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Stress analysis plays an important role in the design of machine or structural members. The stress analysis is mainly intended to estimate the load carrying capacity of the members. Mechanics of Materials and Theory of Elasticity approaches have been developed and successfully applied for the stress analysis of the members.

Mechanics of Materials approach is simple but has limited applications due to the assumptions such as i) plane cross-sections before deformation remain plane after deformation and ii) geometry of the member does not contain any discontinuities. Theory of Elasticity method, on the other hand, is free of these assumptions. It involves the derivation and integration of governing equations of a stress element of load carrying member. The exact closed form solution of these complex governing equations is obtainable only for cases of geometric, loading and material simplicity. While in case of members with complex materials, boundary, geometry and loading the elasticity solutions are impossible / prohibitively expensive. However, the solution of the exact elasticity equations of real life complex mechanics problem is attempted through one of various approximate numerical solution techniques vogue in practice.
For complex situations the finite element method based on the Theory of Elasticity is the versatile numerical technique solution to obtain an approximate solution. The accuracy of the results can be improved by modifying the degrees of freedom of the finite element mesh. In this chapter a brief introduction is given for the following topics relevant to the research project.

i) Introduction to stress analysis

ii) Introduction to fiber-reinforced Composites

iii) Introduction to finite element method

1.2 INTRODUCTION TO STRESS ANALYSIS

In many structures and machines the main function of a member is to resist the external forces, called loads, which are applied to it. In resisting the loads the member must not undergo structural damage, that is, it must not fail to perform its function satisfactorily in the structure or machine. The term structural damage or failure as used here, therefore, does not necessarily mean fracture; it means any action in the member, such as excessive elastic deflection, inelastic deformation or yielding and fracture resulting from the application of loads, which causes the member to cease to function satisfactorily in the structure or machine.

The main consideration or problem in solid mechanics consists in obtaining the relation between the loads applied to a member and its
behavior characterized by tensile stress, shearing stress, elastic strain or deflection, strain energy, etc., which is significant in the action or phenomenon that causes the member to fail in its load-carrying function; the dimensions of the member also are involved in this relationship.

The mode of failure of a member and the quantity that is most closely associated with the failure depend on such factors as nature or properties of the material, type of loading, shape of member, temperature of member, time during which the load acts, medium surrounding the member etc.; for example, the mode of failure of a member made of ductile steel may be very different from one made of brittle material such as concrete; yielding may constitute structural damage in the former and fracture in the latter.

There are, in general, two methods of determining stresses and strains in a member, namely, (a) by analysis of mathematical model and (b) by experimental or mechanical methods, in which either the actual member or a model of the member may be used. The method of mathematical analysis has been responsible for much of the rapid development of rational methods of design of structures and machines.
The mathematical analysis is used to derive the necessary equations by satisfying the following conditions.

1. The equations of equilibrium.
2. The compatibility conditions.
3. The stress-strain relations.
4. The material response.

Two different methods are used to satisfy the conditions (1) and (2): the method of mechanics of materials and the method of general continuum mechanics.

1.2.1 Method of Mechanics of Materials

The method of mechanics of materials is based on simplified assumptions related to the geometry of deformation (condition 2) so that strain variation across a cross section of a member is assumed. A basic assumption is that plane sections before loading remain plane after loading. The assumption can be shown to be exact for axially loaded members of uniform cross sections, for straight torsion members having uniform circular cross sections and for straight beams of uniform cross sections subjected to pure bending. The assumption is approximate for other problems.

1.2.2 Method of Continuum Mechanics - Theory of Elasticity

The method of mechanics of materials cannot be employed to the problems having multi-axial states of stress such as noncircular torsion, thick plates/ beams, thick walled cylinders, contact stresses,
stress concentrations etc. Therefore in such cases, the method of continuum mechanics is used. When we consider small displacements and when we deal with linear elastic material behavior only, the general method of continuum mechanics reduces to the method of Theory of Elasticity.

1.2.3 The Material Response and Stress-Strain Relations

In an isotropic material, properties are the same in all the directions. Thus the material contains an infinite number of planes of materials property symmetry passing through a point. In an anisotropic material, properties are different in different directions so that the material contains no planes of material property symmetry. The orthotropic materials contain three orthogonal planes of material property symmetry, namely, the planes containing 1, 2, and 3 axes as shown in Fig. 1.1.

![Fig. 1.1](image-url) Three planes of symmetry in an orthotropic material
Fig. 1.2 Differences in the deformations of isotropic, orthotropic, and anisotropic materials subjected to uniaxial tension and pure shear stresses

Differences in the mechanical behavior of isotropic, orthotropic, and anisotropic materials are demonstrated schematically in Fig. 1.2. Tensile normal stresses applied in any direction on an isotropic material cause elongation in the direction of the applied stresses and contractions in the two transverse directions. Similar behavior is observed in orthotropic material only if the normal stresses are applied in one of the principal material directions. However, normal stresses applied in any other direction create both extensional and shear deformation. In any anisotropic material, a combination of extensional and shear deformations is produced by a normal stress acting in any direction. This phenomenon of creating both extensional
and shear deformations by the application of either normal or shear stresses is termed "extension – shear coupling" and is not observed in isotropic and orthotropic materials.

The difference in material property symmetry in isotropic, orthotropic, and anisotropic materials is also reflected in the mechanics and design of these types of material. Two examples are given below.

1. The elastic stress-strain characteristics of an isotropic material are described by three elastic constants, its Young's modulus $E$, Poisson's ratio $v$, and shear modulus $G$; only two of these are independent. The number of independent elastic constants required to characterize anisotropic and orthotropic materials are 21 and 9, respectively. For orthotropic material, the nine independent elastic constants are $E_{11}$, $E_{22}$, $E_{33}$, $G_{12}$, $G_{13}$, $G_{23}$, $v_{12}$, $v_{13}$, and $v_{23}$.

2. For an isotropic material, the sign convention for shear stresses and shear strains is of little practical significance, since its mechanical behavior is independent of the direction of shear stress. For an orthotropic or anisotropic material, the direction of shear stress is critically important in determining its strength and modulus.
1.3 INTRODUCTION TO FIBER-REINFORCED COMPOSITES

In the previous section different types of materials and their stress-strain relations have been discussed. This section discusses about the classification and importance of fiber-reinforced composite materials.

In the recent period, there has been a tremendous advancement in the science and technology of fiber-reinforced composite materials. The low density, high strength, high stiffness to weight ratio, excellent durability and design flexibility of fiber-reinforced composites are the primary reasons for their use in many structural components, in aircraft, automotive, marine and other industries.

Fiber-reinforced composites are now used in applications ranging from space craft frames to ladder rails, from aircraft wings to automobile doors, from rocket motor cases to oxygen tanks and from printed circuit boards to tennis rackets. Their use is increasing at such a rapid rate that they are no longer considered advanced materials.

The essence of fiber-reinforced composite technology is the ability to put strong stiff fibers in the right place in the right orientation and right volume fraction.
1.3.1 Definition

Fiber-reinforced composite materials consist of ‘fibers’ of high strength and modulus embedded in or bonded to a ‘weak matrix’, with distinct interface (boundary) between them. In this form, both fibers and matrix retain their physical and chemical identities, yet they produce a combination of properties that cannot be achieved with either of the constituents acting alone. In general, fibers are the principal load carrying members, while the surrounding matrix keeps them in the desired location and orientation, acts as a load transfer medium between them and protects them from environmental damages caused by elevated temperatures, humidity, etc. Thus even though the fibers provide reinforcement for the matrix, the latter also serves a number of useful functions in a fiber-reinforced composite material.

![Diagram of fiber-reinforcement types](image)

Fig. 1.3 Types of fiber-reinforcement
The fibers can be incorporated into a matrix either in continuous lengths or discontinuous forms (Fig. 1.3). The principal fibers in commercial use are various types of glass, carbon and Kevlar fibers. Other fibers such as boron, silicon carbide and aluminum oxide are used in limited quantities. The matrix material may be a polymer, a metal, or a ceramic.

1.3.2 Importance

Many fiber-reinforced composite materials offer a combination of strength and modulus that are either comparable to or better than many traditional metallic materials. Because of their low specific gravities, high strength to weight and modulus to weight ratios, these materials are markedly superior to those of metallic materials. In addition, fatigue strength to weight ratio as well as fatigue damage tolerances of many composite laminates are excellent. For these reasons, fiber-reinforced composites have emerged as a major class of structural materials and are either used or being considered as substitutions of metals in many weight-critical components in aerospace, automotive and other industries.

In general, the properties of a fiber-reinforced composite depend strongly on the direction of measurement. For example, the tensile strength and modulus of a uni-directionally oriented fiber-reinforced lamina are more when these properties are measured in the longitudinal direction of the fibers. At any other angle of
measurement, the properties are lower. The minimum value is observed at 90° to the longitudinal direction. Similar angular dependence is observed for other physical and mechanical properties, such as coefficient of thermal expansion, thermal conductivity, and impact strength. Bi- or multi-directional reinforcement, either in the planar form or in the laminated construction, yields a more balanced set of properties. Although these properties are lower than the longitudinal properties of a unidirectional composite, they still represent a considerable advantage over common structural materials on a unit weight basis.

Coefficients of thermal expansion of many fiber-reinforced composites are much lower than those for metals. As a result, composite structures may exhibit a better dimensional stability over a wide temperature range.

Another unique characteristic of many fiber-reinforced composites is their high internal damping. This leads to better vibration energy absorption within the materials and results in reduced transmission of noise and vibrations to neighbouring structures; high damping capacity can be beneficial in many automotive applications in which noise, vibration, and harshness (NVH) is a critical issue for passenger comfort.
An advantage attributed to fiber-reinforced polymers is their non-corroding behavior. However, many polymeric matrix composites are capable of absorbing moisture from the surrounding environments, which creates dimensional changes as well as adverse internal stresses within the material. As such behavior is undesirable in an application, the composite surface must be protected from moisture diffusion by appropriate paints or casting. Among the other environmental factors that may cause degradation in the mechanical properties of some polymeric matrix composites are elevated temperatures, corrosive fluids and ultraviolet rays. In metal matrix composites, oxidation of the matrix as well as adverse chemical reaction between fibers and matrix are of great concern at high temperature applications.

Increased use of fiber-reinforced composite plates in the modern engineering structures, often under stringent and varying load conditions, demands accurate assessment of their thermo-mechanical behavior. Several attractive properties including high strength (and stiffness) to weight ratio and operability over wide range of temperature have enabled the composites beneficially replace the conventional materials.

Thermoelastic behavior of fiber-reinforced composite plates may therefore be said to essentially dependent on (i) individual properties of the constituent materials, (ii) fiber orientation, (iii) layer orientation,
(iv) layer sequence and (v) geometric aspect rations in addition to (vi)
the thermal and mechanical loading / boundary conditions.

Mechanical and thermal stresses, deformations, and other related
design variables are routinely used in the design of structural and
machine components. Needless to say, an accurate and confident
selection and optimization of the fiber-reinforced composite plates in
engineering structures is possible by a systematic investigation on the
effects of pertinent parameters (i.e., (i) to (vi) enlisted above and on the
design variables).

Whereas experimental investigations in this regard might give a
valuable information, accurate local measurements are formidable
because of experimental uncertainties. An analytical cum numerical
investigation is carried out in the present work with a broad objective
of evaluating thermoelastic behaviors of fiber-reinforced laminated
composites.

1.4 INTRODUCTION TO FINITE ELEMENT METHOD

For problems of complex loading, boundary conditions and
geometry, the direct application of the mechanics of materials method
or Theory of Elasticity method, explained in section 1.1, does not yield
closed form solutions. So more advanced theories (e.g. Elasticity
Theory, Higher-order Theories) and numerical solution techniques are needed.

The finite element method is a numerical procedure for analyzing structures and continua. Usually, the problem addressed is too complicated to be solved satisfactorily by classical analytical methods. The problem may concern stress analysis, heat conduction, or any of several other areas. The finite element procedure produces many simultaneous algebraic equations, which are generated and solved on a digital computer. However, accuracy of the results can be improved by processing increased number of equations; and results accurate enough for engineering purposes are obtainable at reasonable cost.

The finite element method originated as a method of stress analysis. Today, finite elements are also used to analyze problems of heat transfer, fluid flow, lubrication, electric and magnetic fields and many others. Problems that previously were utterly intractable are now solved routinely by finite element method. Finite element procedures are used in the design of buildings, electric motors, heat engines, ships, airframes, and spacecrafts. Manufacturing companies and large design offices typically have one or more in-house large finite element programs. Smaller companies usually have access to a large program through a commercially computing centre or use a smaller program on a personal computer.
The finite element method is a method of piecewise approximation in which the approximating function $\varphi$ is formed by connecting simple functions, each defined over a small region (element). A finite element is a region in space in which a function $\varphi$ is interpolated from nodal values of $\varphi$ on the boundary of the region in such a way that inter-element continuity of $\varphi$ tends to be maintained in the assemblage.

A finite element analysis typically involves the following steps. Steps 1, 4, and 5 require decisions by the analyst and provide input data for the computer program. Steps 2, 3, 6, and 7 are carried automatically by the computer program.

1. Divide the structure or continuum into finite elements. Mesh generation programs, called preprocessors, help the user in doing this work.

2. Formulate the properties of each element. In stress analysis, this means determining nodal loads associated with all element deformation states that are allowed. In Heat transfer, it means determining nodal heat fluxes associated with all element temperature fields that are allowed.

3. Assemble elements to obtain the finite element model of the structure.

4. Apply the known loads: nodal forces and/or moments in stress analysis. Nodal heat fluxes in heat transfer.
5. In stress analysis, specify how the structure is supported. This step involves setting several nodal displacements to known values (which are often zero). In heat transfer, where typically certain temperatures are known impose all known values of nodal temperatures.


7. In stress analysis, calculate element strains from the nodal DOF and element displacement field interpolation, and finally calculate stresses from strains. In heat transfer, calculate element heat fluxes from the nodal temperatures and the element temperature field interpolation. Output interpretation programs, called postprocessors, help the user sort the output and display it in graphical form.

The power of the finite element method resides principally in its versatility. The method can be applied to various physical problems. The body analyzed can have arbitrary shape, loads, and support conditions. The mesh can mix elements of different types, shapes, and physical properties. This great versatility is contained within a single computer program. User-prepared input data controls the selection of problem type, geometry, boundary conditions, element selection, and so on.
Another attractive feature of finite elements is the close physical resemblance between the actual structure and its finite element model. The model is not simply an abstraction. This seems especially true in structural mechanics, and may account for the finite element method having its origins there.

The finite element method also has disadvantages. A specific numerical result is found for a specific problem: a finite element analysis provides no closed-form solution that permits analytical study of the effects of changing various parameters. A computer, a reliable program, and their intelligent use are essential. A general-purpose program has extensive documentation, which cannot be ignored. Experience and good engineering judgment are needed in order to define a good model. Many input data are required and voluminous output must be sorted and understood.