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APPENDIX

PUBLICATION

ABSTRACT

The behaviour of a cross-ply laminated composite skew plate with hole and fibers oriented parallel to the diagonals of the plate, subjected to temperature as well as combined temperature and transverse pressure loading has been investigated in the present analysis. The analysis has been carried out using the finite element software ANSYS. The finite element method, which works on the basis of three-dimensional theory of elasticity, is employed to evaluate the transverse deflection, in-plane stresses and interlaminar stresses. The results obtained by varying the skew angle and the hole diameter are discussed. It has been observed that the magnitude of stresses and deflection in case of combined loading is more when compared with the respective values in case of pure thermal loading. The magnitudes of the transverse deflection and in-plane stresses for combined loading are observed to be less at higher skew angles for larger size of the hole. The solutions of skew structures considered in the present analysis will be useful for the construction of safe and efficient structures like skew bridges and swept wings of aircraft structures.

Key words: FEM, Skew Laminate, Cutout, Interlaminar stresses

NOMENCLATURE

\( E_1 \) = Young's modulus of the lamina in the fiber direction
\( E_2 = E_3 \) = Young's modulus of the lamina in the transverse direction of the fiber
\( G_{12}, G_{13} = \) Shear modulus in the longitudinal plane of the fiber
\( G_{23} = \) Shear modulus in the transverse plane of the fiber
\( \nu_{12}, \nu_{13} = \) Poisson’s ratio in the longitudinal plane of the fiber
\( \nu_{23} = \) Poisson ratio in the transverse plane of the fiber
\( n = \) number of layers = 5 for the present work
\( a_1, a_2 = \) Coefficient of thermal expansion in the fiber direction
\( a_3 = \) Coefficient of thermal expansion in the transverse direction of the fiber

\( \text{EL} = \) Exact Elasticity solution
\( \text{FE} = \) Finite Element solution

Normalized \( \sigma = \frac{\sigma}{P_0 s^2} \), Normalized \( \tau = \frac{\tau}{P_0 s^1} \)

Normalized \( w = \frac{100E_1 w}{p_0 h s^4} \)

\( s = \) Length of the plate (l) / thickness of the plate (h)
\( P_0 = \) The maximum intensity of sinusoidal load
\( a = \) length and width of the square plate
\( 1/2 \) and \( 1/3 \) are the normalized positions along the thickness direction

(Normalized \( z = 2a / h, z \) coordinate measured from middle plane of the plate and \( h \) = total thickness of the plate)

1. INTRODUCTION

The increasing use of fiber reinforced laminates in space vehicles, aircrafts, automobiles, ships and chemical vessels has necessitated the rational analysis of structures for their mechanical response. In addition, the anisotropy and non-homogeneity and larger ratio of longitudinal to transverse moduli of these new materials demand

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improvement in the existing analytical tools. As a result, the analysis of laminated composite structures has attracted many research workers, and has been considerably improved to achieve realistic results. In the design of modern high-speed aircraft and missile structures, swept wing and tail surfaces are extensively employed. Moreover some of the structural elements are provided with cutouts of different shapes to meet the functional requirements like i) for the passage of various cables, ii) for undertaking maintenance work, iii) for fitting auxiliary equipment, etc. Depending upon the nature of application, these structural elements are acted upon by mechanical and thermal loads of varied nature. Usually, the anisotropy in laminated composite structures causes complicated responses under different loading conditions by creating complex couplings between extension, bending, and shear deformation modes. To capture the full mechanical behavior, it must be described by three dimensional elasticity theories.

In solving the three-dimensional elasticity equations of rectangular plates, quite a number of solution approaches has been proposed. Srinivas and Rao [1] and Srinivas et al. [2] presented a set of complete analytical analyses on bending, buckling and free vibration of plates with both isotropic and orthotropic materials. Zhang and Zhang [3] presented a new concise procedure for obtaining the static exact solution of composite laminates with piezo-thermo-elastic layers under cylindrical bending using the basic coupled thermo-electro-elastic differential equations. Setoodeh and Karami [4] employed a three-dimensional elasticity based layer-wise finite element method (FEM) to study the static, free vibration and buckling responses of general laminated thick composite plates. Pagano and Hatfield [5] have given exact solutions for the deflections and stresses of a cross-ply laminated rectangular composites without holes using elasticity theory. Kong and Cheung [6] proposed a displacement-based, three-dimensional finite element scheme for analyzing thick laminated plates by treating the plate as a three-dimensional inhomogeneous anisotropic elastic body. Frasad and Staut [7] presented a closed form solution for the moment distributions around holes in symmetric laminates subjected to bending moments. Ukkadgaonker and Rao [8] gave a general solution for bending of symmetric laminates with holes. Morley [9] developed an elementary bending theory for the small displacements of initially flat isotropic skew plates without hole. Karami et al. [10] has applied Differential Quadrature Method (DQM) for static, free vibration, and stability analysis of skewed and trapezoidal composite thin plates without hole. From the review of available literature it is observed that the thermoelastic analysis of skew plates with cutouts using elasticity theory has not been studied. The thermoelastic behaviour of a laminate with skew edges and having various types of cutouts is different from the one without skew edges and/or cutouts. So it is necessary to analyse this kind of problem using elasticity theory based finite element method to evaluate for the most accurate behaviour of thick laminated skew plates with cutouts.

2. PROBLEM STATEMENT

The present work aims at filling the knowledge gaps in the existing literature. The research problem deals with the thermoelastic analysis of skew laminated plate with cutouts by elasticity theory based finite element method.

2.1 PROBLEM MODELING

2.1.1 Geometric Modeling

Fig.1 shows the in-plane dimensions of the laminate considered for the present analysis. The dimensions for 'l' (length of the skew plate) and 'b' (width of the skew plate) parallel to y' axis are taken as 20mm.

![Fig. 1 Skew laminated composite plate with circular cutout](image)

The value of 'd' (diameter of the hole) is determined from the ratio of b/d and the skew angle 'a' is varied from 0° to 50°; the thickness of the plate is fixed from the length to thickness ratio l/h (=10). The individual layers are arranged so that the total thickness of the layers oriented along one diagonal is equal to the total thickness of the layers oriented along the other diagonal.
2.1.2 Finite Element Meshing

The finite element mesh is generated using a three-dimensional brick element 'SOLID 95'. This element (Fig. 2) is a structural solid element designed based on three-dimensional elasticity theory and is used to model orthotropic solids. The element is defined by 20 nodes having three degrees of freedom per node: translations in the nodal x, y, and z directions. The element may have any spatial orientation.

2.1.3 Boundary Conditions

All the edges of the skew plate are clamped. So, all the three degrees of freedom (Displacements in global x, y and z directions) of the nodes attached to the side faces of the plate are constrained.

2.1.4 Loading

i) A uniform temperature rise of 100°C is applied on the body of the plate in case of thermal loading.

ii) A transverse pressure of 1MPa is applied on the top surface of the plate in addition to the change in temperature in case of combined loading.

2.1.5 Material Properties (Graphite-Epoxy)

- $E_x = 172.72$ GPa, $E_y = 6.909$ GPa
- $G_{xy} = G_{yz} = 3.45$ GPa, $G_{xz} = 1.38$ GPa
- $v_{xy} = v_{yz} = v_{xz} = 0.25$
- $a_1 = 6.57 \times 10^{-4}/°C$, $a_2 = a_3 = 35.6 \times 10^{-4}/°C$

3. VALIDITY OF THE PRESENT ANALYSIS

To validate the finite element results, a square plate with simply supported edges and subjected to a sinusoidal load of $p = p_0 \sin (\pi x/a) \sin (\pi y/b)$, where 'a' and 'b' are the length and width of the plate, is modeled with SOLID95 element. The results obtained from this model are compared with the exact elasticity solution [5] (Table 1). It is observed that the finite element results are in close agreement with the exact elasticity solution.

In the present work, the transverse deflection and stresses (including the inter-laminar stresses at the free edge of the hole) of a clamped skew laminated plate with circular hole at the centre of the plate and subjected to uniform rise in temperature and transverse pressure are evaluated by varying the size of the hole and skew angle.
4. RESULTS AND DISCUSSION

Numerical results are obtained for two different load cases as mentioned above. Variations of the stresses and deflection with respect to the skew angle ($\alpha$) and the ratio of diameter of the hole to the side length of the plate ($d/l$) are shown in Figs. 3 to 9. The following observations are made:

When a skew angle is given to a square plate, two of its corners come close to each other where as other two corners move away from each other. The first effect may cause reduction in stresses and deflection, and the later effect may cause increase in deflection and stresses. In addition to this the variation of the diameter of the hole influences the stresses and deflection. When the radius of the hole increases, the area supporting the load decreases. Since the resultant load acting on the plate is equal to the product of the pressure and the supporting area, the net force acting on the plate decreases due to the increase in the radius of the hole which in turn causes for the reduction in stresses. At the same time the resisting volume of the material decreases and as a result the induced stresses will increase. The resultant effects on the stresses and deflection due to these parameters are explained below.

Fig. 3 shows the variation of $\sigma_y$ with respect to the skew angle $\alpha$. For $d/l = 0.1$ to 0.3, $\sigma_y$ increases with increase in $\alpha$ due to thermal loading. The rate of increase is drastic beyond $20^\circ$ of $\alpha$ for $d/l = 0.1$ and 0.2, and $30^\circ$ of $\alpha$ for $d/l = 0.3$. For $d/l = 0.4$, $\sigma_y$ due to thermal loading increases with increase in $\alpha$ up to $30^\circ$ and later decreases.

In case of combined loading, $\sigma_y$ increases up to $10^\circ$ of $\alpha$ for all $d/l$ ratios. For $d/l = 0.1$ and 0.3, $\sigma_y$ decreases between $10^\circ$ and $40^\circ$ and finally increases for $\alpha = 50^\circ$. When $d/l = 0.2$, the decrease in $\sigma_y$ is between $10^\circ$ and $30^\circ$ and then increases. In case of $d/l = 0.4$, there is a continuous decrease in $\sigma_y$ beyond $\alpha = 10^\circ$.

Table 1 Comparison of present work with exact elasticity theory [5]

<table>
<thead>
<tr>
<th>$d/l$</th>
<th>Normalized $\sigma_y$ ($\sigma_y/\sigma_y^{(0)}$)</th>
<th>Normalized $\tau_{xy}$ ($\tau_{xy}/\tau_{xy}^{(0)}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>EL 0.545</td>
<td>EL 0.223</td>
</tr>
<tr>
<td></td>
<td>-0.545PE</td>
<td>FE 0.209</td>
</tr>
<tr>
<td>0.2</td>
<td>0.537</td>
<td>FE 0.212</td>
</tr>
<tr>
<td></td>
<td>-0.536</td>
<td>FE 0.692</td>
</tr>
</tbody>
</table>

Fig. 4 shows the variation of $\sigma_y$ with respect to the skew angle $\alpha$. In case of thermal loading, for $d/l = 0.1$ and 0.2, $\sigma_y$ increases up to $20^\circ$ of $\alpha$. Beyond this point of $\alpha$, $\sigma_y$ increases for $d/l = 0.1$, increases between $20^\circ$ and $30^\circ$ and later decreases for $d/l = 0.2$. For other two values of $d/l$, $\sigma_y$ decreases with increase in $\alpha$.

In case of combined loading, $\sigma_y$ decreases with increase in $\alpha$ up to $10^\circ$, increases from $10^\circ$ to $30^\circ$ of $\alpha$ and later decreases for $d/l = 0.1$ and 0.2. For $d/l = 0.3$, $\sigma_y$ decreases up to $20^\circ$ of $\alpha$, remains almost constant between $20^\circ$ and $30^\circ$, and later decreases. When $d/l = 0.4$, $\sigma_y$ due to combined loading decreases with increase in $\alpha$.

$\tau_{xy}$ due to thermal loading decreases with increase in $\alpha$ for all the values of $d/l$. The rate of decrease is proportional to the $d/l$ ratio. In case of combined loading $\tau_{xy}$ increases slightly with increase in $\alpha$ up to $10^\circ$ and then decreases for $d/l$ ratios of 0.1 to 0.3. For $d/l = 0.4$, $\tau_{xy}$ decreases with increase in $\alpha$ (Fig. 5).

The interlaminar normal stress ($\sigma_z$) at the free edge of the hole decreases with increase in $\alpha$ up to $30^\circ$ and then increases for both thermal and combined loading (Fig. 6). The interlaminar shear stress $\tau_{yz}$ at the free edge of the hole increases with increase in $\alpha$ for all values of $d/l$ in thermal loading except for $d/l = 0.4$ where it decreases after $\alpha = 40^\circ$. In case of combined loading $\tau_{yz}$ increases with increase in $\alpha$ up to $20^\circ$ and decreases after $\alpha = 20^\circ$ for $d/l = 0.1$ and 0.2 and after $\alpha = 30^\circ$ for other values of $d/l$ (Fig. 7).

The interlaminar shear stress $\tau_{xz}$ at the free edge of the hole decreases with increase in $\alpha$ for thermal loading for all $d/l$ ratios. In case of combined loading this stress remains almost constant up to $10^\circ$ of $\alpha$ and then decreases for all values of $d/l$ (Fig. 8). There is no variation in the transverse deflection ($w$) due to the variation of $\alpha$ for all $d/l$ ratios in thermal loading. "$w$" decreases with increase in $\alpha$ for combined loading for all $d/l$ ratios (Fig. 9).
Fig. 3 Variation of $c_w$ with skew angle

Fig. 4 Variation of $c_y$ with skew angle

Fig. 5 Variation of $c_{yw}$ with skew angle

Fig. 6 Variation of $c_y$ with skew angle

Fig. 7 Variation of $c_{yw}$ with skew angle

Fig. 8 Variation of $c_y$ with skew angle
Fig. 9 Variation of 'w' with skew angle

The contours for the transverse deflection, the in-plane normal stress $\sigma$, and the in-plane shear stress $\tau$ are shown in Figs. 10 to 15.

Fig. 10. Transverse deflection contour (n=5, $\alpha=30^\circ$, d=6mm, Thermal loading)

Fig. 11. Transverse deflection contour (n=5, $\alpha=30^\circ$, d=6mm, Combined loading)

Fig. 12. $\sigma$ contour (n=5, $\alpha=30^\circ$, d=6mm, Thermal loading)

Fig. 13. $\sigma$ contour (n=5, $\alpha=30^\circ$, d=6mm, Combined loading)

Fig. 14. $\tau$ contour (n=5, $\alpha=30^\circ$, d=6mm, Thermal loading)
5. CONCLUSIONS

Thermoelastic analysis of a laminated composite skew plate with a circular hole at the centre of the plate has been carried out in the present work. The transverse deflection, maximum in-plane stresses and maximum interlaminar stresses at the free edge of the hole have been evaluated using 3-dimensional theory of elasticity based finite element Analysis. The results obtained for two different load cases i.e. Uniform temperature loading and combined pressure and uniform temperature loading are analyzed for the variation of skew angle of the plate and size of the hole. It has been observed that the deflection and stresses are more in case of combined loading when compared to their values for thermal loading. The magnitudes of the in-plane normal stresses are greatly affected by the skew angle variation. The present analysis helps for the design of safe and efficient structures like skew bridges and swept wings of aircraft structures. From the present work it is observed that the magnitudes of the transverse deflection and in-plane stresses for combined loading are observed to be less at higher skew angles for larger size of the hole. This method can be extended to the following cases.

i) change in the type of structural loading from uniform pressure to concentrated forces around the hole circumference.

ii) change in the type of thermal loading from uniform temperature to linear and non linear temperature variation across the thickness.

iii) Change in the type of supports from clamped to simply supported.

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