( L \) direction.

4.4 VALIDITY OF THE FEM ANALYSIS

Tungikar and Rao (238) analyzed a cross ply laminate with three layers of equal thickness and obtained an exact solution for steady state heat conduction. The results for the thermal analysis of the present finite element model are compared with the above solution for its validity and a very good agreement between the results (Fig.4.5) is observed.

![Normalized Temperature Plot](image)

Fig 4.5 Comparison of the present FE Model with Analytical solution
To validate the finite element results for structural analysis, a five layered square plate with simply supported edges and subjected to a sinusoidal load of \( p = p_0 \sin \left( \frac{n\pi}{l} \right) \sin \left( \frac{m\pi}{b} \right) \), where \( l \) and \( b \) are the length and width of the plate, is modeled with SOLID95 element. The results obtained from this model are compared with the exact elasticity solution (142) for various lengths to thickness ratios of the plate (Table 4.1). It is observed that the finite element results are in close agreement with the exact elasticity solution.

Table 4.1 Comparison of present work with exact elasticity theory

<table>
<thead>
<tr>
<th>( S = l/h )</th>
<th>Normalized ( \sigma^0 ) (0.5, 1.5, 1/3)</th>
<th>Normalized ( \epsilon^0 ) (0.5, 1.5, 1/3)</th>
<th>Normalized ( \tau_{xy} ) (0, 1/2, 0)</th>
<th>Normalized ( \tau_{xz} ) (0, 1/2, 0)</th>
<th>Normalized ( w ) (0.5, 1/2, 0)</th>
<th>Normalized ( w ) (0.5, 1/2, 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>EL 0.545 -0.545 -0.536</td>
<td>EL 0.430 -0.432 -0.431</td>
<td>EL 0.223</td>
<td>EL 0.258</td>
<td>EL 0.677</td>
<td>EL 0.692</td>
</tr>
<tr>
<td></td>
<td>FE 0.537 -0.537 -0.536</td>
<td>FE 0.431 -0.431 -0.431</td>
<td>FE 0.209</td>
<td>FE 0.212</td>
<td>FE 0.212</td>
<td>FE 0.212</td>
</tr>
<tr>
<td>20</td>
<td>EL 0.539 -0.539 -0.535</td>
<td>EL 0.380 -0.380 -0.378</td>
<td>EL 0.212</td>
<td>EL 0.268</td>
<td>EL 0.4938</td>
<td>EL 0.4838</td>
</tr>
<tr>
<td></td>
<td>FE 0.534 -0.534 -0.535</td>
<td>FE 0.377 -0.377 -0.377</td>
<td>FE 0.218</td>
<td>FE 0.271</td>
<td>FE 0.271</td>
<td>FE 0.271</td>
</tr>
</tbody>
</table>

where

\( EL = \) Exact Elasticity solution (142)

\( FE = \) Finite Element solution

Normalized \( \sigma = \frac{\sigma}{p_0s^2} \)

Normalized \( \tau = \frac{\tau}{p_0s} \)

Normalized \( w = \frac{100E_2w}{p_0hs^4} \)

\( s = \) Length of the plate (l) / thickness of the plate (h)

\( p_0 = \) The maximum intensity of sinusoidal load
Material Properties used for the verification: (142).

\[
\begin{align*}
E_L &= 0.175 \times 10^6 \text{ MPa} \\
G_{LT} &= 0.35 \times 10^4 \text{ MPa} \\
\nu_{LT} &= \nu_{TT} = 0.25 \\
E_T &= 0.7 \times 10^4 \text{ MPa} \\
G_{TT} &= 0.14 \times 10^4 \text{ MPa}
\end{align*}
\]

4.5 Present Work

In the present work the transverse deflection and stresses (including the inter-laminar stresses at the free edge of the hole) of a 5-layered clamped skew laminated plate of length to thickness ratio, \(s = 10\) with circular hole at the centre of the plate and subjected to thermal as well as uniform pressure loads are evaluated for various cases by varying the following parameters

- size of the hole,
- skew angle,
- fiber angle and
- temperature.

The discussion of the results is presented in chapter 5, 6, 7 and 8.

In chapter 5 the results obtained for uniform pressure in combination with uniform temperature load are discussed. Chapter 6 discusses the analysis of results obtained for uniform pressure in combination with linearly varying temperature load. The results obtained for uniform pressure in combination with non-linearly varying temperature across the thickness of the laminate are presented in chapter 7. A comparative study of effect of different types of temperature load is explained in chapter 8.