CHAPTER - III

METALLIC PARTICLES IN A BARE ELECTRODE SYSTEM: THEORY OF PARTICLE MOVEMENT

3.1 METALLIC PARTICLES ON BARE ELECTRODES IN A GIS

The understanding of the dynamics of a metallic particle in a coaxial electrode system is of vital importance for determining the effect of metallic contamination in a Gas Insulated System (GIS). If the motion pattern of a metallic particle is known, the probability of particle crossing a coaxial gap and causing a flashover can be estimated.

This study does not include particles that are stuck to an insulating or energized surface, since the problems with fixed conducting particles are different from those with free conducting particles. However, in order to develop appropriate mathematical models of the particle motion in a GIS, it is important to understand the charging mechanisms of metallic particles, which are in contact with naked, or coated electrodes. The lift-off field, which can be defined as the minimum electrical field required in the vicinity of a resting particle to make it lift from the electrode, can be estimated for a bare electrode system by making simple approximations.

For electrodes, however, experiments made with spiral particles [76] show that it is more difficult to determine the lift-off field, since, in a series of measurements; the first value of the lift-off field obtained is considerably higher than those, which follow. Other measurements performed by Parekh et al. [30] showed that, by conditioning the electrodes for several hours at a
fixed AC voltage, the variance and mean value of the lift-off field were significantly reduced.

### 3.2 CHARGING OF THE METALLIC PARTICLES IN CONTACT WITH BARE ELECTRODES

For particles on bare electrodes, several authors [77 to 82, 30] have suggested expressions for the estimation of charge on both vertical/horizontal wires and spherical particles. The equations are primarily based on the work of Felici et al. [83].

When the electric field surrounding a particle is increased, an uncharged metallic particle resting on a bare electrode will gradually acquire a net charge. The charge accumulated on the particle is a function of the local electrical field, orientation, shape and size of the particle. When the electrostatic force exceeds the gravitational force, the particle will lift. A further increase in the applied voltage will make the charged particle move into the inter-electrode gap. This increases the probability of flashover.

The lift-off field for a particle on the surface of an electrode can be estimated by solving the following equations. The electrostatic force acting on a particle of mass ‘m’ is given by

\[ F_g = mg \]  

Where \( F_g \) = Gravitational force

\( g \) = acceleration due to gravity

When a particle is positioned near or on the surface of the enclosure, the image charges, due to the presence of the grounded enclosure, have to
be considered. This can be realized by including a correction factor, \( K \) [83, 30] in the expression of the electrostatic force.

\[
F_e = KQE;
\]

(3.2)

Where

\( K \) is the correction factor less than unity

\( Q \) is the particle charge

\( E \) is the ambient electric field.

\( E(t) \) in a co-axial electrode system can be expressed as

\[
E(t) = \frac{\hat{V} \sin \omega t}{[r_0 - y(t)] \ln \left( \frac{r_0}{r_i} \right)}
\]

(3.3)

Where

\( \hat{V} \sin \omega t \) is the supply voltage on the inner electrode,

\( r_0 \) is the enclosure radius,

\( r_i \) is the inner conductor radius

\( y(t) \) is the position of the particle which is moving upwards, the distance from the surface of the enclosure towards the inner electrode.

### 3.2.1 Charge Acquired by a Spherical Conducting Particle

The charge of spherical particle lying on a bare electrode with an ambient Electric Field \( E \) can be expressed as [78, 79, 81, 84, 30]

\[
Q_s = \frac{2\pi^3 e_0 r^2 E}{3}
\]

(3.4)

where
$\varepsilon_0$ is the permittivity of free space  

$r$ is the particle radius

For spherical particles the correction factor, $K$, in equation (3.2) is equal to 0.832 \cite{83}. The lift off field can therefore be estimated by combining equations (3.1) through (3.4).

Thus, the lift-off field can be estimated as follows:

$$0.832 \times \frac{2\pi \varepsilon_0 r^3 E_{lo}^2}{3} = \left( \frac{4\pi^3}{3} \right) \rho g.$$  \hspace{1cm} (3.5)

which yields

$$E_{lo} = \frac{0.49 \sqrt{\frac{r \rho g}{\varepsilon_0}}}{l}$$  \hspace{1cm} (3.6)

$\rho = \text{density of the particle material}$  

$g = \text{acceleration due to gravity}$

3.2.2 **CHARGE ACQUIRED ON A HORIZONTAL WIRE PARTICLE**

The charge of a horizontal wire particle on a bare electrode can be expressed as \cite{24, 77, 84, 30}:

$$Q_{hw} = 2\pi \varepsilon_0 \ v \ E$$  \hspace{1cm} (3.7)

where

$r$ is the wire particle radius  

$l$ is the wire particle length.

The lift-off field can be estimated, as in the case of the spherical particle, if the correction factor is known. For horizontal wire particles, the correction factor equals 0.715 \cite{82}. Hence, if the wire particle is considered
as an ideal cylinder, the lift-off field is given by solving the following equation:

\[ 0.715 \left( 2 \pi \varepsilon_0 r l E_{LO} \right) = \pi r^2 \rho g \]  

(3.8)

which gives

\[ E_{LO} = 0.84 \sqrt{\frac{\rho g r}{\varepsilon_0}} \]  

(3.9)

Figure 3.1 shows the Schematic diagram of a typical single-phase isolated conductor gas insulated busduct (GIB).

Figure 3.2 shows the calculated lift-off field of a sphere and a horizontal wire like aluminium particle on the surface of a bare electrode as a function of the particle radius.

### 3.2.3 Charge Acquired by a Vertical Wire Particle

If a wire particle is represented by a semi-ellipsoid, the charge of a vertical wire particle on a bare electrode can be expressed as [78, 77, 84]:

\[ Q_w = \frac{\pi \varepsilon_0 l^2 E}{\ln\left(\frac{2l}{r} \right) - 1} \]  

(3.10)

and the lift-off field can be expressed as:

\[ E_{w} = \sqrt{\frac{r^2 \rho g}{\varepsilon_0 \ln\left(\frac{1}{r} \right) - 0.5}} \]  

(3.11)
i.e. once the particle has lifted from a horizontal to a vertical position, the charge will increase significantly. The sudden increase of charge will most likely lift the particle from the electrode.

There are other methods of estimating the particle charge including charge simulation algorithms, which consider the actual shape of the particle. However, for greater $l/r$ ratios, equations (3.9) and (3.10) represent acceptable approximations of the particle charge and the lift-off field, respectively.

For vertical wire particles, the value of the correction factor, $K$, in equation (3.2) is dependent on the particle length-to-radius ratio ($l/r$). For $l/r$ greater than 20, $K$ is close to unity.

From equations (3.9) and (3.10),

$$Q_{vw} > Q_{hw}$$

Fig 3.3 shows the lift-off field calculated from equation (3.10) for three vertical aluminium particles with lengths 6, 9 and 12 mm, respectively, as a function of their radii.
Fig. 3.1 Schematic diagram of a typical Gas insulated busduct

Fig. 3.2 $E_{10} - r$ plot for horizontal and vertical particles

Fig. 3.3 $E_{10} - r$ plot for three vertical particles
From equation 3.9, it can be concluded that for vertical wire particles the particle charge-to-mass ratio increases with increasing length. This implies that, in general, a longer particle moves higher from the electrode than a shorter one. Experiments show that, for ac voltage, a critical length of the wire particle with respect to flashover in GIS is of the order of few millimeters [85].

3.2.4 SIMULATION OF PARTICLE MOTION

The primary goal of the simulation is the comparison of calculated and simulated results to create a satisfactory mathematical model of particle motion in the GIS bus, which will enable future simulation of the motion of particles with arbitrary shapes.

Several authors [78-81,86] have both experimentally and theoretically, suggested solutions for the particle motion of a sphere or a wire like metallic particle in a coaxial system with bare electrodes under ac voltage. In the previous sections, details of the charge and electrostatic force on the particle were discussed. In the following discussion, a drag force will be presented as an additional force acting on the particle. The consideration of drag force was shown to be important, especially for higher gas pressures and higher velocities of the particle.

In many cases, a satisfactory model of the dynamics is created by solving the motion equation, where the electrostatic force and the gravitational force (mg), as described earlier, are the only involved forces,
\[
\frac{m d^2 y}{dt^2} = F_e - mg \tag{3.12}
\]

where \(y\) is the direction of motion (vertical axis).

If the charge on the particle is based on equations (3.3) to (3.10), no information of gas pressure is included. The model of the particle motion described above gives an acceptable approximation of the motion of the particle, even if the net charge on the particle is considered constant between the impacts. An improved model can be created by considering the drag as an additional force in the calculations, and considering the influence of gas pressure and gas properties. The motion equation of a particle with a mass \(m\), can therefore be expressed as:

\[
\frac{m d^2 y}{dt^2} = F_e - mg - F_d \tag{3.13}
\]

where \(F_d\) is drag force

3.2.5 THE DRAG FORCE

The direction of drag force is always opposed to the direction of motion. In the following equations, elongated particles are assumed to move longitudinally, which has been confirmed by investigations in this work.

Drag results from an energy dissipation in the shockwave near the particle and the skin friction along the surface of the particle. For spherical particles in compressed gases, energy dissipation in the shock waves dominates, whereas skin friction is more significant for particles with increasing length-to-radius ratios such as wire particles (equations).
A wire particle with hemispherical ends as in Figure 3.3, which is moving in a compressible fluid such as SF₆, will encounter two types of drag force components – skin and shock friction.

Fig. 3.4 A wire particle moving in a compressible fluid is subject to both skin and shock friction

For laminar flow, the drag force component around the hemispherical ends of the particle is due to shock and skin frictions [79,80,82,87]:

If the boundary (object) is stationary, the velocity of the fluid at boundary surface will be zero. Thus at the boundary surface the layer of the fluid undergoes retardation. This retarded layer further causes the retardation for the adjacent layers of fluid, there by developing a small region in the immediate vicinity (place) of the boundary surface in which the velocity of flowing fluid increases gradually from zero at the boundary surface to the velocity of the main stream. This region is known as boundary layer.

Inside the boundary layer since the viscous forces are predominant, it is reasonable to assume that the inertial forces and viscous forces are the same order of magnitude in laminar.

\[
\text{The inertial forces per unit volume} = \rho v \frac{\partial v}{\partial x} \quad (3.14)
\]

Where 'v' is the velocity of the fluid
Which is proportional to \( \rho \frac{v^2}{x} \) (since \( \frac{\partial v}{\partial x} = \frac{v}{x} \) ) \( (3.15) \)

Similarly viscous force per unit volume is \( \frac{\partial \tau}{\partial y} \)

For laminar flow which it is equal to \( \frac{\partial}{\partial y} \left[ \mu \frac{\partial v}{\partial y} \right] = \mu \frac{\partial^2 v}{\partial y^2} \) \( (3.16) \)

Therefore \( \frac{\partial \tau}{\partial y} = \frac{\mu v}{\delta^2} \) \( (3.17) \)

If equations (3.15) and (3.17) are proportional to each other then

\[
\rho \frac{v^2}{x} = k \frac{\mu v}{\delta^2} \quad (3.18)
\]

\[
\frac{\delta^2}{x^2} = \frac{k \mu}{\rho vx} \quad (3.19)
\]

where '\( \delta \)' is the thickness of the boundary layer

\[
\delta = \frac{k}{\sqrt{Re}} \quad (\text{Let } \sqrt{k^1} = k) \quad (3.20)
\]

\[
\delta = k \sqrt{\frac{\mu}{\rho v}} \quad (3.21)
\]

By exact analytical solution of the boundary layer equations Blasius obtained the value of 'k' as '5'

For shear stress '\( \tau_0 \)' can be obtained as

\[
\left( \frac{\partial v}{\partial y} \right)_{y=0} = \frac{v}{\delta} \quad (3.22)
\]
Total horizontal force $F_d$ (or) skin friction drag acting on one side of the plate on which laminar boundary layer is developed can be obtained as

Along the sides of the particle, the particle will also encounter a skin friction component:

$$F_{u_2} = \int_{\delta}^{L} \tau_0 Bdx$$ (where 'B' is width of the plate and 'L' is the length)

$$F_{d1} = \int_{0}^{L} 0.664 \sqrt{\frac{\rho v^3 \mu}{x}} Bdx$$  \hspace{1cm} (3.26)

$$= 1.328 \sqrt{\frac{v^3}{2} \rho^{1/2} \mu^{1/2} L^{1/2}} B$$

$$F_{d2} = 1.328(2 \pi r) [\mu \rho_g L]^{0.5} r^{1.5}$$  \hspace{1cm} (3.27)

where

$\rho_g$ is the gas density

$q$ is the particle length

From Stoke's Theorem
\[ F_{d1} = 6 \mu \pi r \dot{y} K_d (\dot{y}) \]  
\( (3.28) \)

\( \mu \) is the viscosity of the gas (fluid)

\( r \) is the particle radius

\( \dot{y} = \frac{dy}{dt} \) is the velocity of the particle

\( K_d (\dot{y}) \) is a dimensionless drag coefficient

Total drag force \('F_d'\) is the sum of \(F_{d1}\) and \(F_d\)

\[ F_d = F_{d1} + F_{d2} \]  
\( (3.29) \)

\[ = 6 \mu \pi r \dot{y} K_d (\dot{y}) + 1.328(2 \pi r) [\mu \rho_g L]^{0.5} (\dot{y})^{1.5} \]

\[ F_d = F_{d1} + F_{d2} = \pi r (6 \mu K_d (\dot{y}) + 2.656 [\mu \rho_g 1 \dot{y}^{0.5}] \]  
\( (3.30) \)

The coefficient \( K_d (\dot{y}) \) in equation (3.30) depends on the Reynold's number, \(Re\), which is a number that characterises the degree of turbulence behind the moving particle.

Fig 3.5 (a) Streamlines of laminar flow,   (b) Streamlines of turbulent flow
The Reynolds number is given by:

\[ R_e = \frac{2 \rho_g \dot{y}}{\mu} \]  \hspace{1cm} (3.31)

For smaller Reynolds numbers (Re<5), the coefficient \( K_d (y) \) is almost constant and equal to unity. For larger numbers, there will be turbulence behind the particle known as wake turbulence. See figure 3.5

The coefficient \( K_d (y) \) for larger Reynolds numbers can be expressed as [79,80];

\[ K_d (y) = e^{[0.1142 + 0.0543 \ln |R_i| + 0.0516 |ln |R_i|^y]} \]  \hspace{1cm} (3.32)

### 3.2.6 INFLUENCE OF GAS PRESSURE ON THE DRAG FORCE

An empirical relationship between gas pressure \( P \) and density \( \rho_g \) is given by [79,80];

\[ \rho_g = 7.118 + 6.332P + 0.2032P^2 \quad 0.1 < P < 1 \text{ (MPa)} \]  \hspace{1cm} (3.33)

Since the pressure is approximately proportional to the gas density and the drag force ratio, \( \frac{F_{d2}}{F_{d1}} \) is almost proportional to \( \frac{\sqrt{P}}{K_d (y)} \)

Hence, the ratio \( F_{d2}/F_{d1} \) is approximately proportional to \( \sqrt{P} \), where \( P \) and \( l \) are the gas pressure and particle length respectively which implies that for longer particles and higher gas pressures, the friction component along the sides of the particle is the dominant part of the drag force.
Computer simulations of the motion of metallic wire particles in a realistic single phase GIS bus with dimensions as shown in Figure 3.1. have been performed.

A conducting particle in motion in an external electrical field will be subjected to a collective influence of several forces. The forces may be divided into:

- Electrostatic force ($F_e$)
- Gravitational force ($mg$)
- Drag force ($F_d$)
- Forces due to space charges formed near the particle and forces due to local ionization near the particle surface (coronal windage effect).

3.2.7 SIMULATED FORCES ON THE PARTICLE

Electrostatic Force:

From the equation 3.9 the charge acquired by a vertical wire particle in contact with a naked enclosure can be expressed as:

$$Q_{\text{net}} = \frac{\pi \varepsilon_0 \rho^2 E(t_p)}{\ln\left(\frac{2l}{r}\right) - 1}$$  \hspace{1cm} (3.34)

where

$Q_{\text{net}}$ is the charge on the particle until the next impact with the enclosure

1 is the particle length,
r is the particle radius,

\( E(t_0) \) is the ambient electrical field at \( t = t_0 \).

The charge carried by the particle between two impacts has been considered constant in the simulations. This is a reasonable assumption when the applied voltage is low or when the gas pressure is high. Disregarding the effect of charges on the particle, the electric field in a coaxial electrode system at position of the particle can be written as:

\[
E(t) = \frac{V \sin \omega t}{[r_0 - y(t)]_\alpha \left[ \frac{r_2}{r_1} \right]}
\]

(3.35)

where

\( V \sin \omega t \) is the supply voltage on the inner electrode,

\( r_0 \) is the enclosure radius,

\( r_1 \) is the inner conductor radius

\( y(t) \) is the position of the particle which is the vertical distance form the surface of the enclosure towards the inner electrode.

From the equation (3.2), the electrostatic force is equal to:

\[
F_r = K Q_\omega E(t)
\]

(3.36)

Where \( K \) is a corrector and is a factor less than unity.

However, as was discussed in section 3.2.3 for length-to-radius ratios greater than 20 the correction factor, \( K \), is close to unity. In the simulations performed in this work, the smallest ratios are 20 (\( l=5 \) mm and \( r=0.25 \) mm); hence the correction factor was neglected in the simulations.
Gravitational Force:

The gravitational force is given by:

\[ mg = \pi r^2 l \rho g \]  \hspace{1cm} (3.37)

Drag force:

\[ F_d = \dot{y} \pi r \left( 6\mu K_d (\dot{y}) + 2.656 \left[ \mu \rho g \dot{y} \right]^{0.5} \right) \]

where

\( \dot{y} \) is the velocity of the particle

\( \mu \) is the viscosity of the fluid (SF6: \( 15.5 \times 10^{-6} \text{ kg/m} \cdot \text{s} \) at 20\(^\circ\)C)

\( r \) is the particle radius

\( \rho_g \) is the gas density

\( l \) is the particle length

\( K_d (\dot{y}) \) is a drag coefficient

Using the above forces, the particle motion equation can be solved.

By considering drag force, the above equation can be modified as

\[ m \frac{d^2 y}{dt^2} = F_e - mg - F_d \]  \hspace{1cm} (3.38)

where \( F_d \) is the drag force

The influence of gas pressure on the drag force is given by empirical formula.

\[ \rho_s = 7.118 + 6.332p + 0.2032p^2 \]  \hspace{1cm} (3.39)

where \( \rho_s \) = density \hspace{1cm} p = Pressure of the gas \hspace{1cm} 0.1 < p < 1 \text{ MPa}
The drag force is given by

\[ F_d = \frac{y\pi r (6\mu K_d(\dot{y}) + 2.656[\mu \rho g I y^2])}{\mu \rho g} \]

where

\[ \dot{y} = \frac{dy}{dt} \text{ is the velocity of the particle} \]
\[ \mu \text{ is the viscosity of the fluid} \]
\[ r \text{ is the particle radius} \]
\[ \rho \text{ is the gas density} \]
\[ I \text{ is the particle length} \]
\[ K_d(\dot{y}) \text{ is a drag coefficient.} \]

The motion of the particles was simulated by using the motion equation

\[ m \frac{d^2y}{dt^2} = F_c - mg - F_d \]  

(3.41)

So, two initial conditions are necessary for a solution:

\[ m \dot{y}(t = 0^+) = -R m \dot{y}(t = 0) \]
\[ \text{and } y(t = 0^+) = 0 \]  

(3.42)

where \( R \) is the restitution coefficient given by the ratio of incoming-to-outgoing impulses.

The restitution coefficient for copper and aluminium particles seem to be in the range of 0.7 to 0.95: \( R=0.8 \) implies that 80% of the incoming impulse of the particle is preserved when it leaves the enclosure.
The motion equation using all forces can therefore be expressed as:

\[
\begin{align*}
\dot{y}(t) &= \left[ \frac{\pi \varepsilon_0 I^2 E(t) \chi}{\ln(2) - 1} \frac{V \sin\omega}{[\rho_0 - y(t)][\ln(\frac{\rho_0}{\rho})]} \right] - mg \\
&\quad - \dot{y}(t) \pi r \left( 6 \mu K_s (\dot{y}) + 2.656 \mu \rho_s \dot{y}^2 \right) 
\end{align*}
\]  

(3.43)

The above equation is a second order non-linear differential equation. It can be solved by using iterative methods. In this thesis, the equation is solved by using Runge-Kutta 4th Order Method. The simulated results are given in chapter-VII. The flow chart is given in the appendix.

3.3 MONTE-CARLO TECHNIQUE

The above simulation yields the particle movement in the radial direction only. However, the configuration at the tip of the particle is generally not sufficiently smooth enough to enable the movement unidirectionally. This decides the movement of particle in axial direction. The randomness of movement can be adequately simulated by Monte-Carlo method. In order to determine the randomness, it is assumed that the particle emanates from its original site at any angle less than \( \theta \), where \( \theta/2 \) is half of the solid angle subtended with the vertical axis. At every step of movement, a new rectangular random number is generated between 0 and 1 and modified to \( \theta \). The angle thus assigned, fixes the position of particle at the end of every time step, and in turn determines the axial and radial positions. The position in the next step is computed on the basis of equation...
of motion with new random angles as described above. The results of the
simulations are given in chapter VII.

3.4 **FINITE ELEMENT METHOD SOLUTION**

Electric field evaluation under steady state conditions in a GIS bus
has been performed using ELECNET software (Finite Element Method). By
defining the cross section (2-dimensional) of the investigated system and
applying necessary boundary conditions, the program calculates
numerically different quantities such as, the electric potential (U), the
electric field strength (E) and the electric flux density (D).

(i) **Specifications for single phase isolated conductor GIS:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applied Voltage</td>
<td>+100 kv (steady state)</td>
</tr>
<tr>
<td>Enclosure potential</td>
<td>0 kv</td>
</tr>
<tr>
<td>Radius of the conductor</td>
<td>55 mm</td>
</tr>
<tr>
<td>Radius of the enclosure</td>
<td>152 mm</td>
</tr>
<tr>
<td>Gas between the electrodes</td>
<td>SF$_6$ ($\epsilon_r = 1.0021$)</td>
</tr>
</tbody>
</table>

(ii) **Specifications for three phase common enclosure GIS:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applied Voltage</td>
<td>+245 kv (steady state)</td>
</tr>
<tr>
<td>Enclosure potential</td>
<td>0 kv</td>
</tr>
<tr>
<td>Radius of the conductor</td>
<td>250 mm</td>
</tr>
<tr>
<td>Radius of the enclosure</td>
<td>32 mm</td>
</tr>
<tr>
<td>Gas between the electrodes</td>
<td>SF$_6$ ($\epsilon_r = 1.0021$)</td>
</tr>
</tbody>
</table>

The ELECNET software simulations have been performed for a single
phase isolated conductor GIS busduct and a three phase common
enclosure busduct. Results from simulations of charged spheres and wire particles at different positions in the lower part near the enclosure surface of the GIS bus are presented. The field plots for different enclosure designs are presented in Annexure II.

3.4 CONCLUSION

In this chapter, mathematical model for metallic particle movement in a single phase isolated GIB has been presented. The study of the motion of moving metallic particles in GIS requires a good knowledge of the charge of the particle. Free conducting particles may have any shape or size, may be spherical or filamentary (wire like) or in the form of fine dust. Particles may be free to move or may be fixed on to the surfaces. They may be of conducting material or of insulating material. Particles of insulating materials are not so harmful as they have little effect on the insulating properties of gases. So wire like particles made of conducting material are more harmful and their effects are more pronounced at higher gas pressures. A conducting particle in motion in an external electric field will be subjected to a collective influence of Electrostatic force, Gravitational force and Drag force. A motion equation is developed using the above forces for the particle movement. In order to determine the axial and radial movement in an enclosure, Monte-Carlo technique has been adopted in conjunction with motion equation.